Announcements

1. **Test corrections due today.**
2. **Today's topics –**

   Conservation of momentum

   Notion of impulse

   Analysis of collisions

      Elastic

      Inelastic

Linear momentum: $\mathbf{p} = m\mathbf{v}$

Newton's second law: $\mathbf{F} = \frac{d\mathbf{p}}{dt}$

Generalization for a composite system:

$$\sum_i \mathbf{F}_i = \sum_i \frac{d\mathbf{p}_i}{dt} \quad \Rightarrow \mathbf{F}_{net} = M\frac{d\mathbf{v}_{com}}{dt}$$

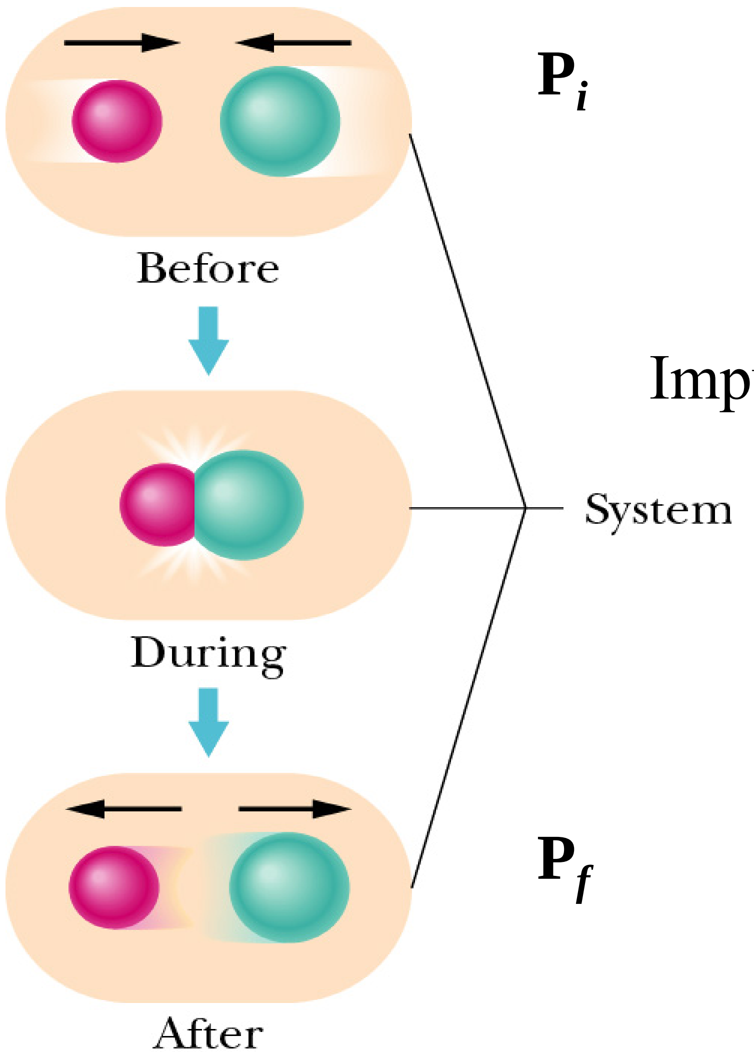
Conservation of momentum:

If $\sum_i \mathbf{F}_i = 0; \Rightarrow \sum_i \frac{d\mathbf{p}_i}{dt} = 0; \Rightarrow \sum_i \mathbf{p}_i = \text{(constant)}$

Energy may also be conserved ("elastic" collision)

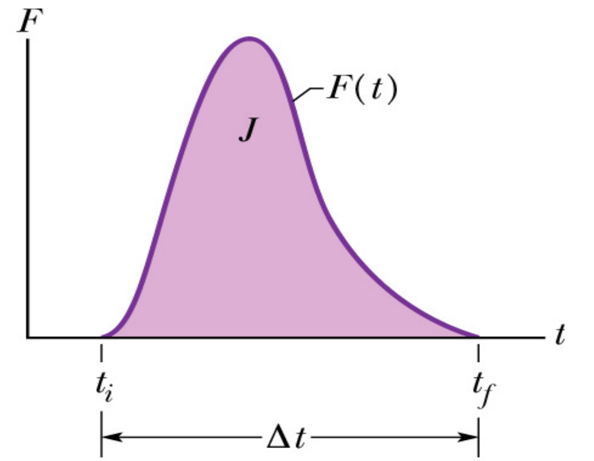
or not conserve ("inelastic" collision)

Snapshot of a collision:

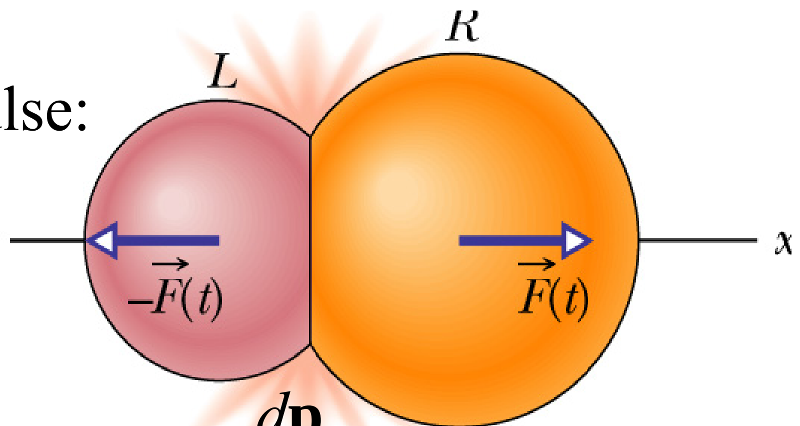


$\mathbf{P}_i$

$\mathbf{P}_f$



Impulse:

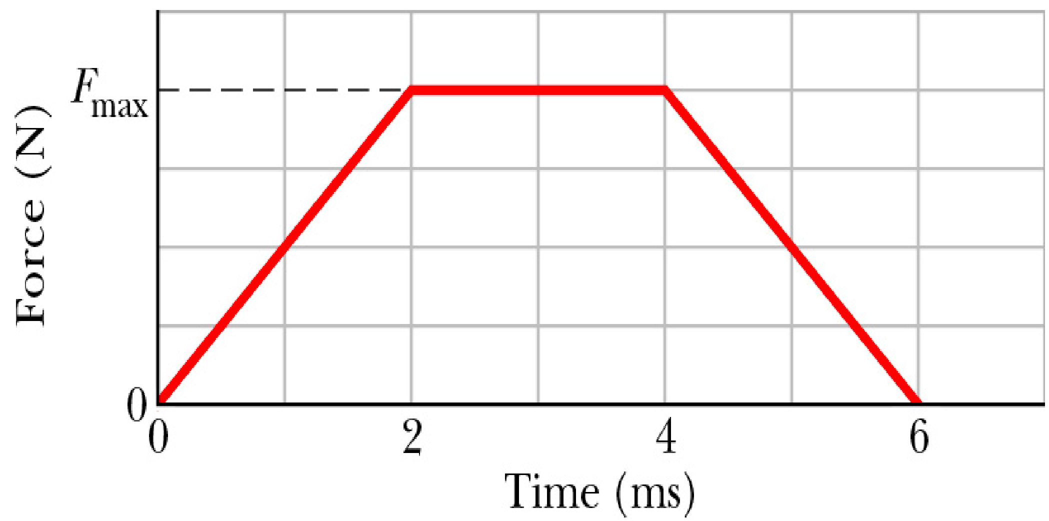


$$\mathbf{F}(t) = \frac{d\mathbf{p}}{dt} \Rightarrow d\mathbf{p} = \mathbf{F}(t)dt$$

$$\int_{t_1}^{t_2} d\mathbf{p} = \int_{t_1}^{t_2} \mathbf{F}(t)dt \equiv \mathbf{J}$$

Example from HW:

Plot shows model of force exerted on ball hitting a wall as a function of time. Given information about $\mathbf{p}_i$ and $\mathbf{p}_f$, you are asked to determine F_{max}.

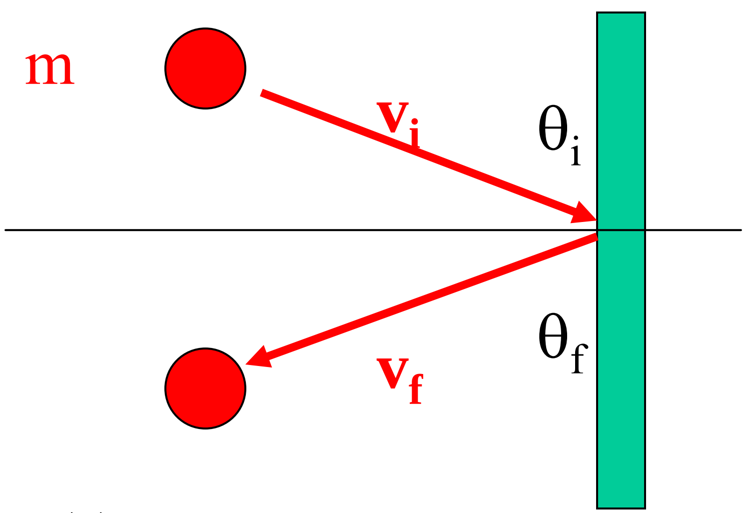


$$\int_{t_1}^{t_2} d\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \int_{t_1}^{t_2} \mathbf{F}(t)dt \equiv \mathbf{J}$$

Notion of impulse:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \quad \Rightarrow \mathbf{F}\Delta t = \Delta\mathbf{p}$$

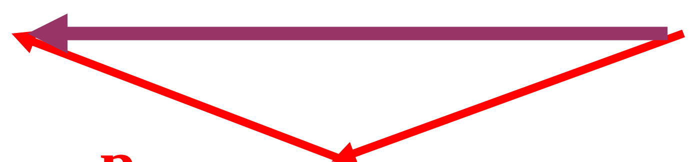
Example:



$$\Delta\mathbf{p} = m\mathbf{v_f} - m\mathbf{v_i}$$

$$\Delta\mathbf{p} = (-mv_f \sin\theta_f - mv_i \sin\theta_i)\ \mathbf{i}$$

$$+(-mv_f \cos\theta_f + mv_i \cos\theta_i)\ \mathbf{j}$$



Notion of impulse:

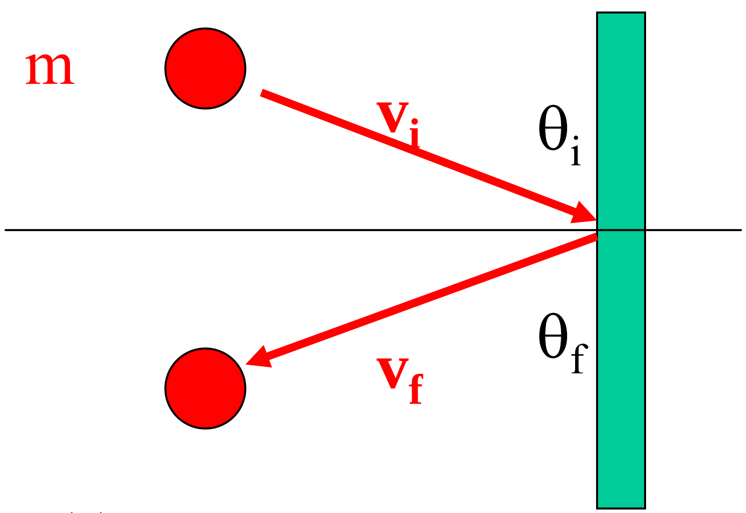
$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

cause ⟷ **effect**

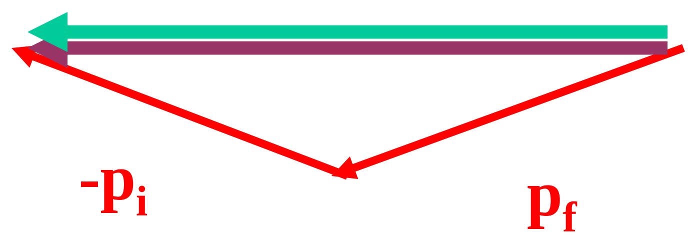
$$\Delta\mathbf{p} = m\mathbf{v_f} - m\mathbf{v_i}$$

Example:

$$\Delta\mathbf{p} = (-mv_f \sin\theta_f - mv_i \sin\theta_i)\,\mathbf{i}$$
$$+(-mv_f \cos\theta_f + mv_i \cos\theta_i)\,\mathbf{j}$$



$$\Delta\mathbf{p} = \mathbf{p_f} - \mathbf{p_i} = \mathbf{F}\,\Delta t$$



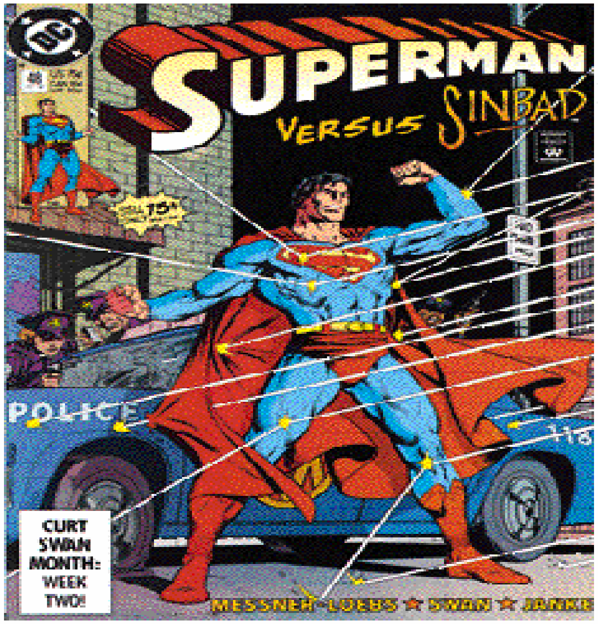
$$F \approx \frac{\Delta p}{\Delta t}$$

Example: Suppose that you (mg= 500 N) jump down a distance of 1m, landing with stiff legs during a time Δt=0.1s. What is the average force exerted by the floor on your bones?

$$\tfrac{1}{2} m v_{floor}^2 = mgh_i$$

$$\Delta p = m v_{floor} - 0 = mg\sqrt{\frac{2h}{g}} = 500N\sqrt{\frac{2 \cdot 1}{9.8}} s = 225.9 N \cdot s$$

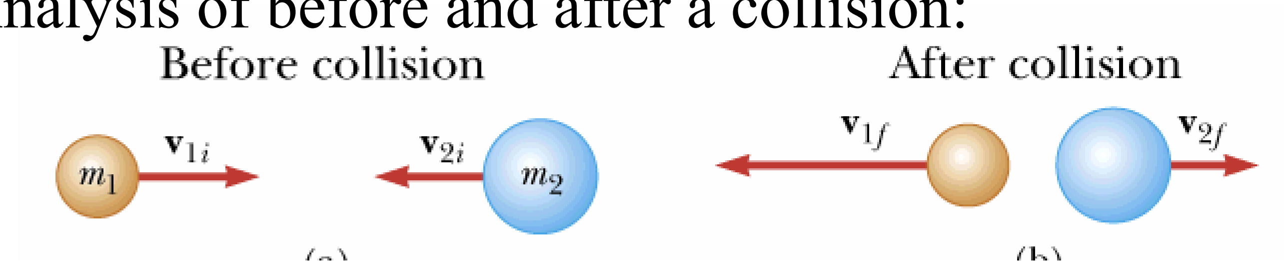
$$F \approx \frac{\Delta p}{\Delta t} = \frac{225.9 N \cdot s}{0.1 s} \approx 2260 N$$



(From HRW text.) It is well known that bullets and other missiles fired at Superman simply bound off his chest. Suppose that a gangster sprays Superman's chest with 0.003 kg bullets at a rate of 100 bullets/min, and that the speed of the bullets traveling directly toward him is 500 m/s. If the bullets rebound straight back with no change in speed, what is the magnitude of the average force on Superman's chest from the stream of bullets?

(a) 3 N
(b) 5 N
(c) 10 N
(d) 15 N
(e) None of these. Please put any comments/explanations in the box below your answers or send email.

Analysis of before and after a collision:



One dimensional case:

Conservation of momentum: $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

If energy (kinetic) is conserved, then:

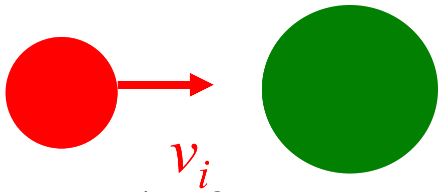
$$\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

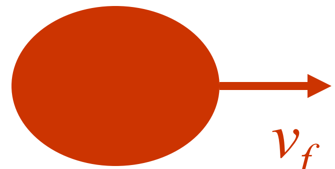
Extra credit: Show that

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

More examples in one dimension:





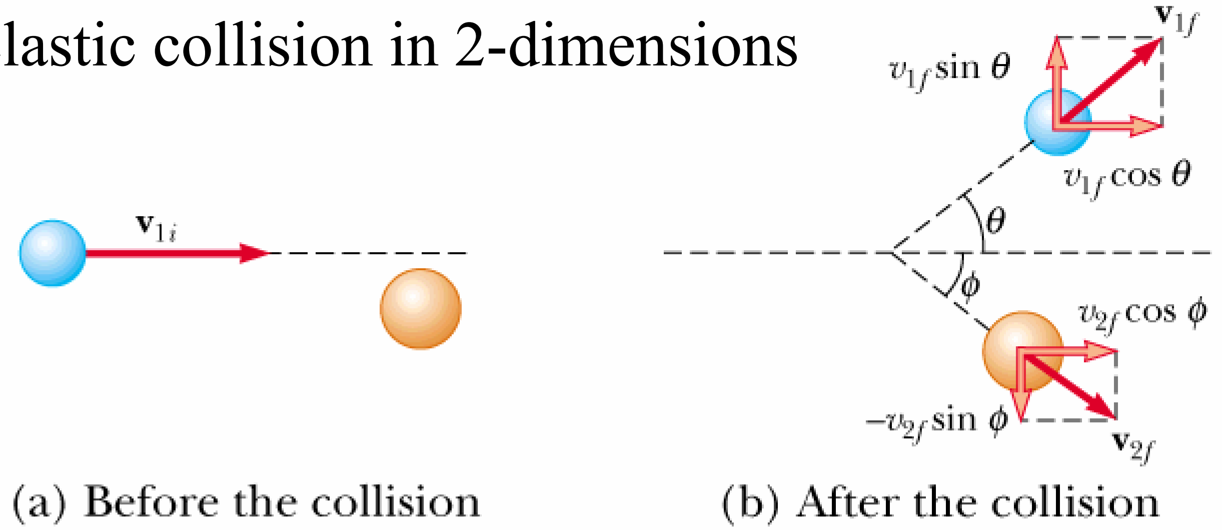
Conservation of momentum: $m_1 v_i = (m_1 + m_2)\, v_f$

$$v_f = \frac{m_1}{m_1 + m_2} v_i$$

Energy lost: $$\Delta E = \tfrac{1}{2} m_1 v_i^2 - \tfrac{1}{2}(m_1 + m_2) v_f^2$$

$$= \tfrac{1}{2} m_1 v_i^2 \left(\frac{m_2}{m_1 + m_2} \right)$$

Elastic collision in 2-dimensions



Statement of conservation of momentum:

$$m_1 v_{1i} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\varphi$$

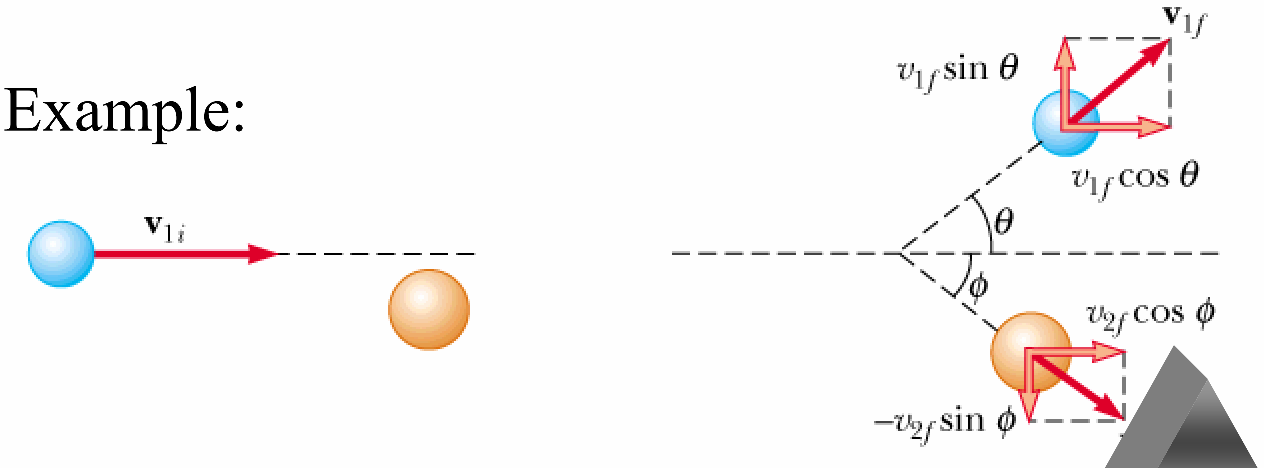
$$0 = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\varphi$$

If mechanical (kinetic) energy is conserved, then:

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

➔ 4 unknowns, 3 equations -- need more information

Example:



(a) Before the collision (b) After the collision

Conservation of momentum

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\varphi$$

$$0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\varphi$$

Detector measures v_{2f} , ϕ

$$m_1 v_{1f} \sin\ \theta = m_2 v_{2f} \sin\varphi$$

$$m_1 v_{1f} \cos\theta = m_1 v_{1i} - m_2 v_{2f} \cos\varphi$$

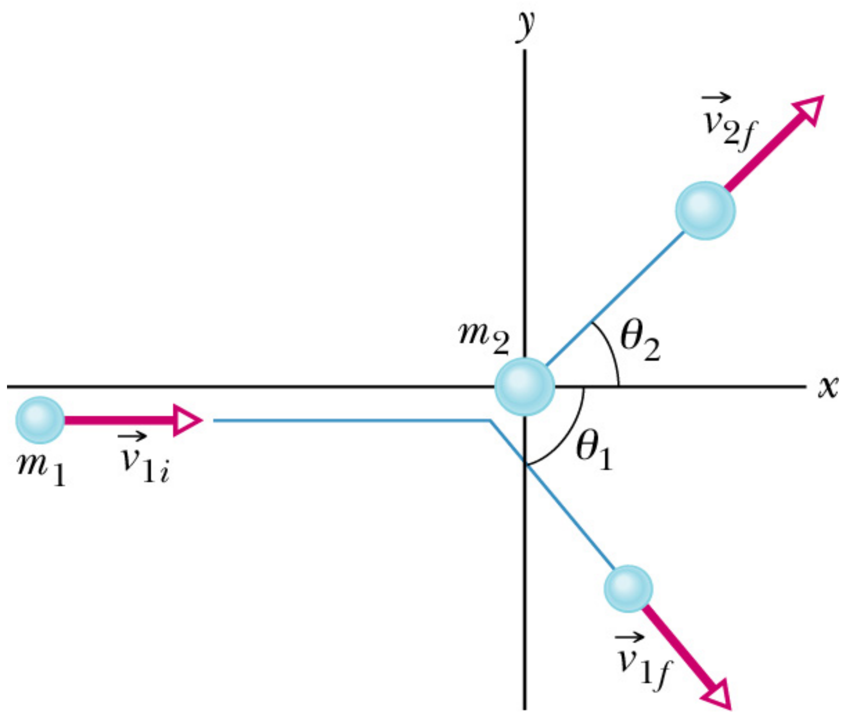
Note: energy is not conserved in this case.

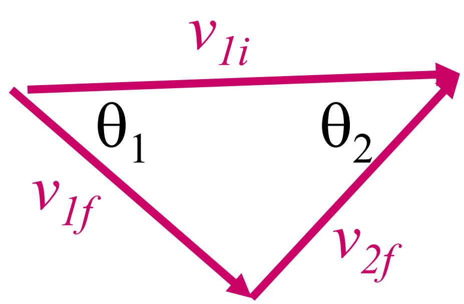
$$\tan\theta = \frac{m_2 v_{2f} \sin\varphi}{m_1 v_{1i} - m_2 v_{2f} \cos\varphi} \qquad v_{1f} = \frac{1}{m_1}\sqrt{\left(m_2 v_{2f} \sin\varphi\right)^2 + \left(m_1 v_{1i} - m_2 v_{2f} \cos\varphi\right)^2}$$

Elastic collision

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2$$

$$0 = m_1 v_{1f} \sin\theta_1 - m_2 v_{2f} \sin\theta_2$$



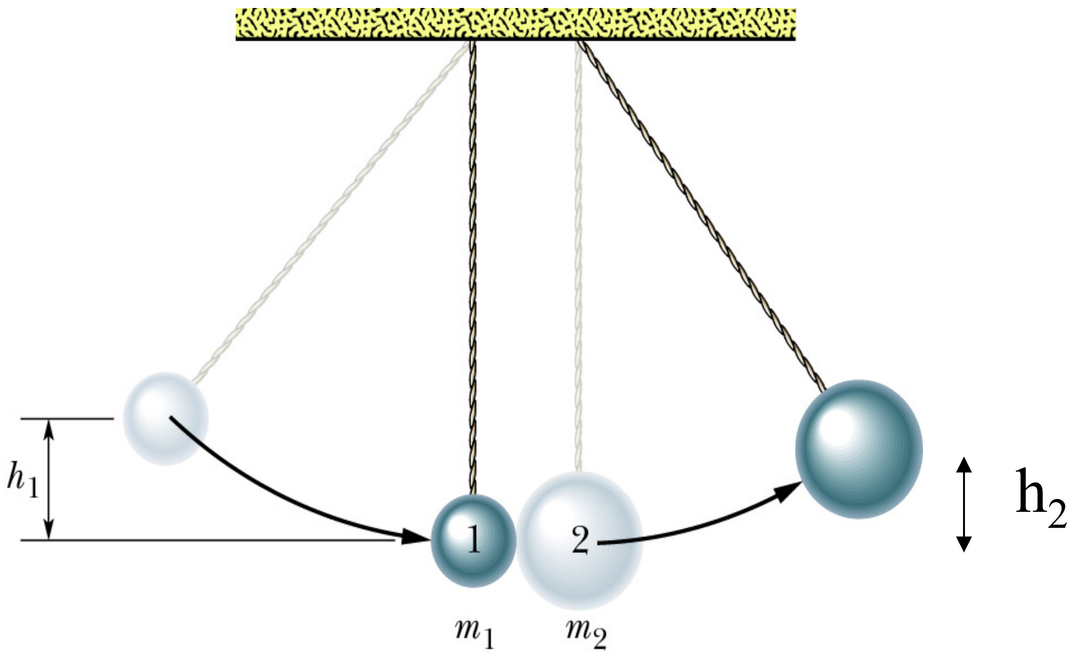


Special case: $m_1 = m_2$

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \Rightarrow v_{1i}^2 = v_{1f}^2 + v_{2f}^2$$

$$\theta_1 + \theta_2 = 90^\circ$$

Other examples:



Suppose you are given h_1, m_1, m_2 -- find h_2
(assume elastic collision)