The Game of Nim

Your name: ____________________________________________

1. THE GAME OF “TAKE-AWAY”

(1) Split your pieces into two equal piles. (Each pile should have 8 pieces.)
(2) Take turns removing pieces as many pieces as you want from one pile at a time.
(3) The person to take the last piece wins.
(4) Try to develop a winning strategy, and record your strategy here:

2. THE RULES OF NIM

(1) Begin with 16 game pieces.
(2) Line up the pieces into four rows, so that the first row has 1 piece, the second row
has 3 pieces, the third row has 5 pieces, and the last row has 7 pieces as shown below.

\[
\begin{array}{ccccc}
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array}
\]

(3) Take turns removing pieces, subject to the following rules:
   (a) Each player may remove pieces from one pile only per turn.
   (b) You may remove as many pieces as you want from ONE row on your turn.
   (c) The player to take the last piece wins.

3. PLAY THE GAME!

(1) Play two rounds of Nim with your partner.
(2) Next, play two more rounds, this time recording the number of pieces in each row
in binary notation using the tables on the back of this sheet. (Fill in the first table
before starting, and then fill in one table after each turn.)
FIRST ROUND:

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SECOND ROUND:

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(3) Can you use the tables to come up with a winning strategy for the game?  
HINT: Look at the sums of the column entries in the tables.