Solutions to practice probs

168. (a) The squares not in the Young diagram of the original partition will fall into rows. Their lengths are nonnegative, and nondecreasing as we move down. Rotating 180° makes them nonincreasing, so we get a Young diagram of a partition.

(b) \( m' n' - k \)

(c) \( m' > m \) and \( n' = n \) or \( m' = m \) and \( n' - n = \) multiplicity of the largest part of the original partition

(d) \( n' > n \) and \( m = m' \) or \( n' = n \) and \( m' - m = \) smallest part of original partition

(e) \( m' = m \) and \( n' - n = \) mult. of largest part of original or \( n' = n \) and \( m' - m = \) smallest part of original

(f) Yes if you use the same rectangle

173. Given a partition \( \lambda = (\lambda_1, \lambda_2, \lambda_3) \) of 7, add 0 to \( \lambda_3 \), 1 to \( \lambda_2 \), and 2 to \( \lambda_1 \). To go backwards, subtract 0 from \( \lambda_3 \), 1 from \( \lambda_2 \), and 2 from \( \lambda_1 \).
3.35
1. (a) \( \binom{n+k-1}{k} = \binom{n+k-1}{n-1} \) (b) \( n^k \)
(c) \( \binom{n}{k} \) (d) \( n^{k-1} \)
(e) none of the above (f) \( \binom{k-1}{n-1} \)

2. (a) \( s^n \) (b) \( s^r \) (c) \( s^r \)
(d) \( \sum_{k=0}^{n} \binom{s}{k} r^k \) or \( \sum_{k=0}^{s} s^k \binom{r}{k} \)
(e) \( \binom{r+s-1}{s-1} \) (f) \( S^r = (r+s-1)^s \)
(g) \( r! \binom{r-1}{s-1} \) (h) \( \binom{r-1}{s-1} \)

3. \( \frac{(n+r+s+t+2)!}{n! \cdot r! \cdot s! \cdot t! \cdot 2!} \)

5. \( \frac{n}{2} \) if \( n \) is even, \( \frac{n-1}{2} \) if \( n \) is odd \( \binom{n}{\frac{n-1}{2}} \)

6. \( 2 : 3+1+\ldots+1 \) or \( 2+2+1+\ldots+1 \)

11. One of the blocks contains \( k \). If that block is of size \( i \), removing it gives a partition of \( k-i \) objects into \( n-1 \) blocks. But the elements removed with \( k \) could be any \( i-1 \) elements, so there are \( \binom{k-1}{i-1} \) options for these.