The Disaggregated New Keynesian Phillips Curve

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Abstract

The New Keynesian Phillips Curve is a widely-used model of inflation dynamics, which predicts that inflation is determined by real marginal cost and expectations of future inflation. However this model can only be exclusively used to estimate what drives economy-wide inflation, using aggregate variables as determinants. This note is an expansion of an appendix to Mazumder (2010) and is meant to provide further clarification for NKPC researchers who are interested in estimating a model of industry-specific inflation.

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1 Introduction

The partial equilibrium, sticky-price New Keynesian Phillips Curve (NKPC) is a prominently-used model of inflation dynamics, which states that economy-wide inflation depends positively on real marginal cost and discounted expectations of future inflation. This model is popular among macroeconomists for several reasons. For example, the model is one which is a macroeconomic relationship borne out of explicit micro-foundations, not to mention that the model has several proponents when it comes to describing inflation empirically (such as Gali and Gertler (1999)).

However the model can only be used to estimate aggregate inflation (defined as the percentage change in the aggregate price level), using an aggregate measure of real marginal cost as the determinant. This leaves us with two important extensions to the model that we are unable to test: first, we cannot examine industry-specific inflation in the traditional NKPC, and second, it becomes difficult to use disaggregated measures of real marginal cost in the model. This is problematic since there are many proxies for real marginal cost such as capacity utilization, for which data is not available at the aggregate level.

Intuitively, there are also other strong reasons that we may wish to estimate a sectoral NKPC. For example, we may have sectoral data that is of better quality than aggregate data, or it may at least match the theoretical model counterparts better than the aggregate data. In this case, a disaggregated NKPC can more precisely pin down the estimated structural parameters of the model. In addition, there is clearly sectoral heterogeneity in price rigidity which the aggregate NKPC does not account for, which a disaggregated model can instead capture. Some authors have indeed attempted to derive this type of model, such as in Imbs, Jondeau, and Pelgrin (2007). However they derive a NKPC equation that does not hold if sectoral real marginal cost is deflated by an aggregate price index, which is an assumption that many would consider to be unrealistic, particularly if we think aggregate prices will affect nominal marginal cost in each sector. Other papers such as Leith and Malley (2007) have also provided useful insights as to how a disaggregated NKPC could be derived, but we argue that this note undertakes this process in a much simpler and more parsimonious way.

Upon deriving an industry-specific version of the NKPC, which is henceforth referred to as the 'disaggregated NKPC', we find that this model turns out to be a generalization of the traditional NKPC and is extremely useful for analysis of industry-level inflation (such as in Mazumder (2010) who applies it to the U.S. manufacturing industry).

2   The NKPC

The three standard equations that we can use to derive the basic NKPC model are (see Woodford (1996, 2003) and Goodfriend and King (1997) for explicit detail, and Mankiw and Reis (2002) who use a similar setup to derive the NKPC):

$$p_t^* = p_t + mc_t \tag{1}$$

$$q_t = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t p_{t+k}^* \tag{2}$$

$$p_t = (1 - \theta)q_t + \theta p_{t-1} \tag{3}$$

where p_t^* is the optimal profit-maximizing price, p_t is the actual price level, mc_t is real marginal cost, and q_t is the firms' adjustment price. Lower case variables represents logs, where all three equations are log-linear approximations around a zero inflation steady state[1]. Also note that these are all aggregate-level variables. (1) states that a firm's desired (or profit-maximizing) price is the sum of the actual aggregate price level and some measure of aggregate marginal cost (see Blanchard and Kiyotaki (1987) for further elaboration). (2) states that the adjustment price for firms will be the weighted average of current and all future optimal prices, where β is the discount factor for firms. Finally (3) is Calvo (1983)'s notion of random price adjustment, which is obtained from expanding the pricing equation of $p_t = (1 - \theta) \sum_{j=0}^{\infty} \theta^j q_{t-j}$. Calvo's equation states that in each period a certain fraction of firms, $(1 - \theta)$, will adjust prices, and the remaining θ of the firms do not adjust prices and continue to use the previous period's price[2]. Solving (1), (2), and (3) for a relationship describing inflation, $\pi_t = p_t - p_{t-1}$ gives the usual aggregate NKPC:

$$\pi_t = \lambda mc_t + \beta E_t\{\pi_{t+1}\} \tag{NKPC}$$

where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

3   The Disaggregated NKPC

To derive a disaggregated version of the NKPC, we must then ask whether (1), (2), and (3) hold at the industry level. First, consider equation (2). This equation originally describes the adjustment price which holds true for any given firm. Thus when we then aggregate across all firms in the economy, we are left with (2). In other words, if this equation holds true for

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[1]Which is also why the markup does not appear in equation (1).

[2]Note that other types of price stickiness can also be used to derive the NKPC, such as staggered contracts.

each firm, it will also hold true when we aggregate across all firms in one particular sector. In other words, we can change all of the variables in (2) to sector-level ones.

The same can be said for (3), which holds true for any group of firms. According to Calvo's model of random price adjustment, we can assume that a fraction of firms in an industry adjusts prices optimally in each period, while the remainder of the firms leave prices unchanged from the previous period. Indeed, using Calvo-type pricing for multiple sectors is often done, such as in Bils and Klenow (2004). Thus the variables in (3) can also be simply changed into industry-specific ones.

However, (1) is not simply a matter of changing aggregate variables to disaggregated ones. (1) comes from assuming Dixit-Stiglitz CES utility which gives us the result that optimal, profit-maximizing price for a given firm j is a markup over nominal marginal cost:

$$P_{j,t}^* = \mu \widehat{MC}_{j,t} \tag{4}$$

where μ is the markup and $\widehat{MC}_{j,t}$ is *nominal* marginal cost for firm j. When we then aggregate (4) across all firms in the economy, convert into real terms, and set the log of the markup equal to zero[3], we are left with (1). Now suppose that we instead aggregate across all firms in a given industry, i. This gives firms in industry i an optimal price of:

$$(P_t^i)^* = \mu \widehat{MC}_t^i \tag{5}$$

where $\sum_k (P_{k,t}^i)^* = (P_t^i)^*$ and $\sum_k (\widehat{MC}_{k,t}^i) = (\widehat{MC}_t^i)$ for k firms in industry i. When we now convert (5) into real terms we must divide by the aggregate price level, which leaves us with $\frac{(P_t^i)^*}{P_t} = \mu MC_t^i$, where we have real marginal cost, MC_t^i, on the right-hand side of the equation. Once again, we then take logs and set the markup term equal to zero, leaving:

$$(p_t^i)^* = p_t + mc_t^i \tag{6}$$

which is the same as changing the variables in (1) to industry-specific ones, with the exception of the aggregate price term, p_t, which remains. Finally this leaves us with the three equations we need to derive the disaggregated NKPC:

$$(p_t^i)^* = p_t + mc_t^i \tag{I}$$

$$q_t^i = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t (p_{t+k}^i)^* \tag{II}$$

$$p_t^i = (1 - \theta) q_t^i + \theta p_{t-1}^i \tag{III}$$

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[3]Which is standard practice in the NKPC literature.

where $(p_t^i)^*$ represents the industry's desired price, p_t is the aggregate price level, mc_t^i is real marginal cost in the given industry, q_t^i is the adjustment price, and p_t^i is the actual industry average price. In other words, the disaggregated model is almost identical to the aggregate one, except in (I) where the aggregate price level, p_t, remains.

To solve for π_t^i, we then break the sum in (2), take expectations at time t, and apply the Law of Iterated Expectations to get an equation for q_t^i. We can also get another equation for q_t^i by rearranging (III), which we also move forward by one period and take expectations at time t to get an equation for $E_t q_{t+1}^i$. We can then use these equations, and some simple algebraic manipulation to get an equation for industry-specific inflation, π_t^i, as:

$$\pi_t^i = \lambda[mc_t^i + (p_t - p_t^i)] + \beta E_t\{\pi_{t+1}^i\} \qquad \text{(Dis. NKPC)}$$

where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ as before. Clearly, we can observe that the disaggregated NKPC is not quite the same as the aggregate NKPC due to the addition of the $(p_t - p_t^i)$ term. In other words, the industry's inflation rate is also dependent on the relative price between that industry and the aggregate economy, as well as the usual NKPC variables.

Intuitively this is not surprising, since the aggregate price will affect nominal marginal cost in each sector, and hence also affects the price charged in that given industry. In other words, we expect the competition between industries to matter when considering inflation within one particular sector. Furthermore, we can say that this disaggregated version of the model is a generalization of the standard NKPC, since if we wish to examine aggregate inflation we would set $p_t^i = p_t$, and are left with the usual NKPC equation.

4   Conclusion

The standard NKPC is a relationship that predicts how aggregate marginal cost determines economy-wide prices. However this model cannot be used to examine industry-specific price changes and cannot be used with industry-specific measures of marginal cost. This note derives a disaggregated version of the NKPC which leaves us with a model that is a generalized version of the standard NKPC, and is one that can be used to examine the issue of industry-specific inflation and its determinants.

References

BILS, M. AND P. KLENOW (2004): "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112, 947–985.

BLANCHARD, O. AND N. KIYOTAKI (1987): "Monopolistic Competition and the Effects of Aggregate Demand," *American Economic Review*, 77, 647–666.

CALVO, G. (1983): "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics*, 12, 383–398.

GALI, J. AND M. GERTLER (1999): "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics*, 44, 195–222.

GOODFRIEND, M. AND R. KING (1997): "The New Neoclassical Synthesis and the Role of Monetary Policy," *NBER Macroeconomics Annual*, 231–283.

IMBS, J., E. JONDEAU, AND F. PELGRIN (2007): "Aggregating Phillips Curves," CEPR Discussion Papers 6184, ECB.

LEITH, C. AND J. MALLEY (2007): "A Sectoral Analysis of Price-Setting Behavior in U.S. Manufacturing Industries," *The Review of Economics and Statistics*, 89, 335–342.

MANKIW, N. AND R. REIS (2002): "Sticky Information Versus Sticky Price: A Proposal to Replace the New Keynesian Phillips Curve," *Quarterly Journal of Economics*, 117, 1295–1328.

MAZUMDER, S. (2010): "The New Keynesian Phillips Curve and the Cyclicality of Marginal Cost," *Journal of Macroeconomics*, 32, 747–765.

WOODFORD, M. (1996): "Control of the Public Debt: A Requirement for Price Stability?" *NBER Working Paper*, 5684.

——— (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.