MTH 752: Nonlinear Dynamics and Asymptotic Analysis
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Course Website: http://users.wfu.edu/gemmerj/math752S26.html
Office Hours: T 9:00-10:00, W 2:30-3:30, Th 1:00-3:00
Meeting Times: MWF 9:00-1:00
Class Location: Kirby-Manchester 124

Material drawn from the following textbooks:
1. Strogatz, S. H. (2024). Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering. Chapman and Hall/CRC.
2. Holmes, M. H. (2012). *Introduction to perturbation methods* (Vol. 20). Springer Science & Business Media.
3. Newell, A. (2018). *Nonlinear optics*. CRC Press.
4. Murray, J. D. (2007). *Mathematical biology: I. An introduction* (Vol. 17). Springer Science & Business Media.

All of these textbooks have available online versions. However, the instructor will provide detailed notes on all of the covered content.

Prerequisites: This course will be taught the PhD level. Technically, this means that the perquisites are an undergraduate degree in applied mathematics/mathematics with undergraduate coursework in analysis, dynamical systems, partial differential equations, complex analysis, and a second course in linear algebra. *However,* as is typical in a graduate course, it is rare that students meet all of this requirements. The grading in this course is designed so that students will be able to fill in this gaps along the way and students will be able to revise their work based on feedback from the instructor.

Course Description: An introduction to advanced techniques in applied mathematics and nonlinear dynamics. Topics will be driven by student and faculty interest. A non-exhaustive list of potential topics are:

1. **Chaotic Dynamics***:* Using the Lorenz equations as motivation, iterated maps, strange attractors, and fractals will be covered. This topic will move fast as it is a direct conniption of MTH 351/651.
2. **Collective Dynamics***:* Using the Kuramoto model as motivation, modern techniques in studying collective behavior such as the use of order parameters and mean-field theory will be covered.
3. **Asymptotic Approximations***:* Using algebraic equations and asymptotic expansions of classic functions as motivation, order symbols and asymptotic sequences and series will be covered.
4. **Calculus of Variations:** Derivation of Euler-Lagrange equations, necessary and sufficient conditions for minimizers, Lagrangian mechanics.
5. **Hamiltonian Dynamics:** Hamiltonian dynamics is a framework for modeling physical systems that describe the conservation and transfer of energy in phase space and can lead to interesting phenomenon including chaotic dynamics.

6. **Matched Asymptotic Expansions in ODEs***:* Starting with singularly perturbed boundary value problems as motivation, boundary layers, matched asymptotic expansions and geometric singular perturbation theory will be covered.
7. **Matched Asymptotic Expansions in PDEs***:* Starting with conservation laws as motivation, the method of characteristics, advection-diffusion equations with weak diffusion, singularly perturbed elliptic equations, Burger's equation, and the viscosity method will be covered.
8. **Weakly Dispersive ODEs and PDEs***:* Using weakly coupled oscillators as motivation, the method of multiple scales, traveling waves, dispersion relations, Laplace's method, the method of stationary phase, and phase and group velocities will be covered.
9. **Reaction-Diffusion and Pattern Formation***:* Using animal dispersal as motivation, reaction diffusion equations will be covered specifically focusing on Turing bifurcations, numerical methods, and asymptotic solutions.

Course Rationale: Many problems in applied mathematics and physics are nonlinear and are intractable to solve using elementary methods. In this course we will systematically develop techniques for obtaining qualitative and quantitative information from nonlinear systems. The techniques used depend on the problem, but several powerful tools include geometric approaches, iterated maps, mean field theory, identification of an underlying variational or Hamiltonian structure, and the exploitation of small parameter/separation of scales. Along the way, we will discuss many applications including nonlinear waves, coupled oscillators, nonlinear optics, fluid dynamics and pattern formation.

Class Delivery: The course material will be delivered through lectures by the faculty member as well as students in the class. Evaluation of the students understanding of the material will be assessed through written homework and by having students lead a lecture on a given topic.

Course Policies:

✦ **Grading:** Your grade will be based on:
- Homework: 70%
- Presentations: 20%
- Participation: 10%

✦ **Homework:** Homework can be submitted on Fridays at the beginning of class and will be assigned on a biweekly schedule. As long as you submitted the original assignment on time, you will be allowed to resubmit the assignment as many times as you would like. However, I will only accept assignment on Fridays and typically grade them over the weekend. When resubmitting an assignment include the original assignment and only resubmit the problems that needed corrections, i.e., there is no need to redo the entire assignment.

✦ **Presentations:** Students will be required to give two twenty to forty minute lectures on a given topic with another ten minutes for discussion and questions from the class. The faculty member will manage an updated list of potential topics on the course website and calendar of student presentations. The presentations will be graded by a rubric that will be posted on the course website.

✦ **Participation:** Students will be graded on attendance during presentations by their classmates.

COURSE ENVIRONMENT

Names/Pronouns

You **deserve** to be addressed in the manner you prefer. To guarantee that I address you properly, you are welcome to tell me your pronoun(s) and/or preferred name at any time, either in person or via email.

Diversity

We embrace diversity of age, background, beliefs, ethnicity, gender, gender identity, gender expression, national origin, religious affiliation, sexual orientation, and other visible and non-visible categories. **I do not tolerate discrimination.**

Accessibility

I want you to succeed in this course and all of your courses. Wake Forest University provides reasonable accommodations to students with disabilities. If you are in need of an accommodation, then please contact me privately as early in the term as possible. Retroactive accommodations may not be provided. Students requiring accommodations must also consult the Center for Learning, Access, and Student Success(118 Reynolda Hall, 336-758-5929, http://class.wfu.edu). For personal issues, stress, health problems or life circumstances see shs.wfu.edu/. Contact me if you have other special circumstances. **I will find resources for you.**

Title IX

You **deserve** a community free from discrimination, sexual harassment, a hostile environment, sexual assault, domestic violence, dating violence, and stalking. If you experience or know of a Title IX violation, you have many options for support and/or reporting; see titleix.wfu.edu/.

The Honor Code

At Wake Forest, we expect you to behave as honorable citizens of the class, the university, and the world as a whole. When you complete an assignment with your name on it, you are representing that everything you are turning in is your own work. That means that you do not copy from other students, textbooks, or websites. If at any time I become aware of cheating or plagiarism in this course, I will submit the information to the honor council.