Marder 5.2

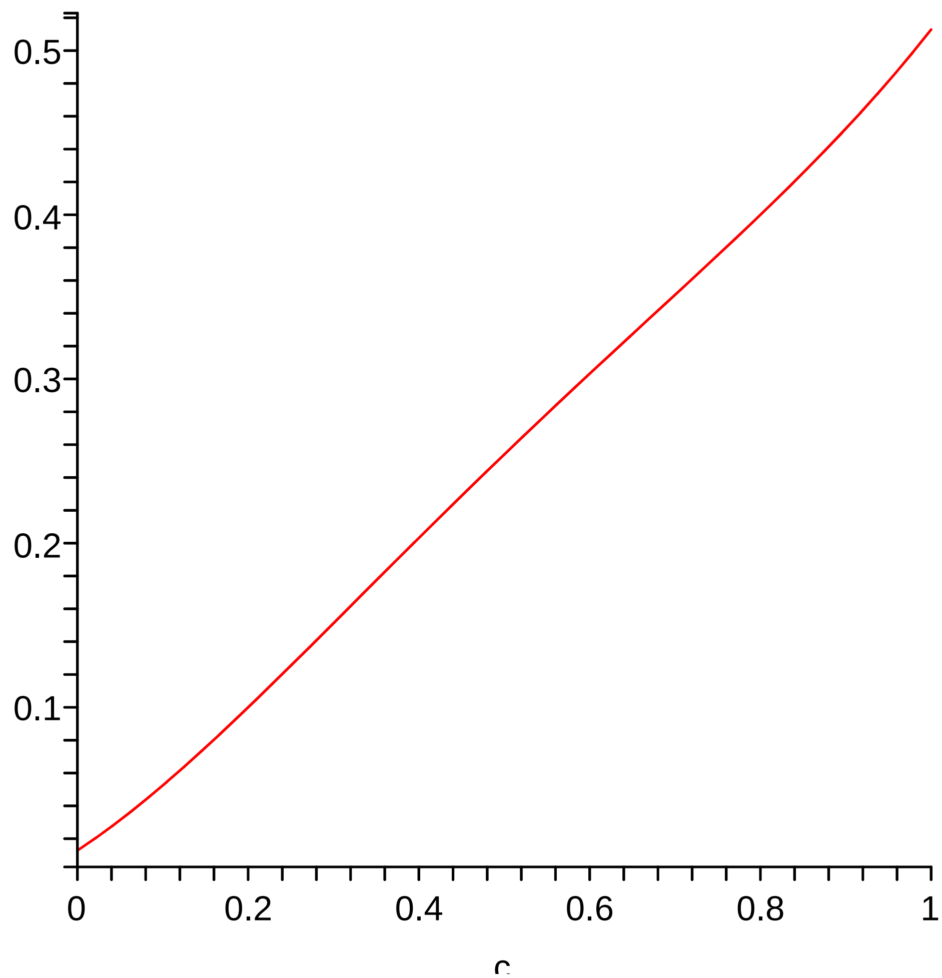
```
> restart;
```

Parts a and b.

```
> F:=(1/2)*(c-0.2)^2*(c-0.8)^2+c/2;
```

$$F := \frac{1}{2}(c - 0.2)^2 (c - 0.8)^2 + \frac{1}{2} c$$

```
> plot(F,c=-0..1);
```



Although there is no minimum in F, there are still phase separations. All that is necessary is for there to be a straight line that is everywhere lower than the curve between the two contact points. **In other words, the line touches in two spots and is always lower than the curve between the two spots. So, just find a a straight line that is tangent at two different concentrations. The condition for tangent is to have the same value and same derivative.**

So in general, we need four equations, not two. We can get by with two only if we know the two tangent points c1 and c2. Let's solve the problem without assuming this.

Let the straight line be y:

```
> y:=alpha*c+beta;
```

$$y := \alpha\, c + \beta$$

Set derivatives equal to each other at concentrations c1 and c2.

```
> eq1:= alpha=subs(c=c1,diff(F,c));
```

$$eq3 := \alpha = (c1 - 0.2)\,(c1 - 0.8)^2 + (c1 - 0.2)^2\,(c1 - 0.8) + \frac{1}{2}$$

```
> eq4:=alpha=subs(c=c2,diff(F,c));
```

$$eq4 := \alpha = (c2 - 0.2)\,(c2 - 0.8)^2 + (c2 - 0.2)^2\,(c2 - 0.8) + \frac{1}{2}$$

Set the functions y and F equal to each other at the same points.

```
> eq5:= subs(c=c1,y)=subs(c=c1,F);
```

$$eq5 := \alpha\, c1 + \beta = \frac{1}{2}\,(c1 - 0.2)^2\,(c1 - 0.8)^2 + \frac{1}{2}\,c1$$

```
> eq6:= subs(c=c2,y)=subs(c=c2,F);
```

$$eq6 := \alpha\, c2 + \beta = \frac{1}{2}\,(c2 - 0.2)^2\,(c2 - 0.8)^2 + \frac{1}{2}\,c2$$

```
> solve({eq3,eq4,eq5,eq6},{alpha,beta,c1,c2});
```

$$\{c1 = c2, \beta = -1.500000000\,c2^4 + 2.\,c2^3 - 0.6600000000\,c2^2 + 0.01280000000,$$
$$\alpha = 2.\,c2^3 - 3.\,c2^2 + 1.320000000\,c2 + 0.3400000000, c2 = c2\},$$
$$\{\beta = 0., c2 = 0.8000000000, \alpha = 0.5000000000, c1 = 0.2000000000\},$$
$$\{c2 = 0.2000000000, c1 = 0.8000000000, \beta = 0., \alpha = 0.5000000000\}$$

The last two solutions are equivalent. The straight line touches the F curve at c=0.2 and c=0.8. At concentrations in between, the minimum energy of the system will be a mixture phases with these two concentrations.

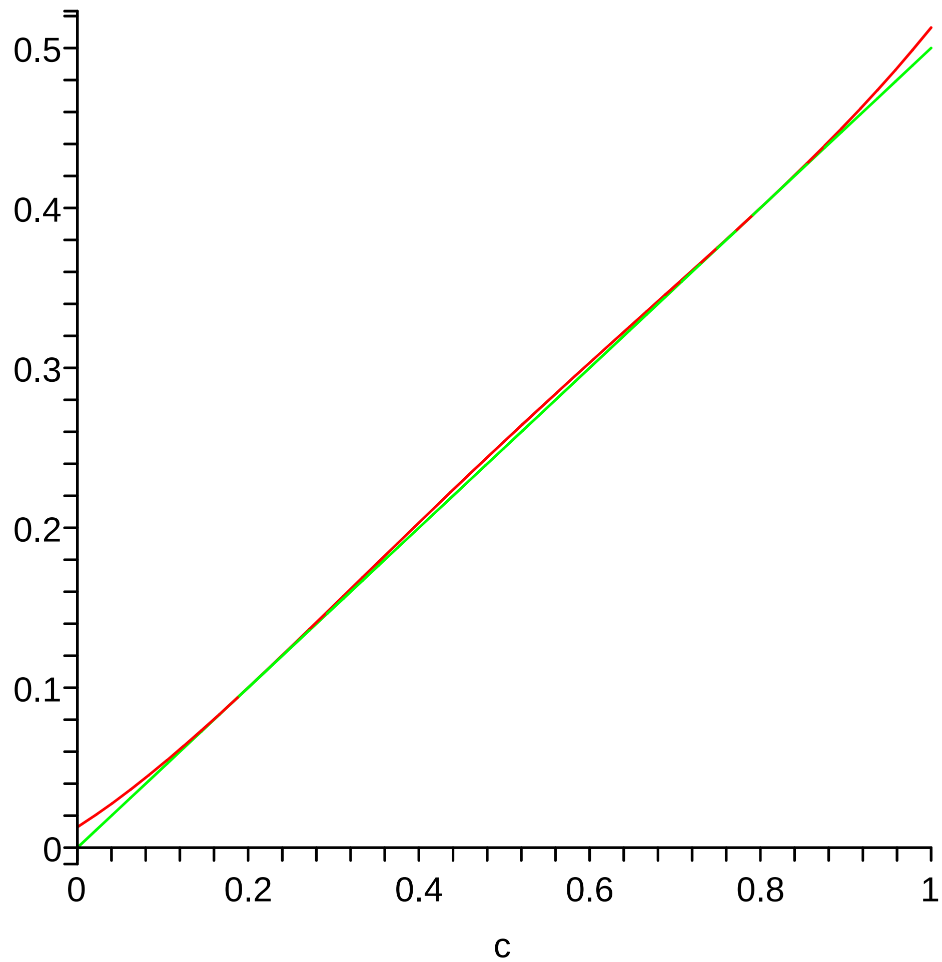
```
> alpha:=0.5;
```

$$\alpha := 0.5$$

```
> beta:=0;
```

$$\beta := 0$$

```
> plot({F,y}, c=0..1);
```



So we have found the two (four) equations and solved them for the tangent straight line.

Part c.

Let f be the fraction in the phase with concentration c1=0.2. Then (1-f) is the fraction in the c2=0.8 phase. The actual concentration is just the weighted sum of the two concentrations, so

```
> 0.2*f+0.8*(1-f)=0.6;
```

$$-0.6 f + 0.8 = 0.6$$

```
> solve(%,f);
```

$$0.3333333333$$

So we have 1/3 of phase c=0.2 and 2/3 of phase c=0.8.