Marder 5.2

> restart;

Parts a and b.

> \( F := \frac{1}{2} (c - 0.2)^2 (c - 0.8)^2 + \frac{1}{2} c \);

> plot(F, c=-0..1);

Although there is no minimum in \( F \), there are still phase separations. All that is necessary is for there to be a straight line that is everywhere lower than the curve between the two contact points. In other words, the line touches in two spots and is always lower than the curve between the two spots. So, just find a straight line that is tangent at two different concentrations. The condition for tangent is to have the same value and same derivative.

So in general, we need four equations, not two. We can get by with two only if we know
the two tangent points $c_1$ and $c_2$. Let's solve the problem without assuming this.

Let the straight line be $y$:

> \( y := \alpha c + \beta \)

Set derivatives equal to each other at concentrations $c_1$ and $c_2$.

> \( eq3 := \alpha = (c_1 - 0.2)(c_1 - 0.8)^2 + (c_1 - 0.2)^2 (c_1 - 0.8) + \frac{1}{2} \)

> \( eq4 := \alpha = (c_2 - 0.2)(c_2 - 0.8)^2 + (c_2 - 0.2)^2 (c_2 - 0.8) + \frac{1}{2} \)

Set the functions $y$ and $F$ equal to each other at the same points.

> \( eq5 := \alpha c_1 + \beta = \frac{1}{2} (c_1 - 0.2)^2 (c_1 - 0.8)^2 + \frac{1}{2} c_1 \)

> \( eq6 := \alpha c_2 + \beta = \frac{1}{2} (c_2 - 0.2)^2 (c_2 - 0.8)^2 + \frac{1}{2} c_2 \)

> \( \text{solve} \{eq3, eq4, eq5, eq6\}, \{\alpha, \beta, c_1, c_2\} \)

\{ \begin{align*}
  c_1 &= c_2, \\
  \beta &= -1.500000000 c_2^4 + 2. c_2^3 - 0.6600000000 c_2^2 + 0.01280000000, \\
  \alpha &= 2. c_2^3 - 3. c_2^2 + 1.3200000000 c_2 + 0.3400000000, \\
  c_2 &= c_2, \\
  \beta &= 0., c_2 = 0.8000000000, \alpha = 0.5000000000, c_1 = 0.2000000000, \\
  c_2 &= 0.2000000000, c_1 = 0.8000000000, \beta = 0., \alpha = 0.5000000000 \}
\}

The last two solutions are equivalent. The straight line touches the F curve at $c=0.2$ and $c=0.8$. At concentrations in between, the minimum energy of the system will be a mixture phases with these two concentrations.

> \( \alpha := 0.5; \)

\( \alpha := 0.5 \)

> \( \beta := 0; \)

\( \beta := 0 \)

> \text{plot}(\{F, y\}, c=0..1);
So we have found the two (four) equations and solved them for the tangent straight line.

**Part c.**

Let $f$ be the fraction in the phase with concentration $c_1=0.2$. Then $(1-f)$ is the fraction in the $c_2=0.8$ phase. The actual concentration is just the weighted sum of the two concentrations, so

\[ 0.2f + 0.8(1-f) = 0.6; \]

\[ -0.6f + 0.8 = 0.6 \]

\[ > \text{solve}(%); \]

\[ 0.3333333333 \]

So we have 1/3 of phase $c=0.2$ and 2/3 of phase $c=0.8$. 