Announcements

1. **Physic Colloquium today --The Physics and Analysis of Non-invasive Optical Imaging**
2. Today's lecture –

   Brief review of momentum & collisions

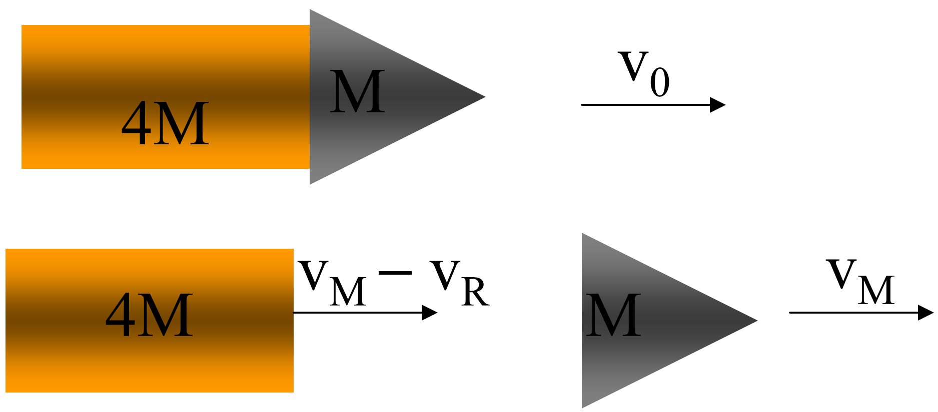
   - Example HW problems

   Introduction to rotations

   - Definition of angular variables
   - Moment of inertia
   - Energy associated with rotations

From HW 9 –

6. HRW6 9.P.031. A space vehicle is traveling at v_0 = 5700 km/h relative to the Earth when the exhausted rocket motor is disengaged and sent backward with a speed of v_R = 85 km/h relative to the command module. The mass of the motor is four times the mass of the module. What is the speed of the command module relative to Earth after the separation?
[ ] km/h



$$5Mv_0 = 4M(v_M - v_R) + Mv_M$$

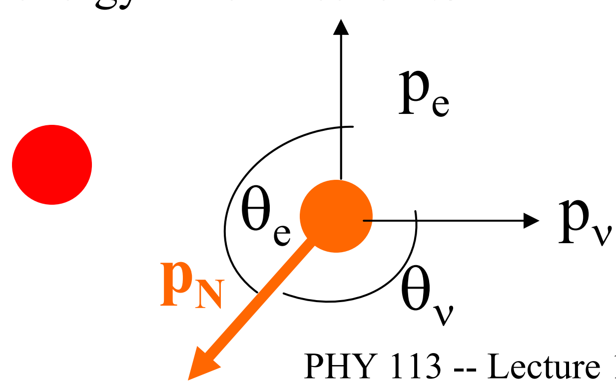
$$\Rightarrow \text{solve for } v_M$$

From HW 9 –

7. HRW6 9.P.035. A certain radioactive nucleus can transform to another nucleus by emitting an electron and a neutrino. (The neutrino is one of the fundamental particles of physics.) Suppose that in such a transformation, the initial nucleus is stationary, the electron and neutrino are emitted along perpendicular paths, and the magnitudes of the linear momenta are 1.4×10^{-22} kg · m/s for the electron and 6.1×10^{-23} kg · m/s for the neutrino. As a result of the emissions, the new nucleus moves (recoils).

(a) What is the magnitude of its linear momentum?
[ ] kg · m/s
(b) What is the angle between its path and the path of the electron?
[ ] °
(c) What is the angle between its path and the path of the neutrino?
[ ] °

(d) What is its kinetic energy if its mass is 5.7×10^{-26} kg?
[ ] J



From HW 10 –

6. HRW6 10.P.046. Two 2.0 kg masses, *A* and *B*, collide. The velocities before the collision are $\mathbf{v}_A = 12\mathbf{i} + 30\mathbf{j}$ and $\mathbf{v}_B = -5\mathbf{i} + 15.0\mathbf{j}$. After the collision, $\mathbf{v}'_A = -6.0\mathbf{i} + 22\mathbf{j}$. All speeds are given in meters per second.

(a) What is the final velocity of *B*? [ ] m/s **i** + [ ] m/s **j**
(b) How much kinetic energy was gained or lost in the collision? [ ] J

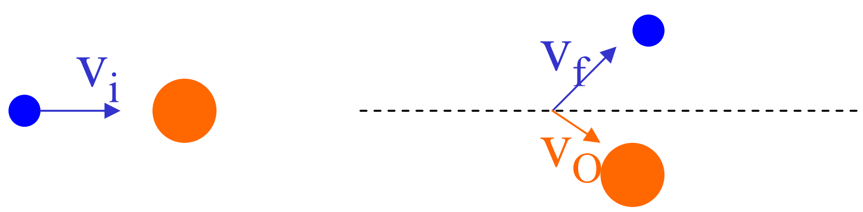
$$m(\mathbf{v}_A + \mathbf{v}_B) = m(\mathbf{v}'_A + \mathbf{v}'_B)$$
$$(12\mathbf{i} + 30\mathbf{j}) + (-5\mathbf{i} + 15.0\mathbf{j}) = (-6.0\mathbf{i} + 22\mathbf{j}) + \mathbf{v}'_B$$
$$\Delta E = \tfrac{1}{2} m\left(v_A'^2 + v_B'^2\right) - \tfrac{1}{2} m\left(v_A^2 + v_B^2\right)$$

From HW 10 –

7. HRW6 10.P.047. An alpha particle collides with an oxygen nucleus, initially at rest. The alpha particle is scattered at an angle of 64.0° above its initial direction of motion, and the oxygen nucleus recoils at an angle of 47.0° on the opposite side of that initial direction. The final speed of the nucleus is 1.10×10^5 m/s. In atomic mass units, the mass of an alpha particle is 4.0 u. The mass of an oxygen nucleus is 16 u.

(a) Find the final speed of the alpha particle. [ ] m/s

(b) Find the initial speed of the alpha particle. [ ] m/s



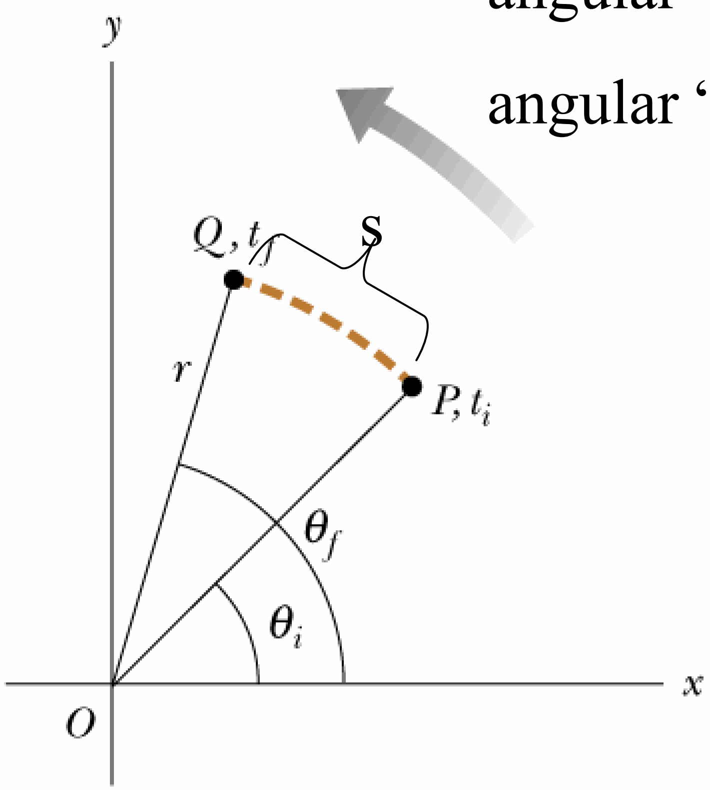
Angular motion

angular "displacement" ➔ $\theta(t)$

angular "velocity" ➔ $\omega(t) = \frac{d\theta}{dt}$

angular "acceleration" ➔ $\alpha(t) = \frac{d\omega}{dt}$

Serway, Physics for Scientists and Engineers, 5/e
Figure 10.2



Harcourt, Inc.

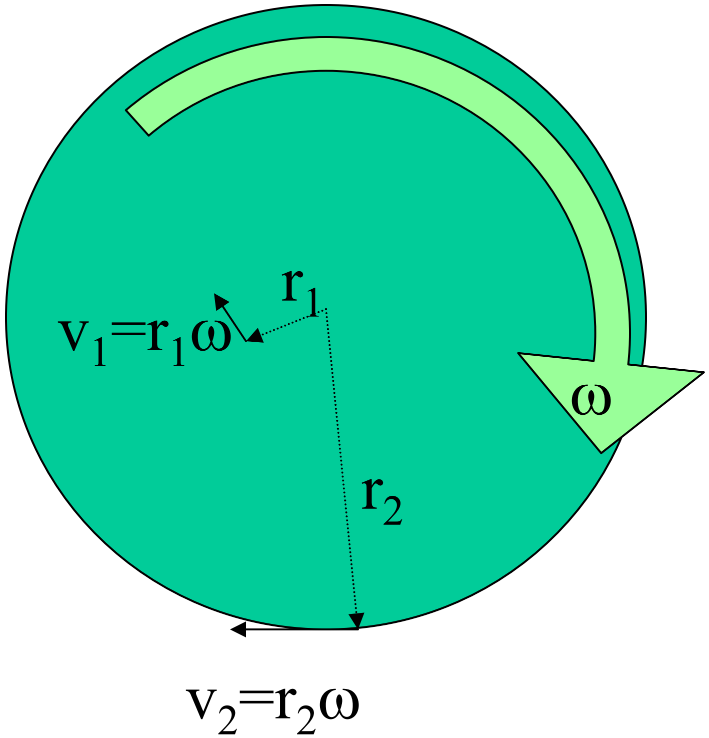
"natural" unit == 1 radian

Relation to linear variables:

$s_\theta = r\,(\theta_f - \theta_i)$

$v_\theta = r\,\omega$

$a_\theta = r\,\alpha$



Special case of constant angular acceleration: $\alpha = \alpha_0$:

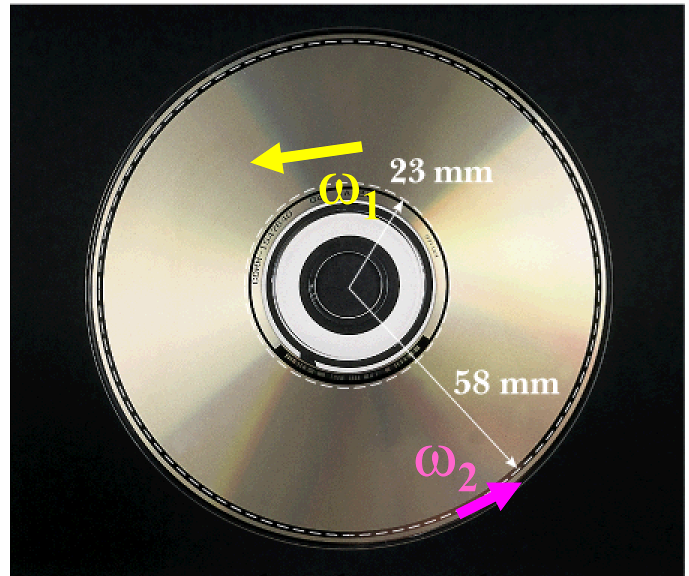
$$\omega(t) = \omega_i + \alpha_0 t$$

$$\theta(t) = \theta_i + \omega_i t + ½ \alpha_0 t^2$$

$$(\omega(t))^2 = \omega_i^2 + 2 \alpha_0 (\theta(t) - \theta_i)$$

Serway, Physics for Scientists and Engineers, 5/e
Figure 10.6

Example: Compact disc motion



Harcourt, Inc.

In a compact disk, each spot on the disk passes the laser-lens system at a constant linear speed of $v_\theta = 1.3$ m/s.

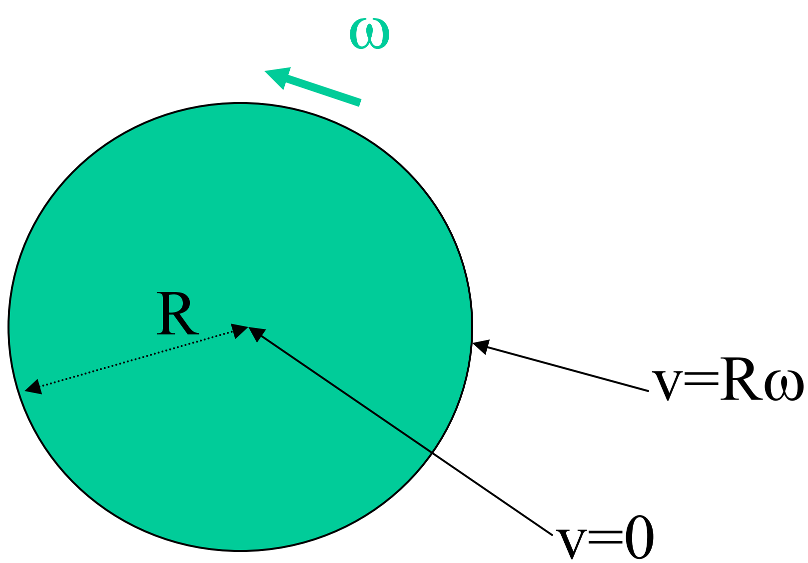
$\omega_1 = v_\theta / r_1 = 56.5$ rad/s

$\omega_2 = v_\theta / r_2 = 22.4$ rad/s

What is the average angular deceleration of the CD over the time interval $\Delta t = 4473$ s?

$\alpha = (\omega_2 - \omega_1)/\Delta t = -0.0076$ rad/s^2

Object rotating with constant angular velocity ($\alpha = 0$)

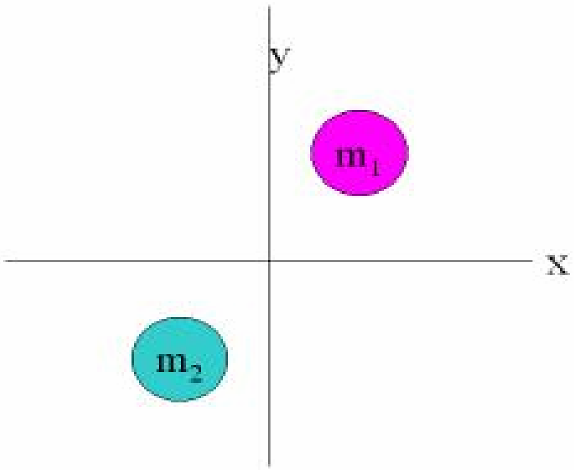


Kinetic energy associated with rotation:

$$K = \sum_i \tfrac{1}{2} m_i v_i^2 = \sum_i \tfrac{1}{2} m_i r_i^2 \omega^2 \equiv \tfrac{1}{2} I \omega^2;$$

$$\text{where}: I \equiv \sum_i m_i r_i^2$$ "moment of inertia"

Online Quiz for Lecture 11
Calculating the moment of inertia -- Oct. 9, 2003

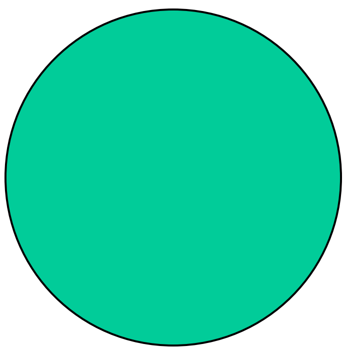


Consider the two masses shown in the figure. Assume that they are held rigidly by massless supports with **r_1**=0.1 m **i**+ 0.1 m **j** for mass 1 and **r_2**=-0.1 m **i**- 0.1 m **j** for mass 2. Suppose that each mass is 1 kg.

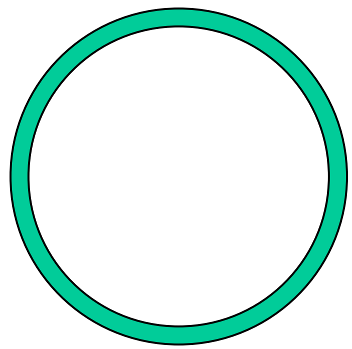
1. What is the moment of inertia (in units of kg m^2) for rotating about the x axis? (a) 0.01 (b) 0.02 (c) 0.04 (d) 0.08
2. What is the moment of inertia (in units of kg m^2) for rotating about the y axis? (a) 0.01 (b) 0.02 (c) 0.04 (d) 0.08
3. What is the moment of inertia (in units of kg m^2) for rotating about the z axis (which is perpendicular to the plane of the figure)? (a) 0.01 (b) 0.02 (c) 0.04 (d) 0.08

Peer instruction question:

Suppose each of the following objects each has the same total mass M and outer radius R and each is rotating counter-clockwise at an constant angular velocity of ω=3 rad/s. Which object has the greater kinetic energy?

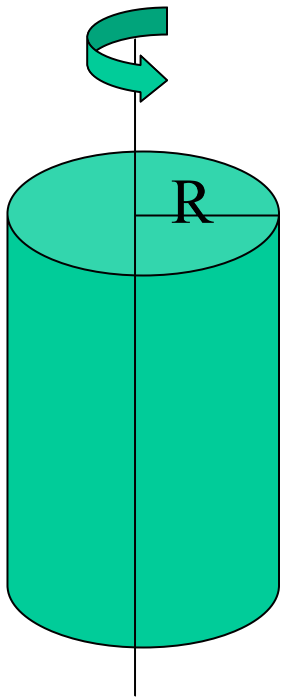


(a) (Solid disk)



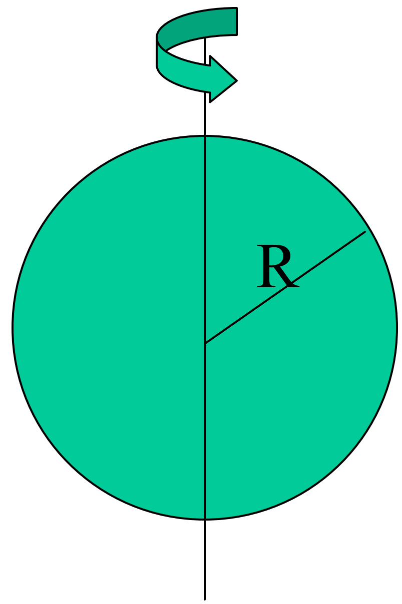
(b) (circular ring)

Various moments of inertia:



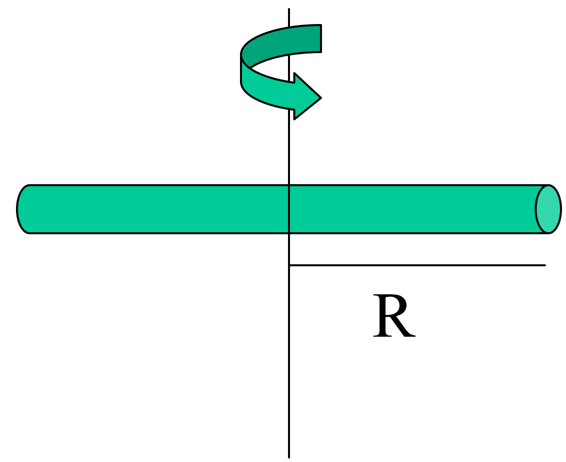
solid cylinder:

$I=1/2\ MR^2$



solid sphere:

$I=2/5\ MR^2$

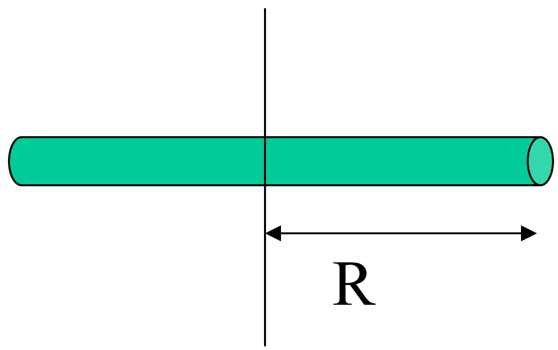


solid rod:

$I=1/3\ MR^2$

Calculation of moment of inertia:

Example -- moment of inertia of solid rod through an axis perpendicular rod and passing through center:



$$I = \sum_i m_i r_i^2 = \int_{-R}^{R} \left(\frac{M}{2R}\right) dr\ r^2 = \left(\frac{M}{2R}\right) \int_{-R}^{R} r^2 dr = \frac{1}{3} M R^2$$

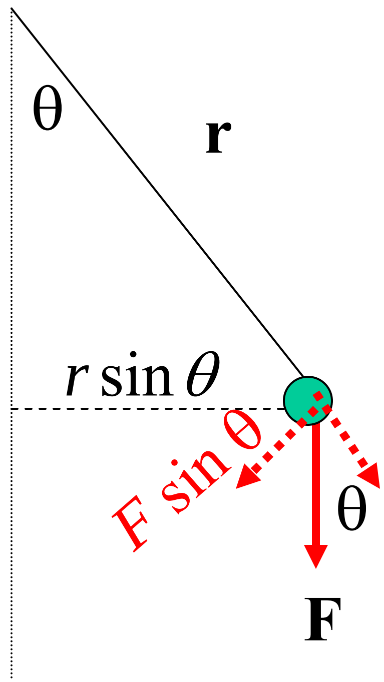
Extra credit: Write out the evaluation of *I* for another shape.

How to make objects rotate.

Define torque:

$\tau = \mathbf{r} \times \mathbf{F}$

$\tau = rF \sin\theta$



$$\mathbf{F} = m\mathbf{a}$$

$$\mathbf{r} \times \mathbf{F} \equiv \boldsymbol{\tau} = \mathbf{r} \times m\mathbf{a} = I\boldsymbol{\alpha}$$

From HW 11 --

6. HRW6 11.P.052. [51818] In Fig. 11-39 a cylinder having a mass of 2.0 kg can rotate about its central axis through point O. Forces are applied as shown. F_1 = 9.0 N, F_2 = 6.0 N, F_3 = 3.0 N, and F_4 = 4.0 N. Also, R_1 = 5.0 cm and R_2 = 12 cm.

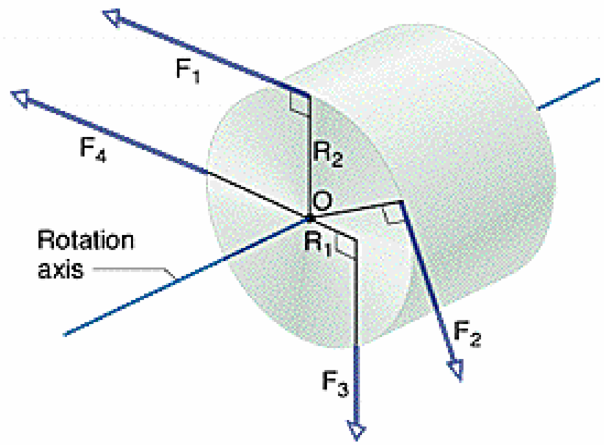


Figure 11-39

Find the magnitude and direction of the angular acceleration of the cylinder. (During the rotation, the forces maintain their same angles relative to the cylinder.)

Magnitude

[0.125] ______ rad/s^2

Direction

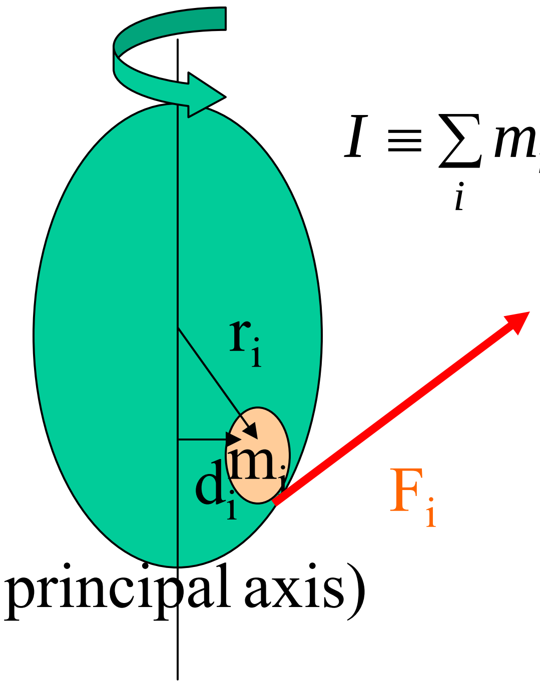
- ○ counterclockwise
- ○ clockwise [0.125]

Newton's second law applied to center-of-mass motion

$$\sum_i \mathbf{F}_i = \sum_i m_i \frac{d\mathbf{v}_i}{dt} \Rightarrow \mathbf{F}_{total} = M\frac{d\mathbf{v}_{CM}}{dt}$$

Newton's second law applied to rotational motion

$$\mathbf{F}_i = m_i \frac{d\mathbf{v}_i}{dt} \Rightarrow \mathbf{r}_i \times \mathbf{F}_i = \mathbf{r}_i \times m_i \frac{d\mathbf{v}_i}{dt}$$



$$I \equiv \sum_i m_i d_i^2$$

$$\boldsymbol{\tau}_i = \mathbf{r}_i \times \mathbf{F}_i$$

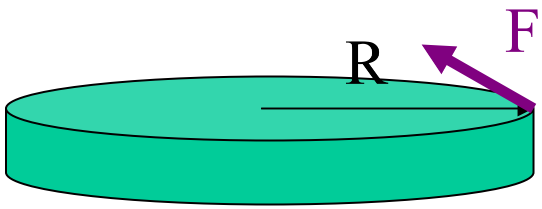
$$\mathbf{v}_i = \boldsymbol{\omega} \times \mathbf{r}_i$$

$$\Rightarrow \boldsymbol{\tau}_i = m_i \mathbf{r}_i \times \frac{d(\boldsymbol{\omega} \times \mathbf{r}_i)}{dt}$$

$$\Rightarrow \boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = I\boldsymbol{\alpha}$$ (for rotating about principal axis)

Another example:

A horizontal 800 N merry-go-round is a solid disc of radius 1.50 m and is started from rest by a constant horizontal force of 50 N applied tangentially to the cylinder. Find the kinetic energy of solid cylinder after 3 s.

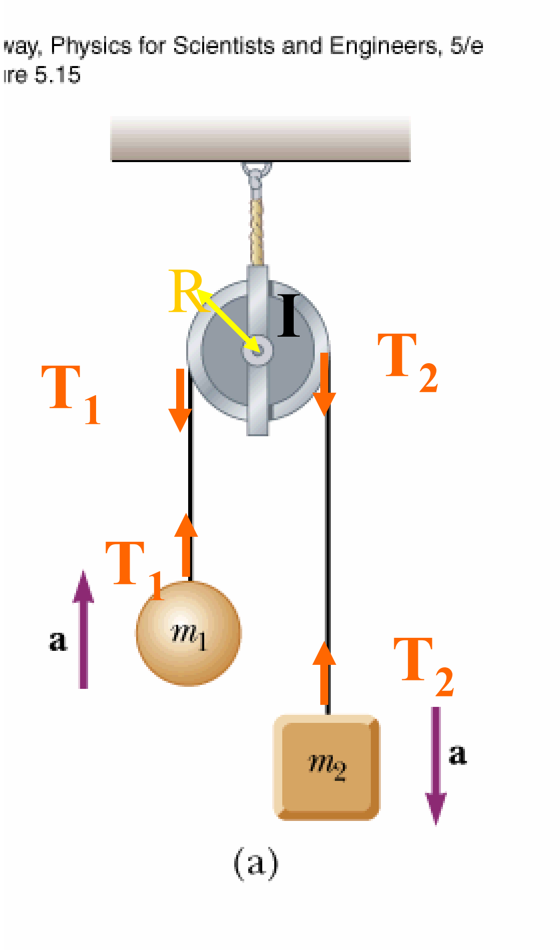


$K = \frac{1}{2} I \omega^2$ $\tau = I \alpha$ $\omega = \omega_i + \alpha t = \alpha t$

In this case $I = \frac{1}{2} m R^2$ and $\tau = FR$

$$K = g\frac{F^2}{mg}t^2 = 9.8\text{m/s}^2\frac{(50N)^2}{800N}(3s)^2 = 275.625J$$

Re-examination of "Atwood's" machine



$$T_1 - m_1 g = m_1 a$$

$$T_2 - m_2 g = -m_2 a$$

$$\tau = T_2 R - T_1 R = I\,\alpha = I\,a/R$$

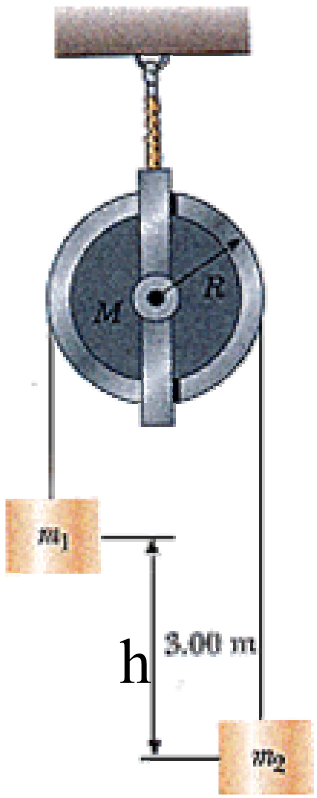
$$a = g\left(\frac{m_2 - m_1}{m_2 + m_1 + I/R^2}\right)$$

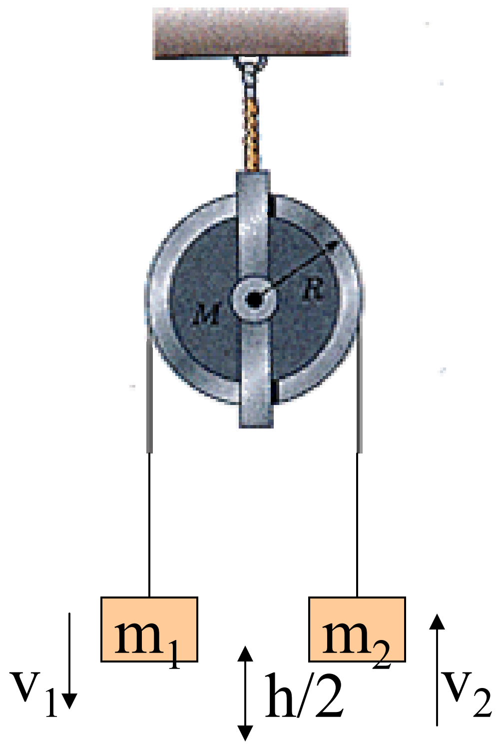
$$\tau = \frac{Ig}{R}\left(\frac{m_2 - m_1}{m_2 + m_1 + I/R^2}\right)$$

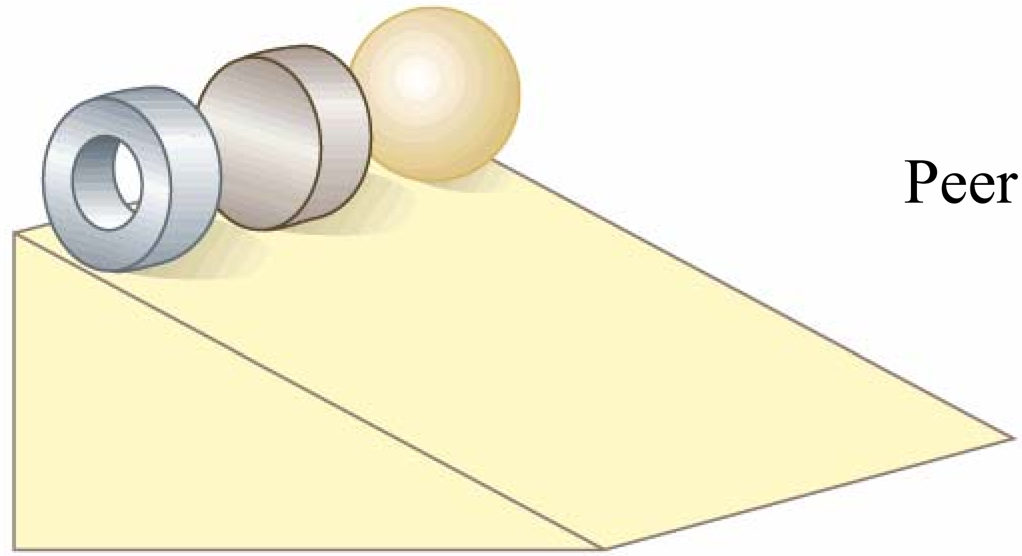
Conservation of energy:

$K_f + U_f = K_i + U_i$

Another example:







Peer instruction question

Three objects of uniform density – a solid sphere (a), a solid cylinder (b), and a hollow cylinder (c) -- are placed at the top of an incline. If they all are released from rest at the same elevation and roll without slipping, which object reaches the bottom first?

(a) solid sphere (b)solid cylinder (c)hollow cylinder