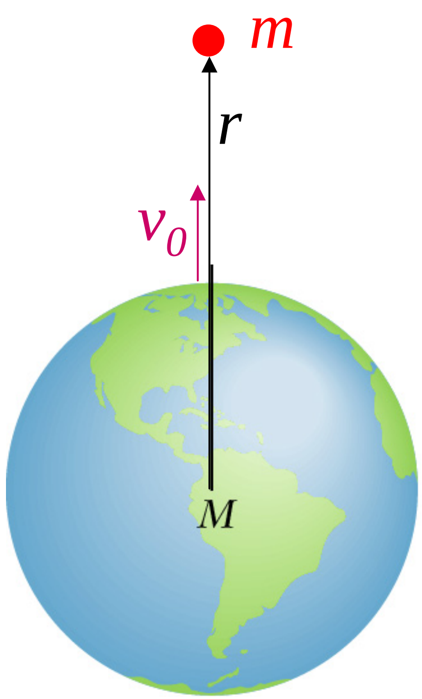
Announcements

1. Remember -- Tuesday, Oct. 28th, 9:30 AM – Second exam (covering Chapters 9-14 of HRW) – Bring the following:
   a) 1 equation sheet
   b) Calculator
   c) Pencil
   d) Clear head
   e) Note: If you have kept up with your HW, you may drop your lowest exam grade
2. Today --Thursday, Oct. 23th, 4 PM – Physics Colloquium by Professor Bernd Schüttler, Dept. of Physics, U. Ga – will discuss the analysis of biological systems in terms of a physical and mathematical model
3. Today's lecture – review Chapters 9-14, problem solving techniques

Gravitational forces and energy



Potential energy: $U(r) = -\frac{GMm}{r}$

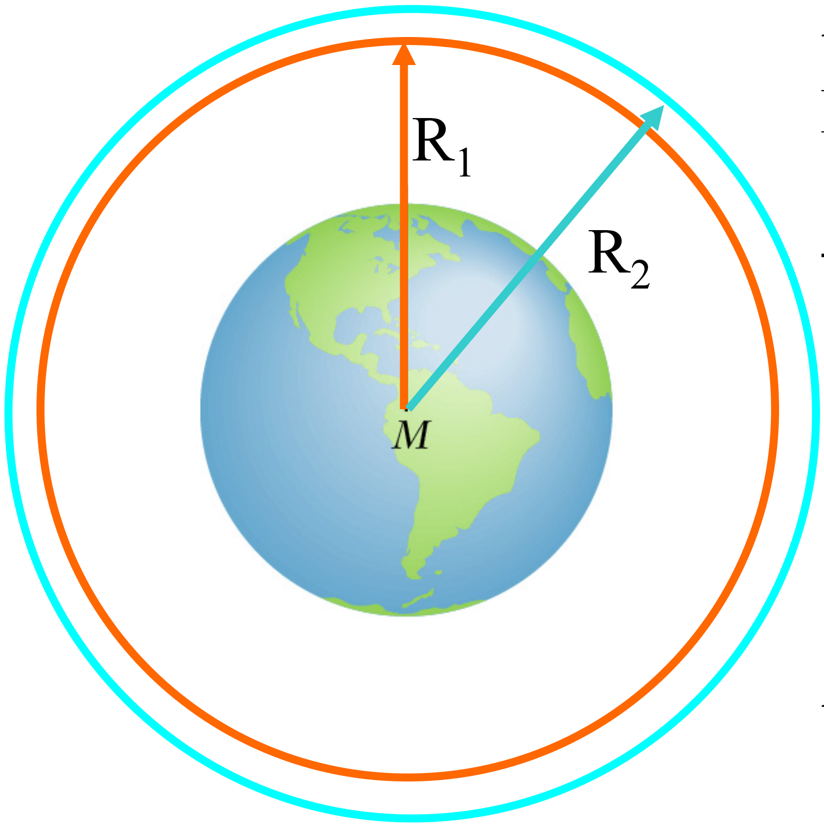
Energy needed to escape Earth's gravitational field, assuming an initial velocity v_0 :

$$K_i + U_i = K_f + U_f$$

$$\tfrac{1}{2} m v_0^2 - \frac{GMm}{R_E} = 0$$

$$\Rightarrow v_0 = \sqrt{\frac{2GM}{R_E}} \approx 25000 \text{ km/h}$$

Energy needed to go from one stable circular orbit to another:



$$E = K + U$$

From Newton's law for cirular orbit :

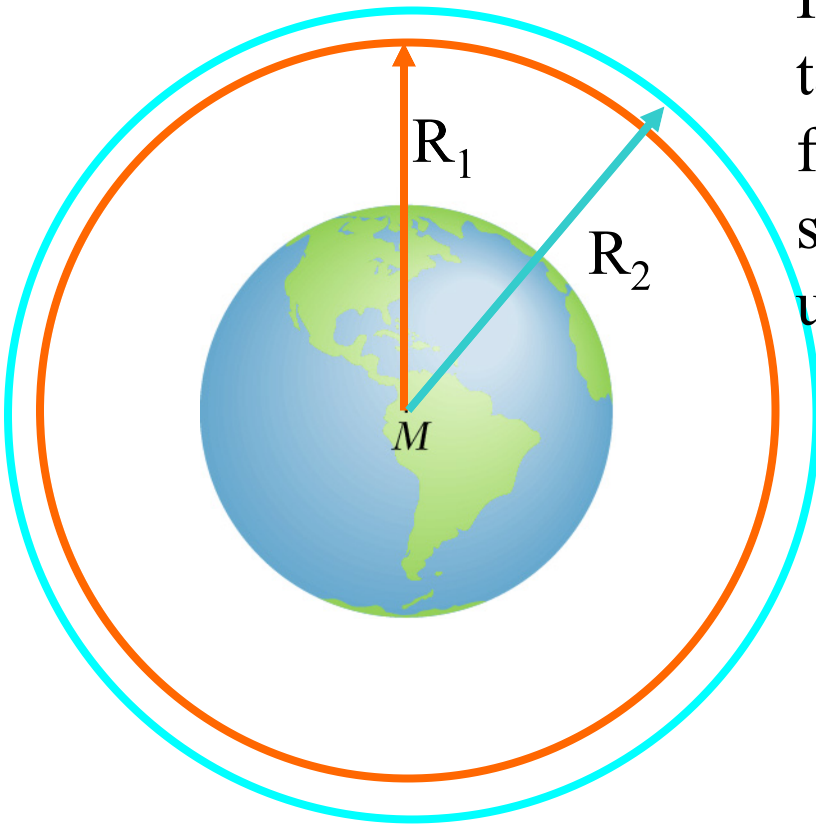
$$-m\frac{v^2}{r} = -\frac{GMm}{r^2} \quad \Rightarrow mv^2 = \frac{GMm}{r}$$

$$\Rightarrow E = -\frac{GMm}{2r}$$

$$E_2 - E_1 = -\frac{GMm}{2R_2} + \frac{GMm}{2R_1}$$

Energy needed to go from one stable circular orbit to another --

Example:



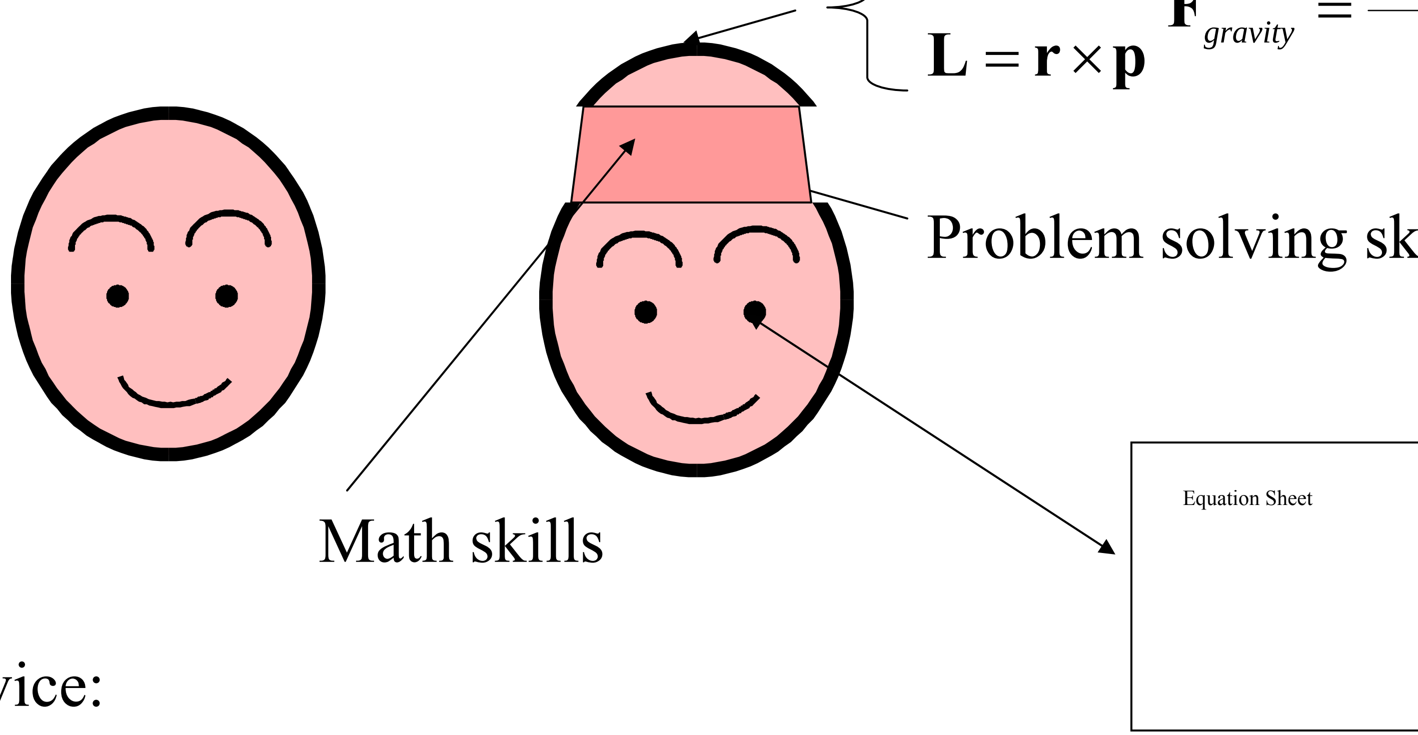
How much energy is needed to take a satellite of mass m=100kg from the international space station (R_1=R_E+390 km) to its usual orbit (R_2=R_E+600 km)?

$$E_2 - E_1 = -\frac{GMm}{2}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

$$\approx 9 \times 10^7 \, J$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$
$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

$$\mathbf{F}_{gravity} = \frac{Gm_1m_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$



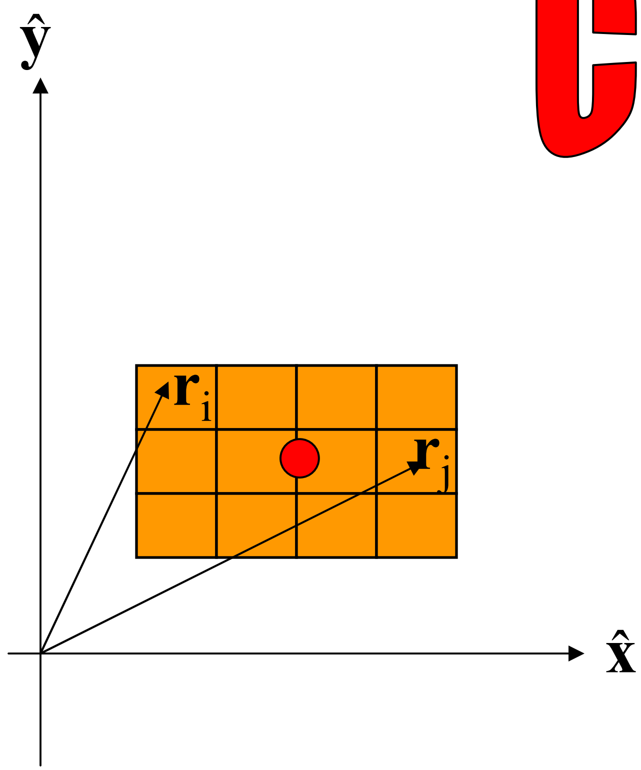
Advice:

1. Keep basic concepts and equations at the top of your head.
2. Practice problem solving and math skills
3. Develop an equation sheet that you can consult.

Problem solving steps

1. Visualize problem – labeling variables
2. Determine which basic physical principle applies
3. Write down the appropriate equations using the variables defined in step 1.
4. Check whether you have the correct amount of information to solve the problem (same number of knowns and unknowns.
5. Solve the equations.
6. Check whether your answer makes sense (units, order of magnitude, etc.).

Center of mass



$$\mathbf{r}_{COM} \equiv \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}$$

Position of the center of mass: $$\mathbf{r}_{com} \equiv \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}$$

Velocity of the center of mass: $$\mathbf{v}_{com} \equiv \frac{\sum_i m_i \mathbf{v}_i}{\sum_i m_i}$$

Acceleration of the center of mass: $$\mathbf{a}_{com} \equiv \frac{\sum_i m_i \mathbf{a}_i}{\sum_i m_i}$$

Physics of composite systems:

$$\sum_i \mathbf{F}_i = \sum_i m_i \mathbf{a}_i = \sum_i \frac{dm_i \mathbf{v}_i}{dt} = \sum_i \frac{d\mathbf{p}_i}{dt}$$

Center-of-mass velocity:

$$\mathbf{v}_{com} \equiv \frac{\sum_i m_i \mathbf{v}_i}{\sum_i m_i} \equiv \frac{\sum_i m_i \mathbf{v}_i}{M}$$

Note that:

$$\sum_i \mathbf{F}_i \equiv \mathbf{F}_{total} = M \frac{d\mathbf{v}_{\text{com}}}{dt}$$

A new way to look at Newton's second law:

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt} \equiv \frac{d\mathbf{p}}{dt}$$

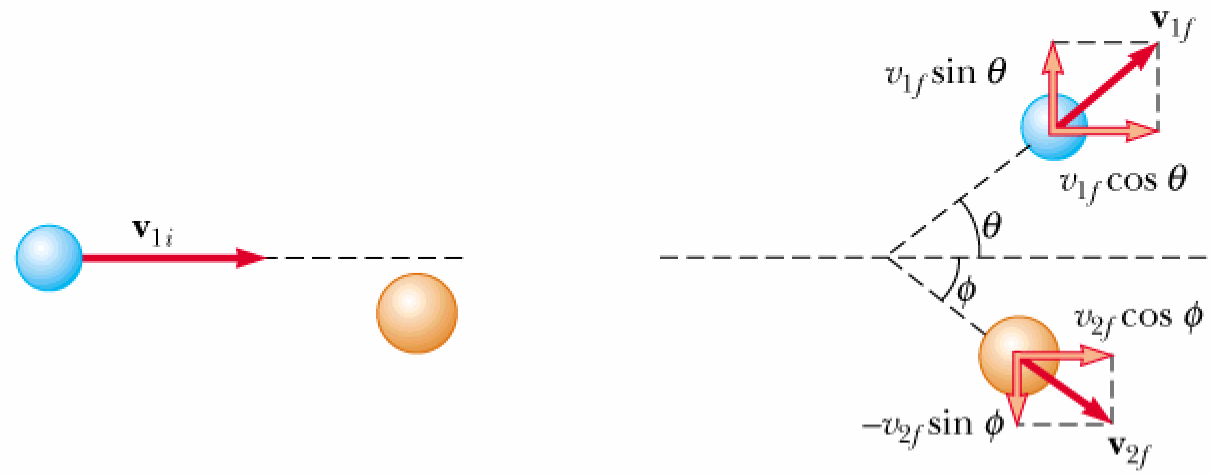
Define linear momentum **p** = m**v**

Consequences:

1. If $\mathbf{F} = 0$ ➔ $\frac{d\mathbf{p}}{dt} = 0$ ➔ $\mathbf{p}$ = constant

2. For system of particles: $\sum_i \mathbf{F}_i = \sum_i \frac{d\mathbf{p}_i}{dt}$

   If $\sum_i \mathbf{F}_i = 0 \quad \Rightarrow \sum_i \frac{d\mathbf{p}_i}{dt} = 0 \quad \Rightarrow \sum_i \mathbf{p}_i = \text{constant}$



(a) Before the collision (b) After the collision

Statement of conservation of momentum:

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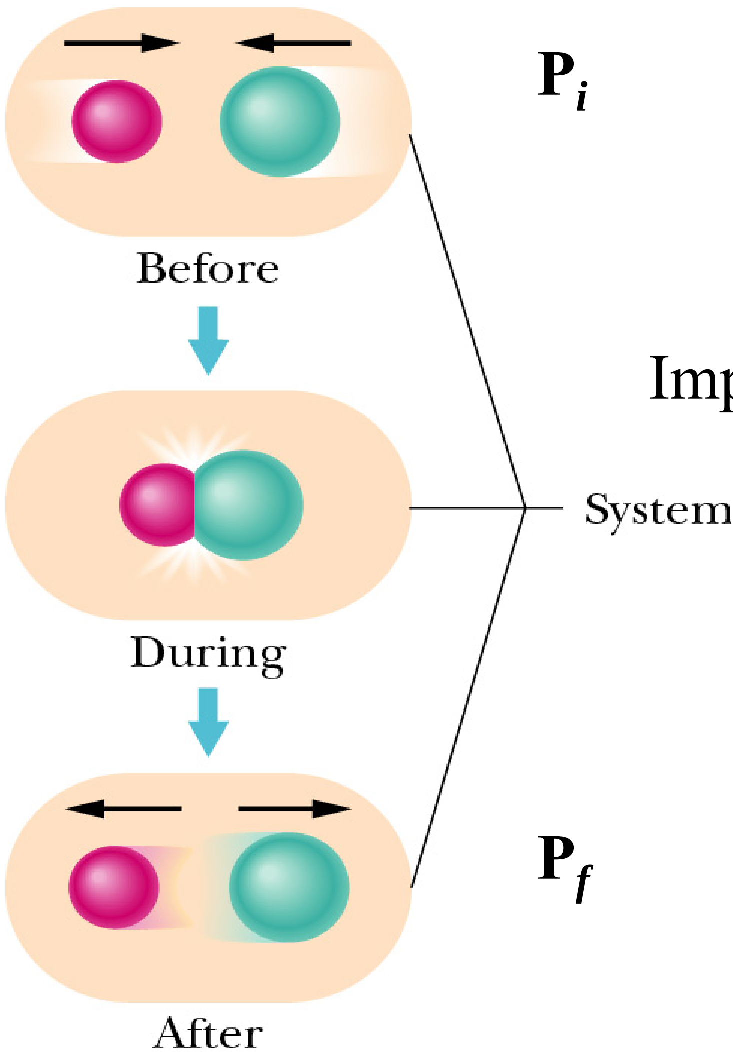
$$m_1 v_{1i} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\varphi$$

$$0 = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\varphi$$

If mechanical (kinetic) energy is conserved, then:

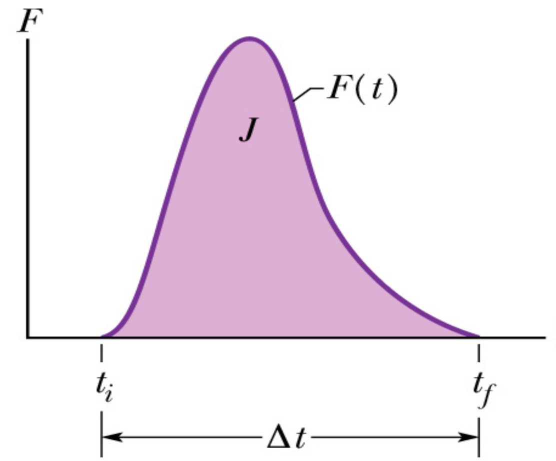
$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

Snapshot of a collision:

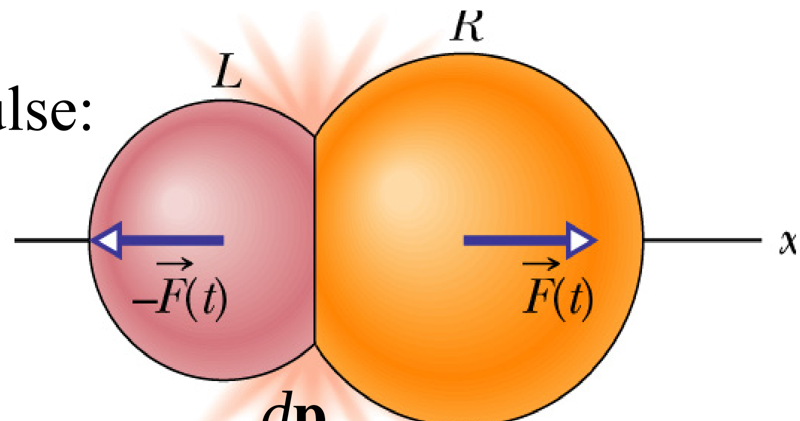


$\mathbf{P}_i$

$\mathbf{P}_f$



Impulse:



$$\mathbf{F}(t) = \frac{d\mathbf{p}}{dt} \Rightarrow d\mathbf{p} = \mathbf{F}(t)dt$$

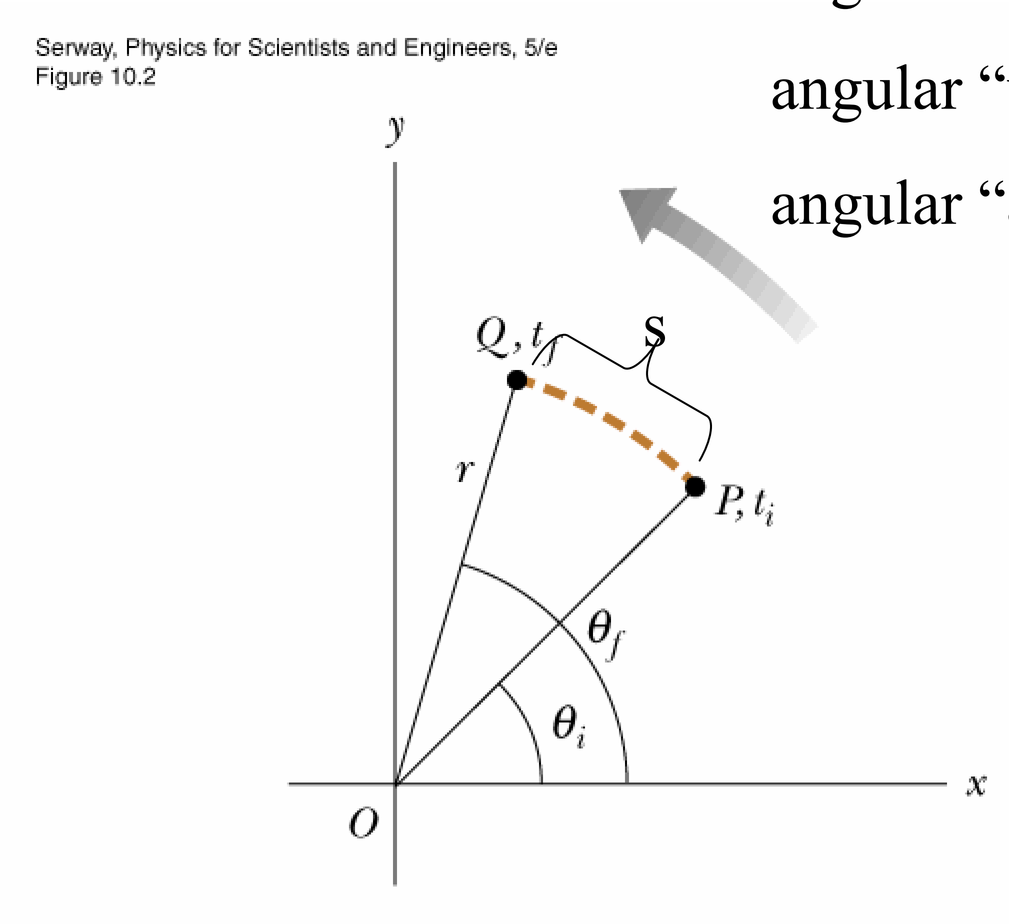
$$\int_{t_1}^{t_2} d\mathbf{p} = \int_{t_1}^{t_2} \mathbf{F}(t)dt \equiv \mathbf{J}$$

Angular motion

angular "displacement" ➔ $\theta(t)$

angular "velocity" ➔ $\omega(t) = \frac{d\theta}{dt}$

angular "acceleration" ➔ $\alpha(t) = \frac{d\omega}{dt}$



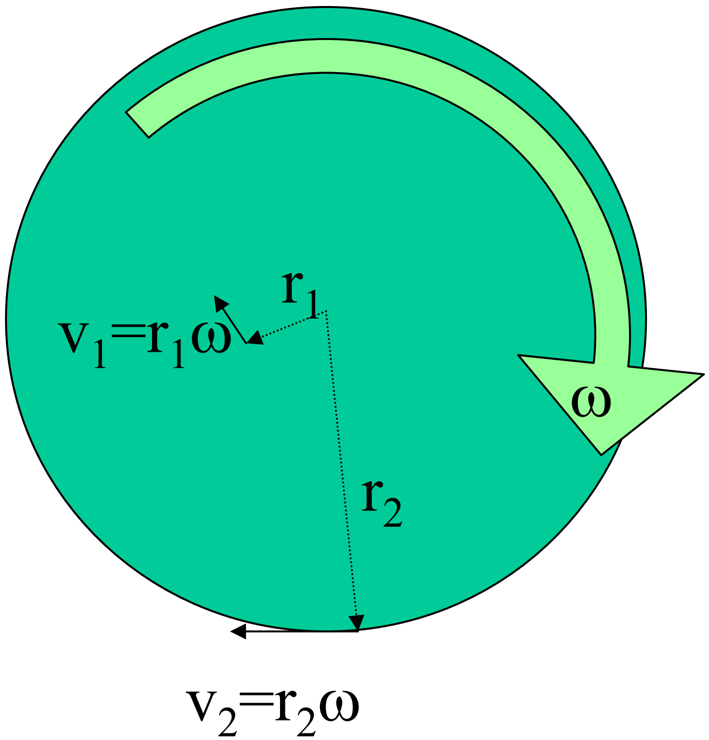
"natural" unit == 1 radian

Relation to linear variables:

$s_\theta = r\,(\theta_f - \theta_i)$

$v_\theta = r\,\omega$

$a_\theta = r\,\alpha$



Special case of constant angular acceleration: $\alpha = \alpha_0$:

$$\omega(t) = \omega_i + \alpha_0\, t$$

$$\theta(t) = \theta_i + \omega_i\, t + \tfrac{1}{2}\, \alpha_0\, t^2$$

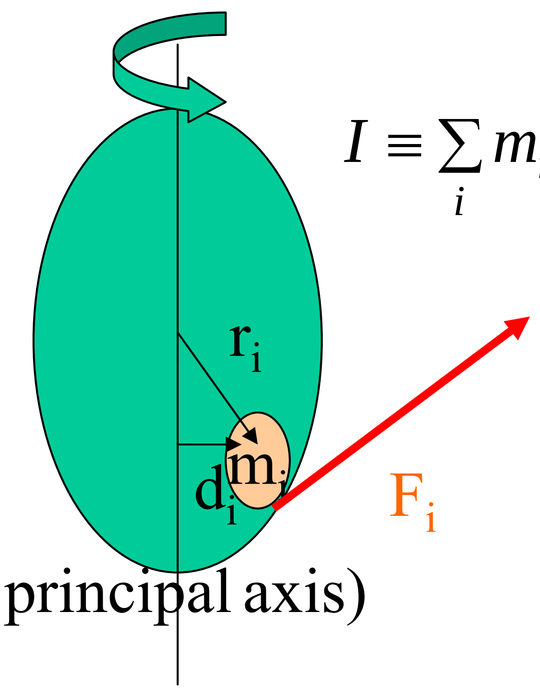
$$(\omega(t))^2 = \omega_i^2 + 2\, \alpha_0\, (\theta(t) - \theta_i)$$

Newton's second law applied to center-of-mass motion

$$\sum_i \mathbf{F}_i = \sum_i m_i \frac{d\mathbf{v}_i}{dt} \Rightarrow \mathbf{F}_{total} = M\frac{d\mathbf{v}_{CM}}{dt}$$

Newton's second law applied to rotational motion

$$\mathbf{F}_i = m_i \frac{d\mathbf{v}_i}{dt} \Rightarrow \mathbf{r}_i \times \mathbf{F}_i = \mathbf{r}_i \times m_i \frac{d\mathbf{v}_i}{dt}$$



$$I \equiv \sum_i m_i d_i^2$$

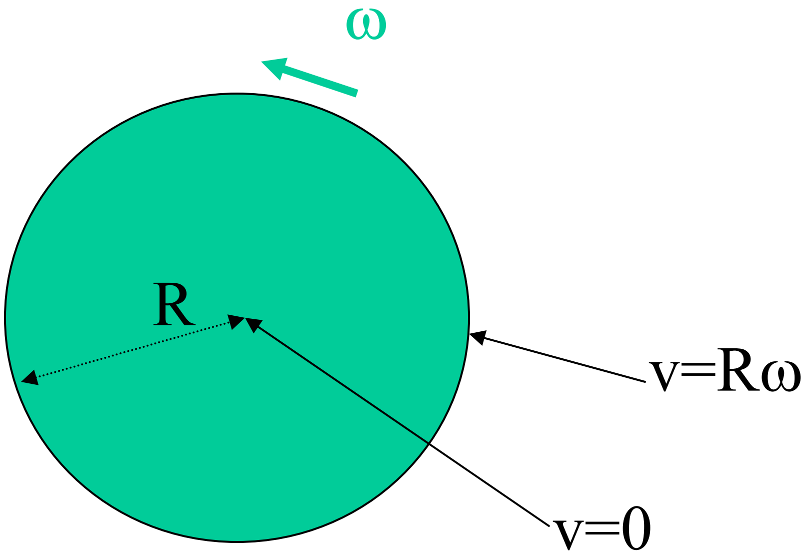
$$\boldsymbol{\tau}_i = \mathbf{r}_i \times \mathbf{F}_i$$

$$\mathbf{v}_i = \boldsymbol{\omega} \times \mathbf{r}_i$$

$$\Rightarrow \boldsymbol{\tau}_i = m_i \mathbf{r}_i \times \frac{d(\boldsymbol{\omega} \times \mathbf{r}_i)}{dt}$$

$$\Rightarrow \boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = I\boldsymbol{\alpha}$$ (for rotating about principal axis)

Object rotating with constant angular velocity ($\alpha = 0$)



Kinetic energy associated with rotation:

$$K = \sum_i \tfrac{1}{2} m_i v_i^2 = \sum_i \tfrac{1}{2} m_i r_i^2 \omega^2 \equiv \tfrac{1}{2} I \omega^2;$$

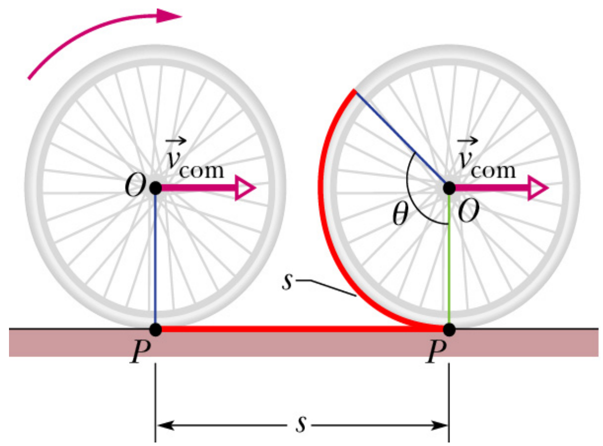
$$\text{where}: I \equiv \sum_i m_i r_i^2$$ "moment of inertia"

Kinetic energy associated with rolling without slipping:

$$K_{rot} = \tfrac{1}{2} I\omega^2 \qquad I \equiv \sum_i m_i r_i^2$$

Distance to axis of rotation

Rolling:



$$K_{tot} = K_{com} + K_{rot}$$

If there is no slipping : $v_{com} = R\omega$

$$\Rightarrow K_{tot} = \tfrac{1}{2} M\left(1 + \frac{I}{MR^2}\right) v_{com}^2$$

Torque and angular momentum

Define angular momentum: $\mathbf{L} \equiv \mathbf{r} \times \mathbf{p}$

For composite object: L = Iω

Newton's law for torque:

$$\boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = \frac{d\mathbf{L}}{dt}$$

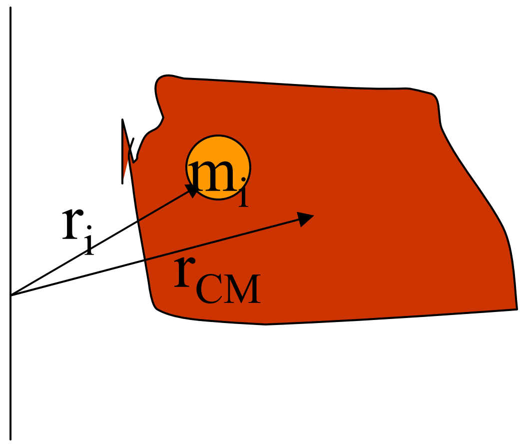
➔ If $\tau_{total} = 0$ then $\mathbf{L}$ = constant

In the absence of a net torque on a system,

angular momentum is conserved.

Center-of-mass $\mathbf{r}_{CM} \equiv \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}$

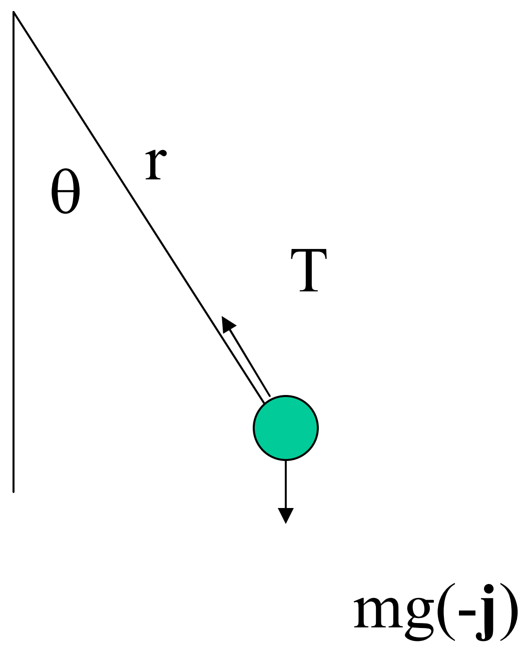
Torque on an extended object due to gravity (near surface of the earth) is the same as the torque on a point mass M located at the center of mass.



$$\boldsymbol{\tau} = \sum_i \mathbf{r}_i \times \{m_i g(-\mathbf{j})\} = \mathbf{r}_{CM} \times \{Mg(-\mathbf{j})\}$$

Notion of equilibrium: $\sum_i \mathbf{F}_i = 0$ $\sum_i \boldsymbol{\tau}_i = 0$

Notion of stability:



$\mathbf{F}=m\mathbf{a}$ ➔ $T - mg\cos\theta = 0$

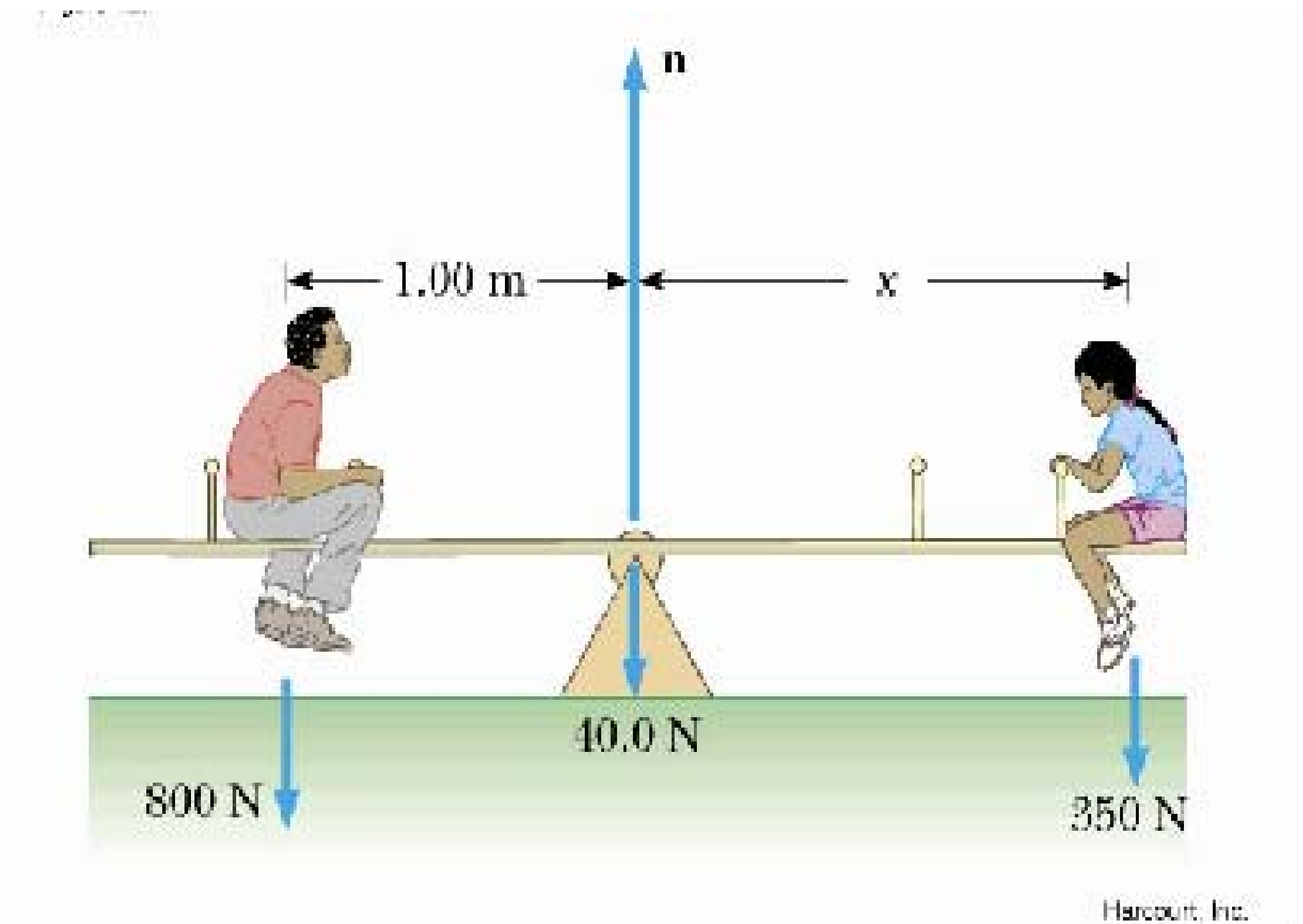
$-mg\sin\theta = -ma_\theta$

$\tau = I\alpha$ ➔ $r\,mg\sin\theta = mr^2\alpha = mra_\theta$

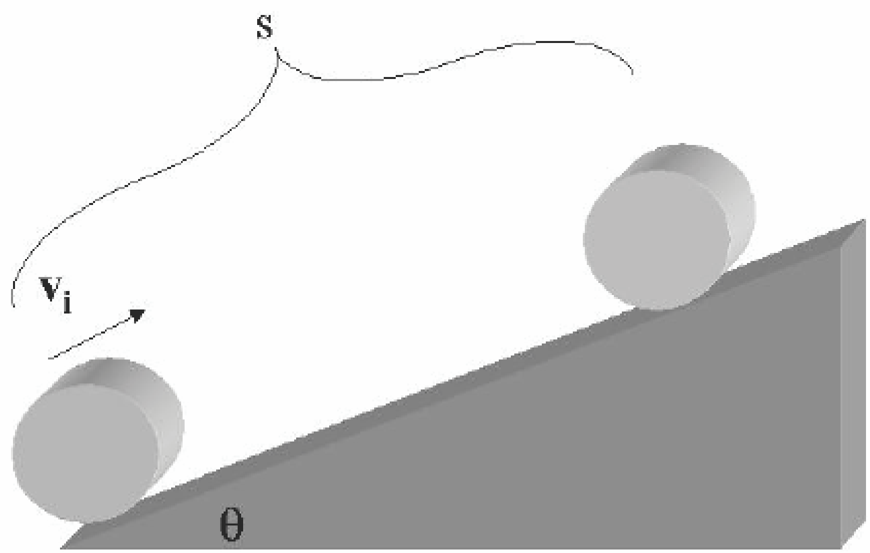
Example of stable equilibrium.

Analysis of stability:

$$\sum_i \mathbf{F}_i = 0 \qquad \sum_i \boldsymbol{\tau}_i = 0$$

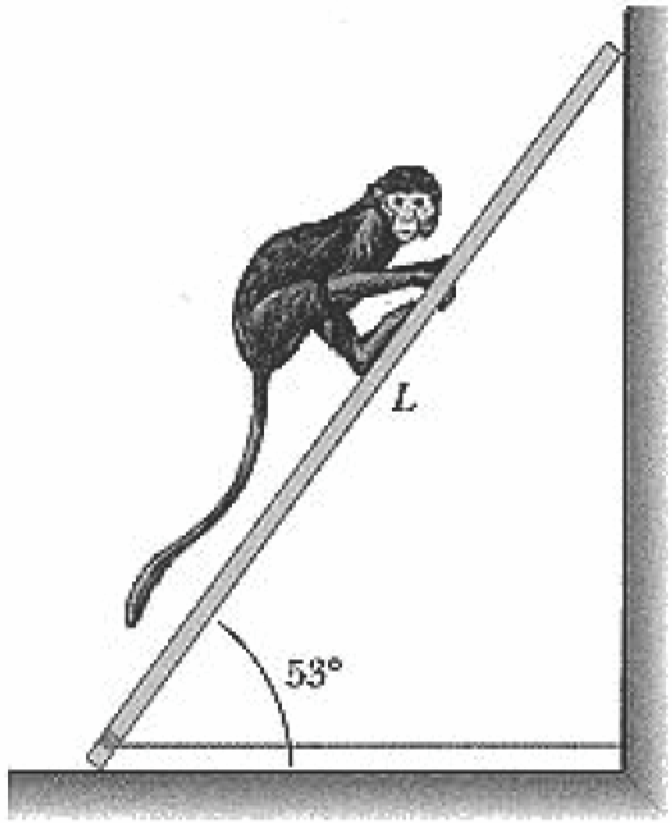


6.

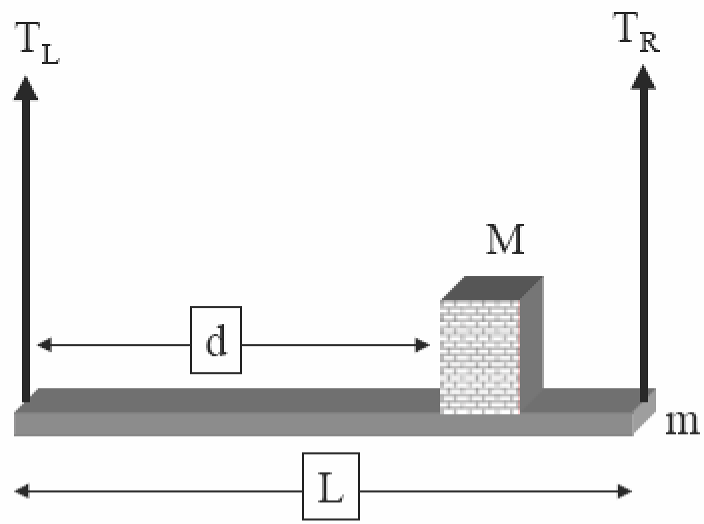


In the figure on the left, the inclined plane is assumed to be stationary with $\theta = 12^o$. The object, having a cylindrical shape with a radius of $r = 0.05$ m, a total mass $M = 10$ kg, and a moment of inertia of $I = 0.02$ kg·m^2, starts at the bottom of the incline with an inital speed of $v_i = 0.5$ m/s and rolls *without slipping* up the incline to a maximum distance s, before rolling back down. What is the distance s? (Note: the mass in the cylindrical object is distributed non-uniformly.)

8.



The figure shown on left illustrates a monkey having a mass of $M_M = 20$ kg which is climbing up a ladder which has a uniformly distributed mass of $M_L = 10$ kg and a length of $L = 3$ m. Suppose that both the floor and wall which support the ladder and monkey are frictionless, but that the bottom of the ladder is held by a horizontal rope fastened to the wall as shown. The ladder makes an angle of 53^o with respect to the floor. Find the tension in the rope when the monkey is has climbed a distance $\frac{2}{3}L$ as measured from the bottom.



6. The figure above shows a system in static equilibrium consisting of a plank with mass $m = 30$ kg, distributed uniformly, and length $L = 2m$ and a box having mass $M = 100$ kg, placed a distance $d = 1.5m$ from the left side. The system is supported by two massless ropes with tensions T_L and T_R. Find the magnitudes of the two tensions T_L and T_R.

7. The figure above shows a before and after picture of a collision process, where $m_1 = 2$ kg and $m_2 = 1$ kg. The magnitudes of the velocities before the collision are measured to be $v_{1i} = 5m/s$ and $v_{2i} = 10m/s$, while the magnitudes of the velocities after the collision are measured to be $v_{1f} = 8.619m/s$ and $v_{2f} = 2.619m/s$. Using this data, determine whether momentum was conserved during this collision. Discuss the implications of your results.

8. The International Space Station makes a complete orbit about the Earth once every $5500 s$. If the orbit were exactly circular, what would be the corresponding centripetal acceleration?