MATH117 Homework 5: due Friday, 7 October

Remember to justify your answers!

(1) When an integer $b$ is divided by 12, the remainder is 5. What is the remainder when $8b$ is divided by 12?

(2) Prove that the square of any integer has the form $4k$ or $4k + 1$ for some integer $k$.

(3) For each of the following statements, either prove it or find a counterexample.
   
   (a) For all real numbers $x$ and $y$, $\lceil xy \rceil = \lceil x \rceil \lfloor y \rfloor$.
   
   (b) For all real numbers $x$, $\lfloor x^2 \rfloor = \lfloor x \rfloor^2$.

(4) Prove (by contradiction) that for all real numbers $x$ and $y$, if $x$ is irrational and $y$ is rational, then $x - y$ is irrational.

(5) Prove that the reciprocal of any irrational number is irrational. (The reciprocal of a nonzero real number $x$ is $1/x$.)

(6) For each of the following, either prove it or provide a counterexample.
   
   (a) The sum of any two positive irrational numbers is irrational.
   
   (b) The product of any two irrational numbers is irrational.