Announcements

1. **HW sets 12, 13, and 14 now available**
2. **Mid-term grades – will be calculated from your first exam only**
3. **Today's lecture**

   Rotations continued

   Torques continued

   Rotational energy

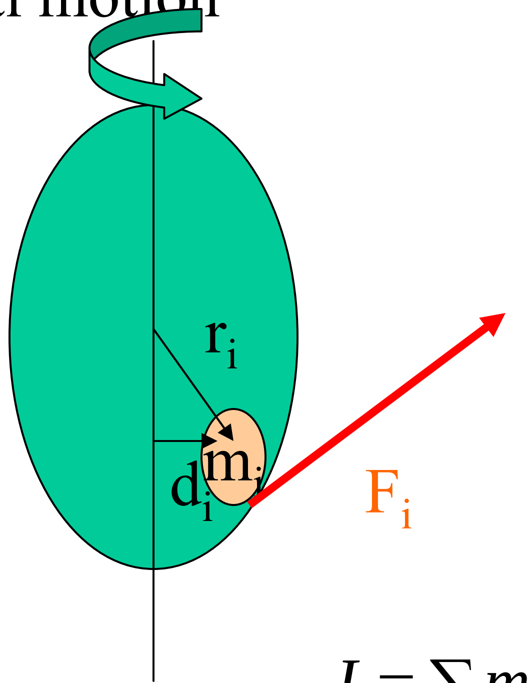
   Conservation of angular momentum

Newton's second law applied to center-of-mass motion

$$\sum_i \mathbf{F}_i = \sum_i m_i \frac{d\mathbf{v}_i}{dt} \Rightarrow \mathbf{F}_{total} = M\frac{d\mathbf{v}_{CM}}{dt}$$

Newton's second law applied to rotational motion

$$\mathbf{F}_i = m_i \frac{d\mathbf{v}_i}{dt} \Rightarrow \mathbf{r}_i \times \mathbf{F}_i = \mathbf{r}_i \times m_i \frac{d\mathbf{v}_i}{dt}$$



$$\boldsymbol{\tau}_i = \mathbf{r}_i \times \mathbf{F}_i$$

$$\mathbf{v}_i = \boldsymbol{\omega} \times \mathbf{r}_i$$

$$\Rightarrow \boldsymbol{\tau}_i = m_i \mathbf{r}_i \times \frac{d(\boldsymbol{\omega} \times \mathbf{r}_i)}{dt}$$

$$I \equiv \sum_i m_i d_i^2$$

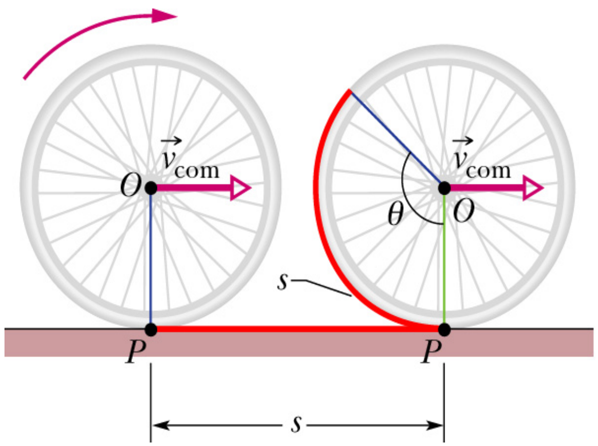
$$\Rightarrow \boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = I\boldsymbol{\alpha}$$ (for rotating about principal axis)

Kinetic energy associated with rotation:

$$K_{rot} = \tfrac{1}{2} I\omega^2 \qquad I \equiv \sum_i m_i r_i^2$$

Distance to axis of rotation

Rolling:



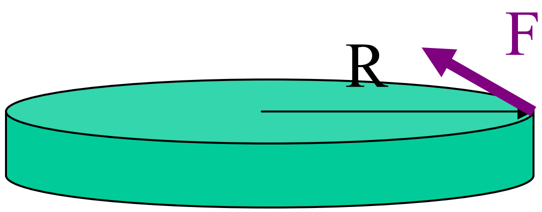
$$K_{tot} = K_{com} + K_{rot}$$

If there is no slipping : $v_{com} = R\omega$

$$\Rightarrow K_{tot} = \tfrac{1}{2} M\left(1 + \frac{I}{MR^2}\right)v_{com}^2$$

Example problem:

A horizontal 800 N merry-go-round is a solid disc of radius 1.50 m and is started from rest by a constant horizontal force of 50 N applied tangentially to the cylinder. Find the kinetic energy of solid cylinder after 3 s.



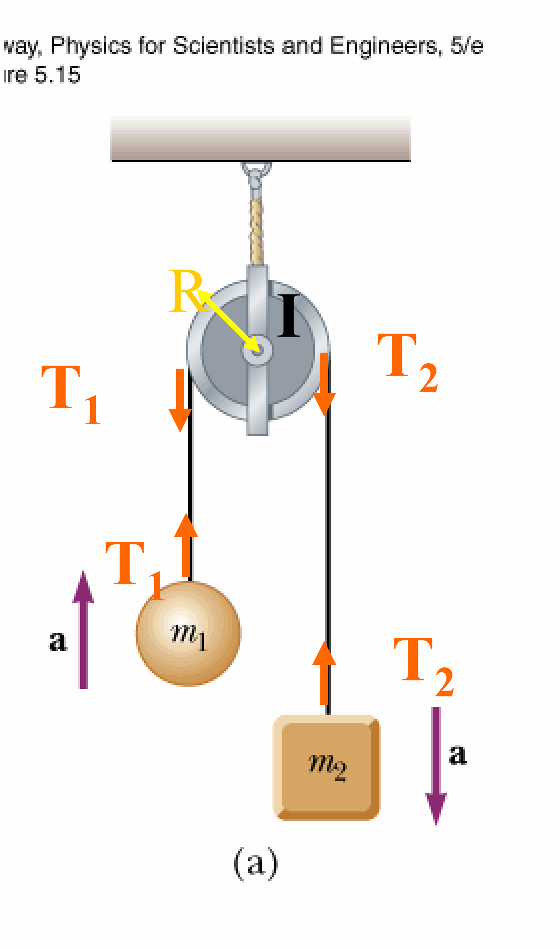
$K = \frac{1}{2} I \omega^2$ $\tau = I \alpha$ $\omega = \omega_i + \alpha t = \alpha t$

In this case $I = \frac{1}{2} m R^2$ and $\tau = FR$

$$K = \frac{1}{2} I (\alpha t)^2 = \frac{1}{2} I \left(\frac{FR}{I} t \right)^2 = \frac{1}{2} \frac{(FRt)^2}{I} = \frac{1}{2} \frac{(FRt)^2}{\frac{1}{2} mR^2} = \frac{(Ft)^2}{m}$$

$$K = g \frac{F^2}{mg} t^2 = 9.8 \mathrm{m/s}^2 \frac{(50N)^2}{800N} (3s)^2 = 275.625J$$

Re-examination of "Atwood's" machine



(a)

$$T_1 - m_1 g = m_1 a$$

$$T_2 - m_2 g = -m_2 a$$

$$\tau = T_2 R - T_1 R = I\,\alpha = I\,a/R$$

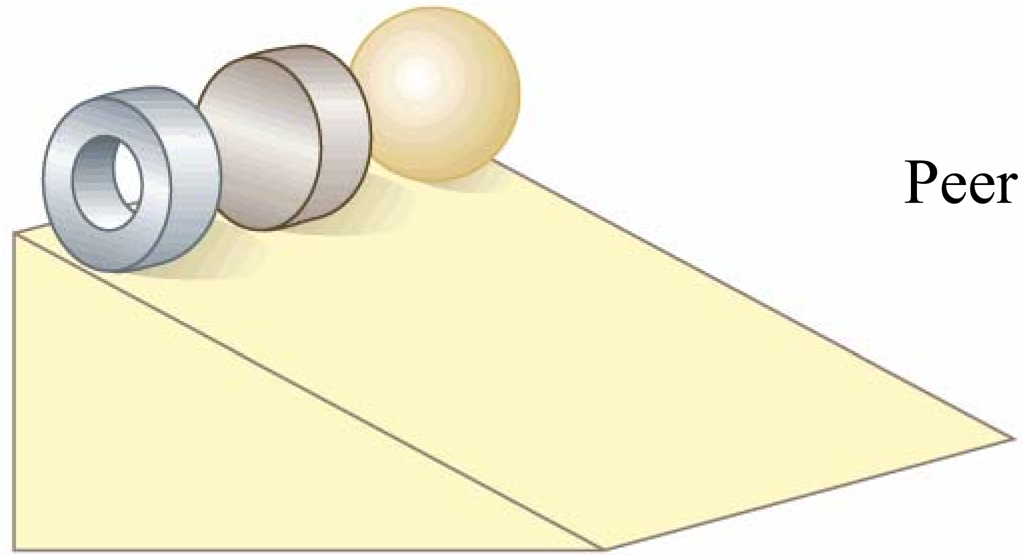
$$a = g\left(\frac{m_2 - m_1}{m_2 + m_1 + I/R^2}\right)$$

$$\tau = \frac{Ig}{R}\left(\frac{m_2 - m_1}{m_2 + m_1 + I/R^2}\right)$$

Online Quiz for Lecture 12
The Physics of Rolling -- Oct.14, 2003

Suppose that you have the following 4 objects each having the same mass and outer radius. Each object is rolling without slipping on its round surface and has the same center of mass kinetic energy. Choose (a) for the object with the largest total kinetic energy, (b) for the object with the next largest total kinetic energy, (c) for the object with the next largest total kinetic energy, (d) for the object with the smallest total kinetic energy. (You may want to refer to Table 11-2 of your text. For the purpose of this problem, you may assume that the "hollow" objects are infinitely thin.)

1. A solid sphere → $I = \frac{2}{5} MR^2$
2. A hollow sphere → $I = \frac{2}{3} MR^2$
3. A solid cylinder → $I = \frac{1}{2} MR^2$
4. A hollow cylinder → $I = MR^2$

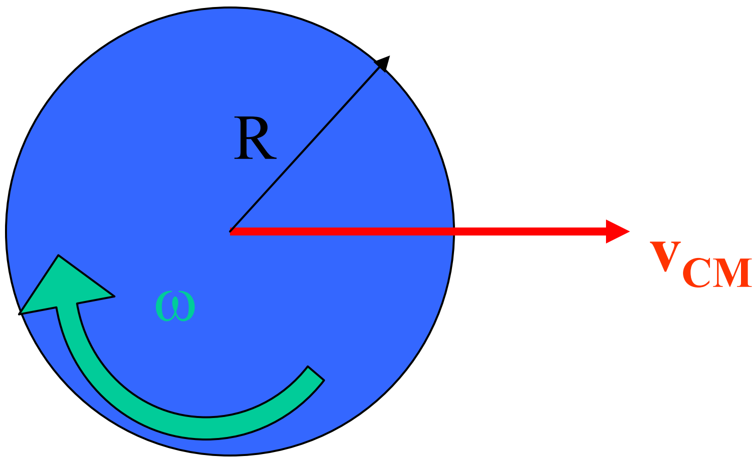


Peer instruction question

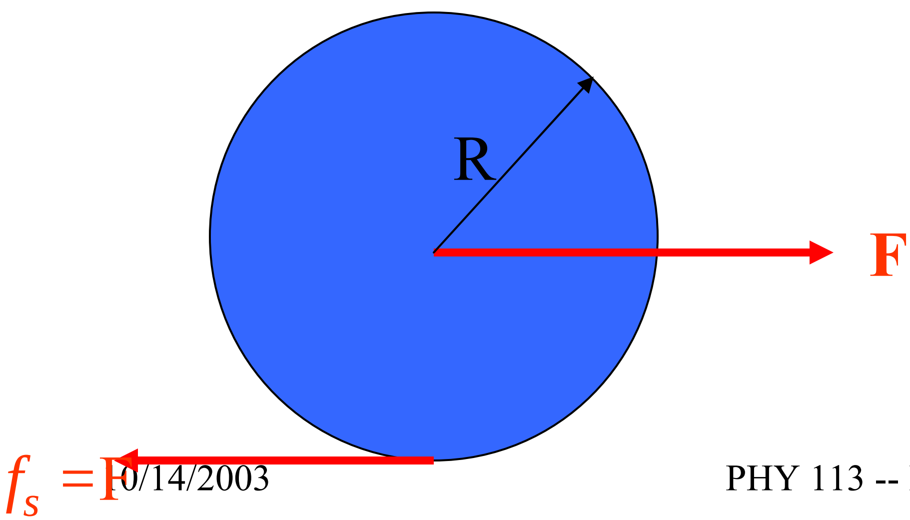
Three objects of uniform density – a solid sphere (a), a solid cylinder (b), and a hollow cylinder (c) -- are placed at the top of an incline. If they all are released from rest at the same elevation and roll without slipping, which object reaches the bottom first?

(a) solid sphere (b)solid cylinder (c)hollow cylinder

Rolling without slipping



$\omega = v_{CM}/R$



$\tau = R\, f_s$

Torque and angular momentum

Define angular momentum: $\mathbf{L} \equiv \mathbf{r} \times \mathbf{p}$

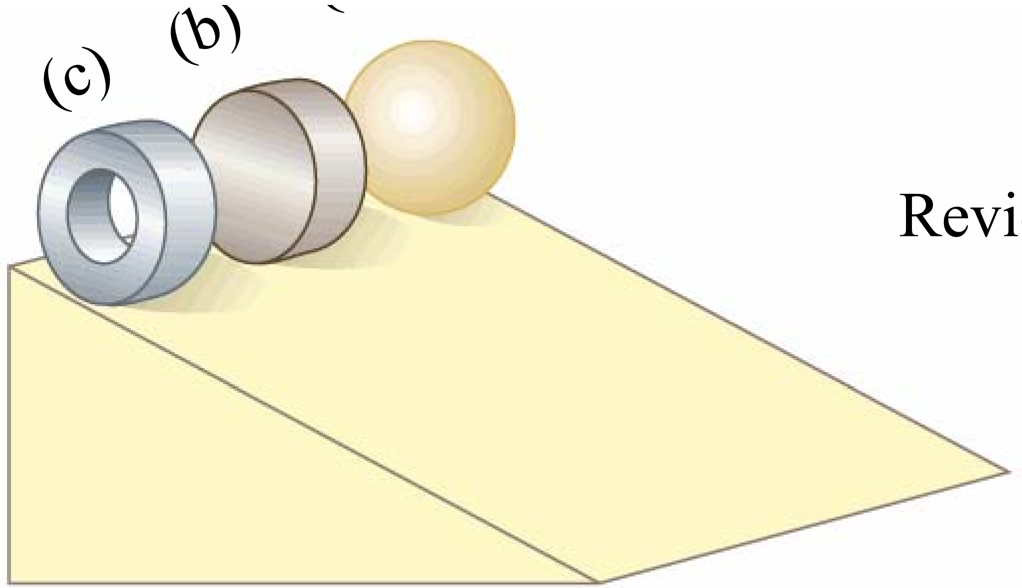
For an extended object: $L = I\omega$

Newton's law for torque:

$$\boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = \frac{d\mathbf{L}}{dt}$$

➔ If $\tau_{total} = 0$ then $\mathbf{L}$ = constant

In this case: $I_1\omega_1 = I_2\omega_2$



Review

$I_a = 2/5\ MR^2$

$I_b = 1/2\ MR^2$

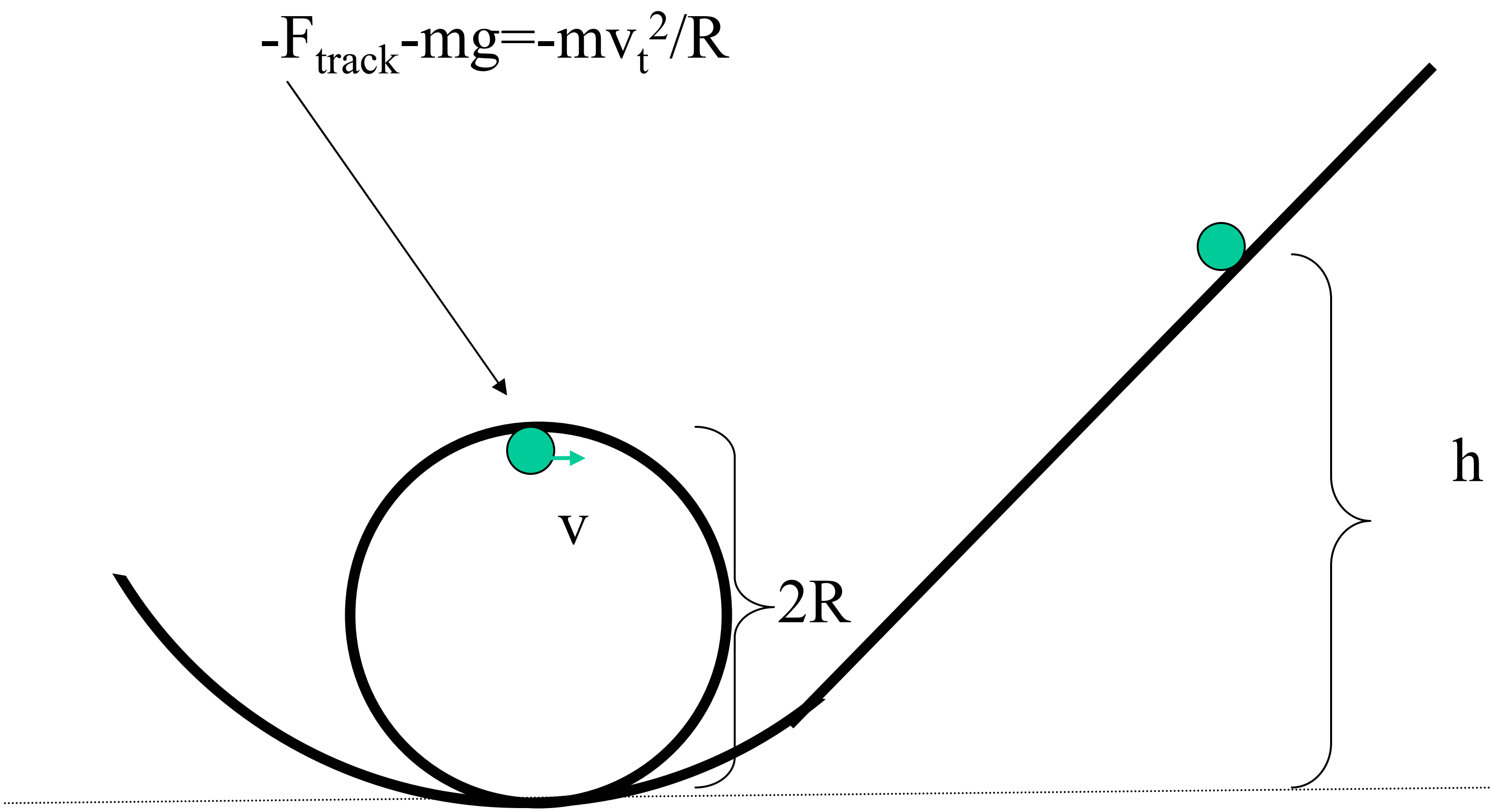
$I_c = MR^2$

Three objects of uniform density – a solid sphere (a), a solid cylinder (b), and a hollow cylinder (c) -- are placed at the top of an incline. If they all are released from rest at the same elevation and roll without slipping, which object reaches the bottom first?

$K_{rot} = \frac{1}{2} I \omega^2$ ➔ For fixed ω, (c) has the largest K_{rot}

➔ For fixed K_{rot}, (a) has the largest ω

$$K_{tot} = K_{CM} + K_{rot}$$



If $F_{track}=0$, then $\frac{1}{2} mv_t^2=\frac{1}{2} mgR$

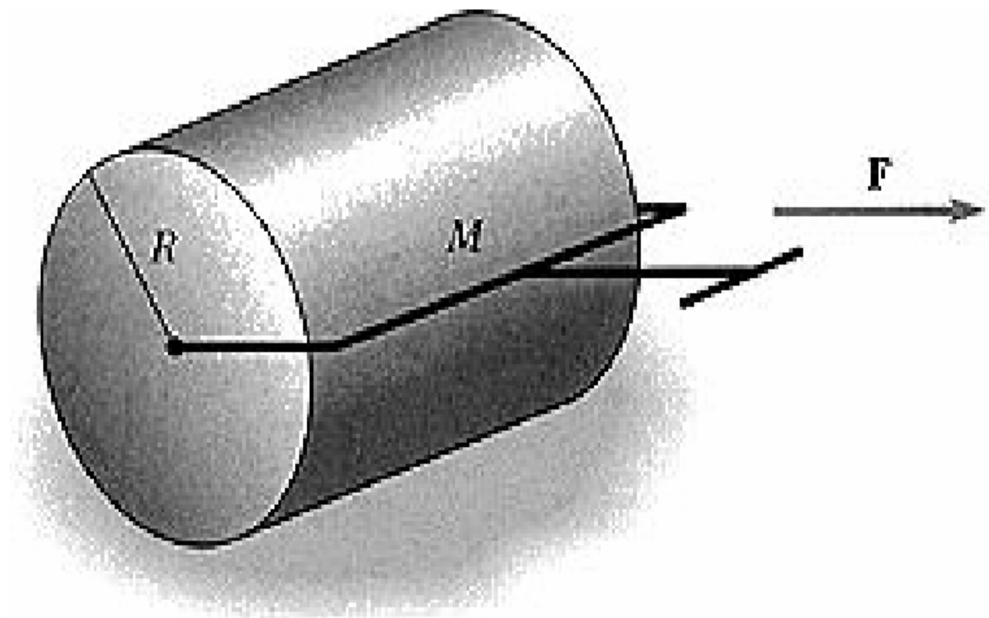
Without rolling: $W_{net} = K_f - K_i$ ➔ $mg(h-2R)= \frac{1}{2} mv_t^2$

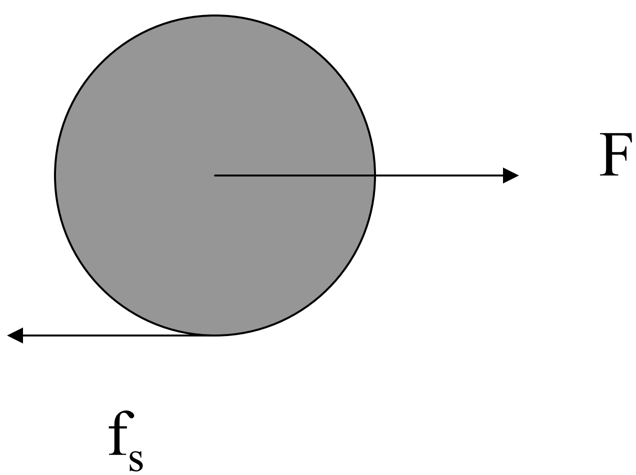
➔ $h=5/2R$

With rolling: $mg(h\text{-}2R) = \frac{1}{2} m\left(1+\frac{I}{MR^2}\right)v_t^2 = \frac{1}{2} m\left(1+\frac{2}{5}\right)v_t^2 \Rightarrow h = \frac{27}{10} R$

Newton's law for torque:

$$\boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = I\boldsymbol{\alpha}$$



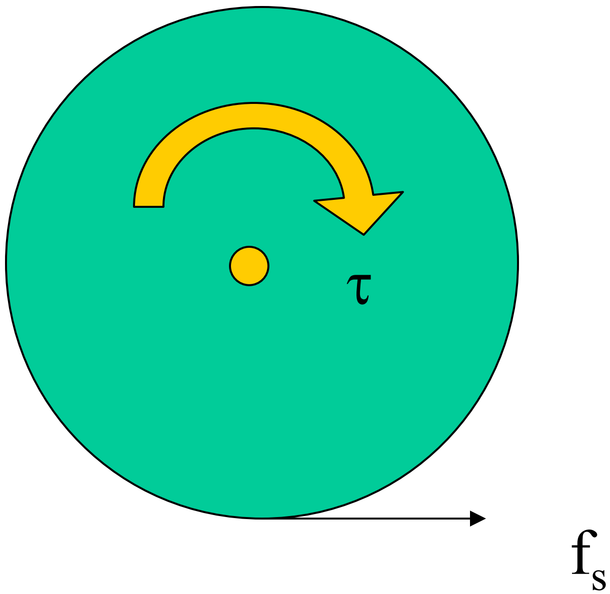


$F - f_s = M\, a_{CM}$

$R f_s = I\, \alpha = I\, a_{CM}/R$

$$f_s = F\left(\frac{I/(MR^2)}{1 + I/(MR^2)}\right)$$

Bicycle or automobile wheel:



$$f_s = Ma_{CM}$$

$$\tau - Rf_s = I\alpha = Ia_{CM} / R$$

$$f_s = \frac{\tau/R}{\left(1 + I/MR^2\right)}$$

Torque and angular momentum

Define angular momentum: $\mathbf{L} \equiv \mathbf{r} \times \mathbf{p}$

For composite object: $L = I\omega$

Newton's law for torque:

$$\boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = \frac{d\mathbf{L}}{dt}$$

➔ If $\tau_{\text{total}} = 0$ then $\mathbf{L}$ = constant

In the absence of a net torque on a system, angular momentum is conserved.

Peer instruction question

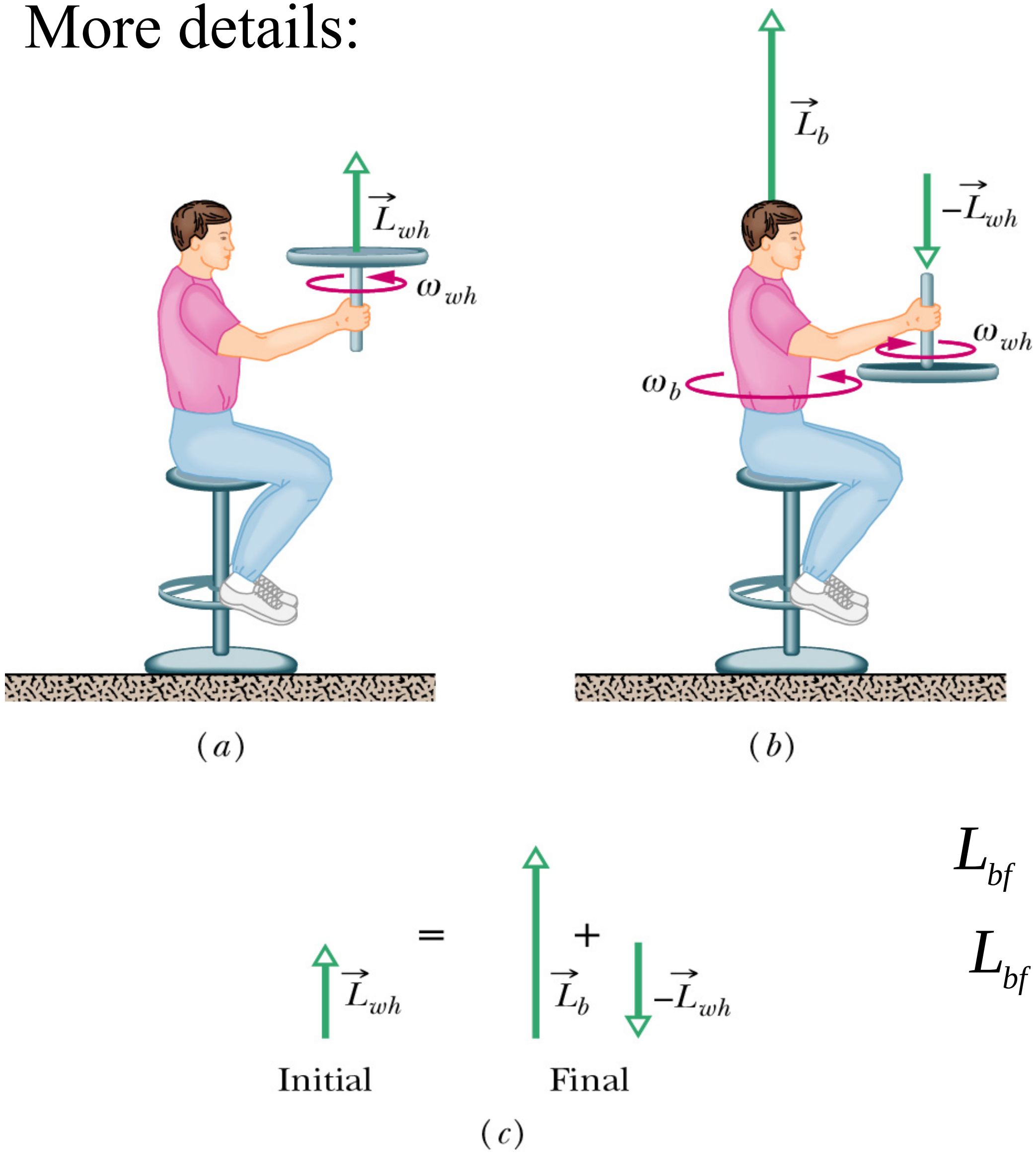
A student sits on a rotatable stool holding a spinning bicycle wheel with angular momentum $\mathbf{L}_i$. What happens when the wheel is inverted?

(a) The student will remain at rest.

(b) The student will rotate counterclockwise.

(c) The student will rotate clockwise.

More details:

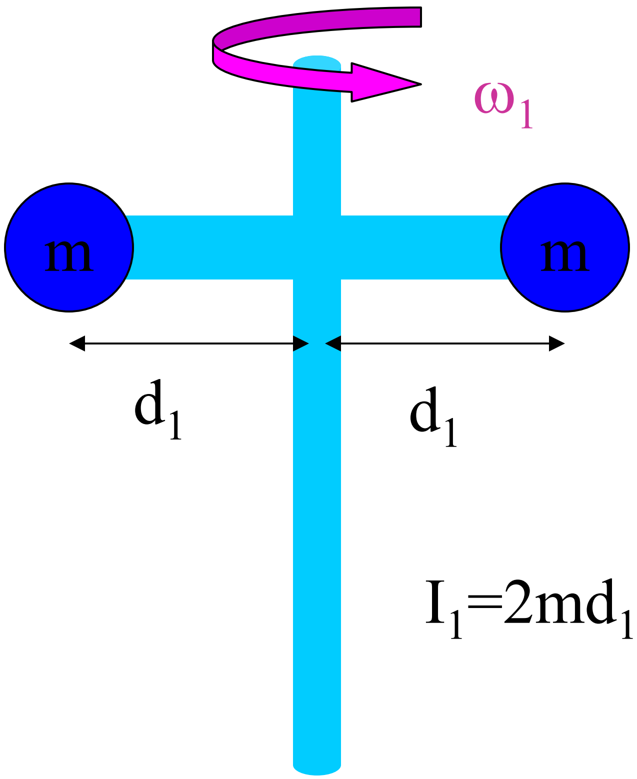


$$L_{bf} + L_{wheelf} = L_{bi} + L_{wheeli}$$

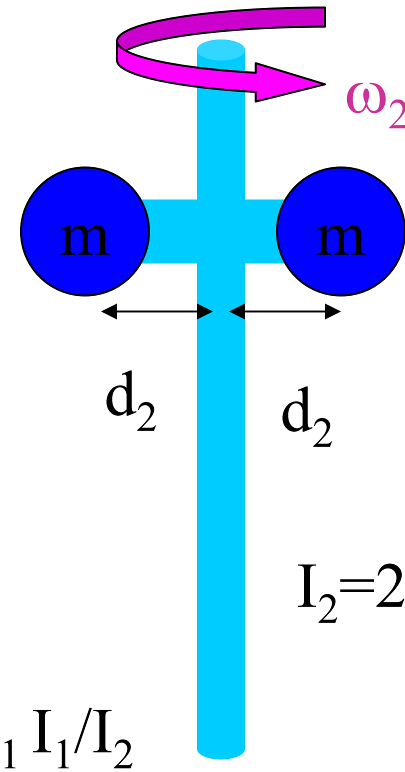
$$L_{bf} - L_{wheel} = 0 + L_{wheel}$$

$$L_{bf} = 2L_{wheel}$$

Other examples of conservation of angular momentum



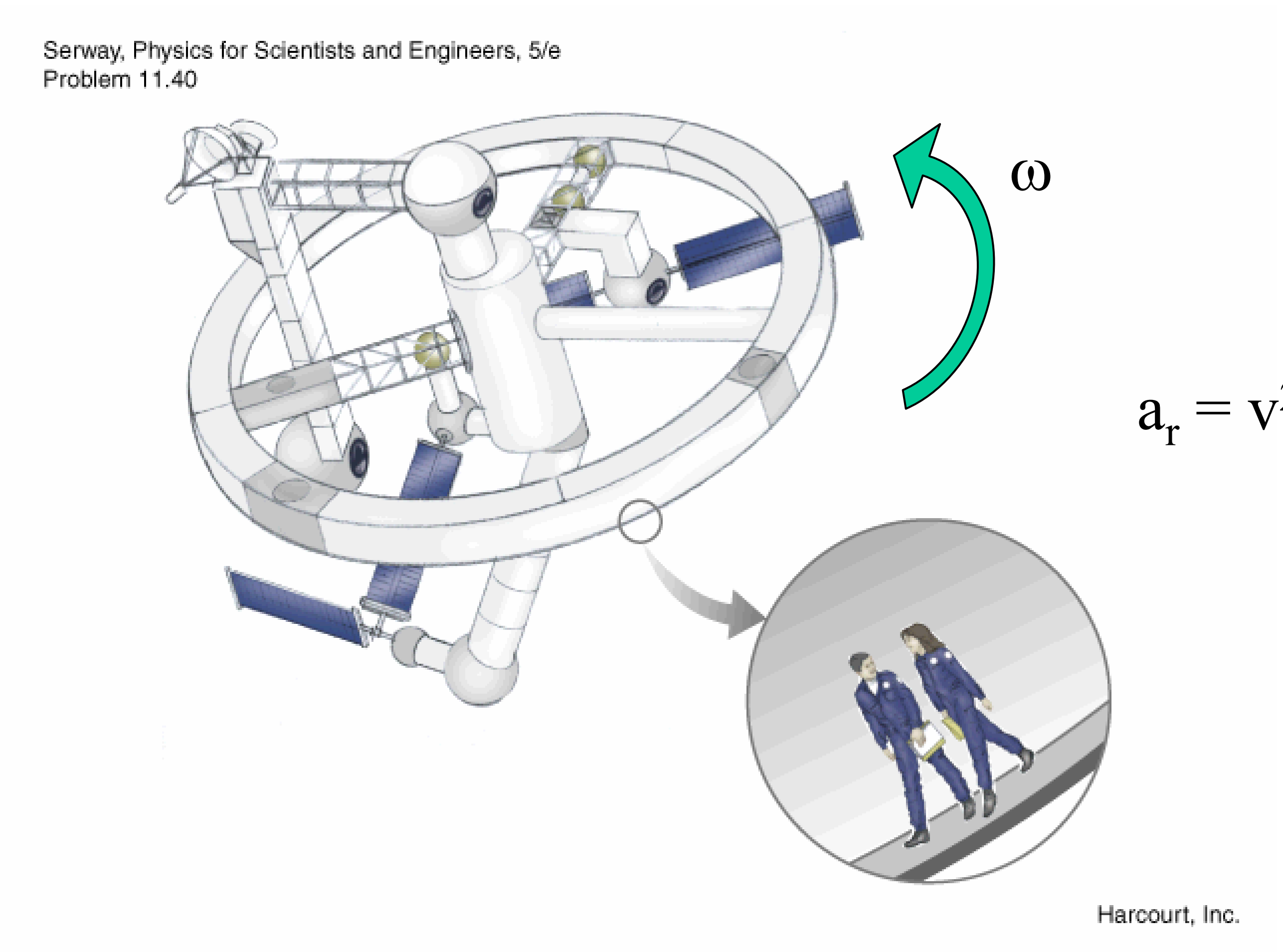
$I_1=2md_1^2$



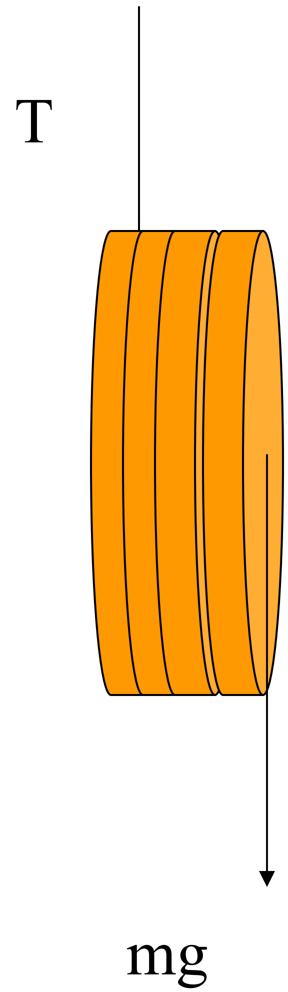
$I_2=2md_2^2$

$I_1\omega_1=I_2\omega_2 \rightarrow \omega_2=\omega_1 I_1/I_2$

What about centripetal acceleration?



$$a_r = v^2/R = \omega^2 R$$



$T - mg = -ma_{CM}$

$RT = I\alpha = Ia_{CM}/R$

$$a_{CM} = \frac{g}{\left(1 + I/mR^2\right)}$$

6. HRW6 12.P.043. [51881] In a playground, there is a small merry-go-round of radius 1.20 m and mass 220 kg. Its radius of gyration (see Problem 58 of Chapter 11) is 91.0 cm. A child of mass 44.0 kg runs at a speed of 2.00 m/s along a path that is tangent to the rim of the initially stationary merry-go-round and then jumps on. Neglect friction between the bearings and the shaft of the merry-go-round.

(a) Calculate the rotational inertia of the merry-go-round about its axis of rotation.

[0.1052632] ______ $kg \cdot m^2$

(b) Calculate the magnitude of the angular momentum of the child, while running, about the axis of rotation of the merry-go-round.

[0.1052632] ______ $kg \cdot m^2/s$

(c) Calculate the angular speed of the merry-go-round and child after the child has jumped on.

[0.1052632] ______ rad/s

7. HRW6 12.P.044. [51882] The rotational inertia of a collapsing spinning star changes to one-third its initial value. What is the ratio of the new rotational kinetic energy to the initial rotational kinetic energy?

- 1:9
- 1:3
- 3:1
- 9:1 [0.1052632]

2. HRW6 12.P.009. [51853] A homogeneous sphere starts from rest at the upper end of the track shown in Fig. 12-32 and rolls without slipping until it rolls off the right-hand end. If H = 11.0 m and h = 4.0 m and the track is horizontal at the right-hand end, how far horizontally from point A does the sphere land on the floor?

[0.1052632] [ ] m

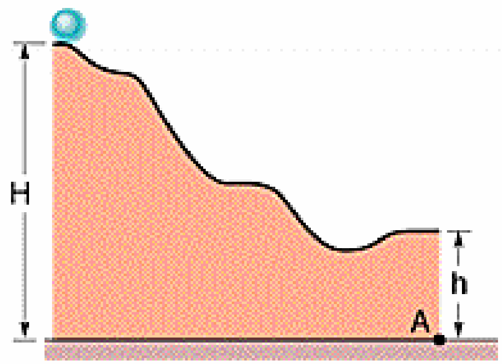


Figure 12-32