Fibonacci numbers.
M117, October 10, 2011 (due October 12, 2011)

Your names: __________________________________________

With your partner, you will conjecture and prove a statement about Fibonacci numbers. The Fibonacci numbers are defined by the following:

\[ f_1 = 1, \quad f_2 = 1, \quad f_n = f_{n-1} + f_{n-2}, \quad \text{for } n \geq 3 \]

(1) Write down the first 12 Fibonacci numbers.

(2) For which values of \( n \) is \( f_n \) even?

(3) Replace \( k \) with the appropriate number in the following statement:

“If \( n \) is divisible by \( k \), then \( f_n \) is even.”

(4) Prove the statement you made in (3), using mathematical induction.