Announcements

1. Second exam scheduled for Oct. 28th -- practice exams now available --
   http://www.wfu.edu/~natalie/f03phy113/extrapractice/

2. Thursday – review of Chapters 9-14

3. Today’s lecture –
   
   Universal law of gravitation
   Gravity near the planet’s surface
   Gravitational potential energy
   Planetary and satellite motion
Newton’s law of gravitation: \( m_2 \) attracts \( m_1 \) according to:

\[
F_{12} = \frac{G m_1 m_2 \hat{r}_{12}}{r_{12}^2}
\]

where \( G = 6.67 \times 10^{-11} \) N m\(^2\)/kg\(^2\).

Example: \( m_1 = m_2 = 70 \) kg; \( r_{12} = 2 \) m:

\[
F = \frac{6.67 \times 10^{-11} \cdot 70 \cdot 70}{2^2} \quad N = 8.17 \times 10^{-8} \quad N
\]
Vector nature of Gravitational law:

\[ \mathbf{F}_1 = G \frac{m_1}{d^2} (m_2 \mathbf{i} + m_3 \mathbf{j}) \]
Gravitational force of the Earth

\[ F = \frac{GM_E m}{R_E^2} \]

\[ \Rightarrow g = \frac{GM_E}{R_E^2} = \frac{6.67 \times 10^{-11} \cdot 5.98 \times 10^{24}}{(6.37 \times 10^6)^2} \text{ m/s}^2 = 9.8 \text{ m/s}^2 \]
Question:

Suppose you are flying in an airplane at an altitude of 35000 ft~11 km above the Earth’s surface. What is the acceleration due to Earth’s gravity?

\[ F = \frac{G M_E m}{(R_E + h)^2} = ma \]

\[ a = \frac{G M_E}{(R_E + h)^2} = \frac{6.67 \times 10^{-11} \cdot 5.98 \times 10^{24}}{((6.37 + 0.011) \times 10^6)^2} \text{ m/s}^2 = 9.796 \text{ m/s}^2 \]

\[ a/g = 0.997 \]
Attraction of moon to the Earth:

\[ F = \frac{G M_E M_M}{R_{EM}^2} = \frac{6.67 \times 10^{-11} \cdot 5.98 \times 10^{24} \cdot 7.36 \times 10^{22}}{(3.84 \times 10^8)^2} N = 1.99 \times 10^{20} N \]

Acceleration of moon toward the Earth:

\[ F = M_M a \ \Rightarrow \ a = \frac{1.99 \times 20^{20} N}{7.36 \times 10^{22} \text{kg}} = 0.0027 \text{ m/s}^2 \]
Stable circular orbit of two gravitationally attracted objects (such as the moon and the Earth)

\[ a = \frac{v^2}{R_{EM}} = \frac{GM_E}{R_{EM}^2} \]

\[ v = \omega R_{EM} = \frac{2\pi}{T} R_{EM} \]

\[ T = 2\pi \sqrt{\frac{R_{EM}^3}{GM_E}} \]

\[ = 2\pi \sqrt{\frac{(3.84 \times 10^8)^3}{6.67 \times 10^{-11} \cdot 5.98 \times 10^{24}}} \]

\[ = 2367353.953 \text{ s} = 27.4 \text{ days} \]
Peer instruction question

In the previous discussion, we saw how the moon orbits the Earth in a stable circular orbit because of the radial gravitational attraction of the moon and Newton’s second law: $F=ma$, where $a$ is the centripetal acceleration of the moon in its circular orbit. Is this the same mechanism which stabilizes airplane travel? Assume that a typical cruising altitude of an airplane is 11 km above the Earth’s surface and that the Earth’s radius is 6370 km.

(a) Yes          (b) No
Stable (??) circular orbit of two gravitationally attracted objects (such as the airplane and the Earth)

\[ \frac{a}{R_{Ea}^2} \approx \frac{GM_E}{2GM_E} = \frac{v^2}{2R_{Ea}} \]

\[ \approx \frac{10^3}{7.9} \times 10^5 \text{ km/s} \approx 1.4 \times 10^8 \text{ km/s} \]
Newton’s law of gravitation:  \[ \mathbf{F}_{12} = \frac{Gm_1 m_2 \mathbf{r}_{12}}{r_{12}^2} \]

Earth’s gravity:

\[ F = \frac{GM_Em}{R_E^2} \]

\[ \Rightarrow g = \frac{GM_E}{R_E^2} = \frac{6.67 \times 10^{-11} \cdot 5.98 \times 10^{24}}{(6.37 \times 10^6)^2} \text{m/s}^2 = 9.8 \text{m/s}^2 \]

Stable circular orbits of gravitational attracted objects:
More details

If we examine the circular orbit more carefully, we find that the correct analysis is that the stable circular orbit of two gravitationally attracted masses is about their center of mass.
Radial forces on $m_1$:

$$F_{r1} = \frac{Gm_1 m_2}{(R_1 + R_2)^2} = m_1 a_{r1} = m_1 \frac{\nu_1^2}{R_1}$$

$$\nu_1 = \frac{2\pi R_1}{T_1}$$

$$T_1 = 2\pi \sqrt{\frac{R_1 (R_1 + R_2)^2}{Gm_2}}$$

$T_2$ ?

Tangential forces ?
Peer instruction question

What is the relationship between the periods $T_1$ and $T_2$ of the two gravitationally attracted objects rotating about their center of mass? (Assume that $m_1 < m_2$.)

(A) $T_1 = T_2$  (B) $T_1 < T_2$  (C) $T_1 > T_2$
\[ m_1 R_1 = m_2 R_2 \]
\[ m_1 \frac{v_1^2}{R_1} = m_1 R_1 \omega_1^2 = \frac{G m_1 m_2}{(R_1 + R_2)^2} = m_2 R_2 \omega_2^2 \]
\[ \Rightarrow \omega_1 = \omega_2 \]

\[ T_1 = T_2 = 2\pi \sqrt{\frac{(R_1 + R_2)^3}{G(m_1 + m_2)}} \]
What is the physical basis for stable circular orbits?

1. Newton’s second law? \( F = ma = \frac{dp}{dt} \)

2. Conservation of mechanical energy? \( E = K + U = (\text{const}) \)

3. Conservation of linear momentum? \( p = (\text{const}) \)

4. Torqued motion? \( \tau = I \alpha \) ?

5. Conservation of angular momentum? \( L = (\text{const}) \)
\[ \tau = \frac{dL}{dt} = 0 \]
\[ \Rightarrow L = \text{(const)} \]

\[ L_1 = m_1 v_1 R_1 \]
\[ L_2 = m_2 v_2 R_2 \]

**Question:**

How are the magnitudes of \( L_1 \) and \( L_2 \) related?
The potential energy associated with the gravitational force.

\[
U(r) = - \int_{r_{ref}}^{r} \frac{Gm_1 m_2}{r^2} dr = - \int_{\infty}^{r} \frac{Gm_1 m_2}{r^2} dr = - \frac{Gm_1 m_2}{r}
\]
Total mechanical energy for circular orbits:

(assume $M \gg m$)

\[ E = \frac{1}{2} mv^2 + U(r) \]

\[ \frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v^2 = \frac{GM}{r} \]

\[ U(r) = -\frac{GMm}{r} \]

\[ E = -\frac{GMm}{2r} \]
Peer instruction question

What is wrong with the previous analysis?

A. Nothing is wrong. (The description of circular motion due to gravitational attraction is complete.)

B. E depends on r and therefore must not be constant.

C. E can only be constant if r is constant, but it is not obvious why r is constant.

D. Conservation of angular moment will come to the rescue.
Angular momentum: \( \mathbf{L} = \mathbf{r} \times m\mathbf{v} \)

For circular orbit:

\[
L = R_{ME} M_M v \\
v = \frac{L}{M_M R_{EM}} \\
K = \frac{1}{2} M_M v^2 \\
= \frac{L^2}{2 M_M R_{EM}^2}
\]
\[ \frac{L^2}{2mr^2} \]

E for elliptical orbit

E for circular orbit

U(r)
Circular orbit:
\[ \frac{x^2}{r_0^2} + \frac{y^2}{r_0^2} = 1 \]

Elliptical orbit:
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]
Satellites orbiting earth (approximately circular orbits):

\[ T = 2\pi \sqrt{\frac{R_E^3}{GM_E}} (1 + h / R_E)^{3/2} = 5058(1 + h / R_E)^{3/2} \text{ s} \]

\( R_E \sim 6370 \text{ km} \)

Examples:

<table>
<thead>
<tr>
<th>Satellite</th>
<th>h (km)</th>
<th>T (hours)</th>
<th>v (mi/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geosynchronous</td>
<td>35790</td>
<td>~24</td>
<td>6900</td>
</tr>
<tr>
<td>NOAA polar orbiter</td>
<td>800</td>
<td>~1.7</td>
<td>16700</td>
</tr>
<tr>
<td>Hubble</td>
<td>600</td>
<td>~1.6</td>
<td>16900</td>
</tr>
<tr>
<td>Inter. space station*</td>
<td>390</td>
<td>~1.5</td>
<td>17200</td>
</tr>
</tbody>
</table>

*Link: [http://liftoff.msfc.nasa.gov/temp/StationLoc.html](http://liftoff.msfc.nasa.gov/temp/StationLoc.html)
Sample question:

Suppose that the space shuttle (m=10^5 kg) was initially in the same orbit as the International space station (h_i=390 km) and the engines are fired to give it exactly the amount of energy ΔW to raise it to the same orbit as the Hubble space telescope (h_f=600 km). What is the energy ΔW?

You can show that the energy of a satellite of mass m in a circular orbit of height h above the Earth’s surface is given by:

\[ E_{mech} = K + U = -\frac{GMmE^2}{2(R_E + h)} \]

\[ \Delta W = -\frac{GMmE^2}{2R_E} \left( \frac{1}{1 + \frac{h_f}{R_E}} - \frac{1}{1 + \frac{h_i}{R_E}} \right) = 89,000 J \]