Remember to justify your answers!

(1) A menagerie consists of seven brown dogs, two black dogs, six gray cats, ten black cats, five blue birds, six yellow birds, and one black bird. Determine which of the following statements are true and which are false.
   (a) Every animal in the menagerie is brown or gray or black.
   (b) There is an animal in the menagerie that is neither a cat nor a dog.
   (c) No animal in the menagerie is blue.
   (d) There are in the menagerie a dog, a cat, and a bird that all have the same color.

(2) Let the domain of $x$ be the set of geometric figures in the plane, and let $\text{Square}(x)$ mean “$x$ is a square” and $\text{Rect}(x)$ mean “$x$ is a rectangle”. Rewrite each of the following statements without using quantifiers or variables. Indicate which are true and which are false.
   (a) $\exists x$ such that $\text{Rect}(x) \land \text{Square}(x)$.
   (b) $\exists x$ such that $\text{Rect}(x) \land \lnot \text{Square}(x)$.
   (c) $\forall x, \text{Square}(x) \rightarrow \text{Rect}(x)$.

(3) Consider the statement “There are no simple solutions to life’s problems.” Write an informal negation for the statement, and then write the statement formally using quantifiers and variables.

(4) Give an example to show that a universal conditional statement is not logically equivalent to its inverse.

(5) Rewrite each of the following statements formally using quantifiers and variables. Then write a negation for the statement.
   (a) Everybody trusts somebody.
   (b) Somebody trusts everybody.

(6) Indicate which of the following statements are true and which are false. Justify your answers as best you can.
   (a) $\exists x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}, x = y + 1$.
   (b) $\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}^+$ such that $xy = 1$.
   (c) $\exists u \in \mathbb{R}^+$ such that $\forall v \in \mathbb{R}^+, uv < v$.
   (d) $\forall v \in \mathbb{R}^+, \exists u \in \mathbb{R}^+$ such that $uv < v$.

(7) Is the following argument valid or invalid? (If it is valid, justify your answer by stating which argument form it uses. If it is invalid, state which error it exhibits.)

   If an infinite series converges, then the terms go to 0.

   The terms of the infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ go to 0.

   $\therefore$ The infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ converges.
A conclusion follows from the given premises, but it is difficult to see because the premises are jumbled up. Reorder the premises to make it clear that a conclusion follows logically, and state the valid conclusion that can be drawn. (It may be helpful to rewrite some of the statements in if-then form and to replace some statements by their contrapositives.)

(a) All writers who understand human nature are clever.
(b) No one is a true poet unless he/she can stir the human heart.
(c) Shakespeare wrote *Hamlet*.
(d) No writer who does not understand human nature can stir the human heart.
(e) None but a true poet could have written *Hamlet*. 