MTH 352
Quiz #6

1. Suppose $f(x)$ is the piecewise function defined by

$$f(x) = \begin{cases} 1 & |x| \le \frac{\pi}{2}, \\ 0 & x > \frac{\pi}{2}. \end{cases}$$

On the interval $[-\pi, \pi]$ the set of functions $\{1, \cos(x), \sin(x), \cos(2x), \sin(2x), \ldots\}$ is a complete orthogonal basis. With respect to this basis, $f(x)$ has the following Fourier series

$$f(x) \sim \frac{1}{2} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{2}{n} \sin\left(\frac{n\pi}{2}\right) \cos(nx).$$

(a) For $x \in [-\pi, \pi]$ let

$$S_N(x) = \frac{1}{2} + \frac{1}{\pi} \sum_{n=1}^{N} \frac{2}{n} \sin\left(\frac{n\pi}{2}\right) \cos(nx).$$

What is $\lim_{N\to\infty} S_N(x)$?

$$\lim_{N\to\infty} S_N(x) = \begin{cases} 1 & |x| < \pi/2 \\ 1/2 & |x| = \pi/2 \\ 0 & |x| > \pi/2 \end{cases}$$

(b) Show that

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \ldots\right).$$

$$f(0) = 1 = \frac{1}{2} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{2}{n} \sin\left(\frac{n\pi}{2}\right)$$

$$\Rightarrow \frac{\pi}{4} = \sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n} = 1 - 1/3 + 1/5 - 1/7 + 1/9 + \ldots$$

$$\Rightarrow \pi = 4(1 - 1/3 + 1/5 - 1/7 + 1/9 + \ldots)$$