Announcements

1. **Tests will be available Tuesday – Nov. 4th**
2. **Chapter 15 – fluids will be discussed on Tuesday – Nov. 4th**
3. **Today's lecture –**

   Simple harmonic motion

      Mass connected to a spring

      Pendulum

   Notion of resonance (not treated very well in your text)

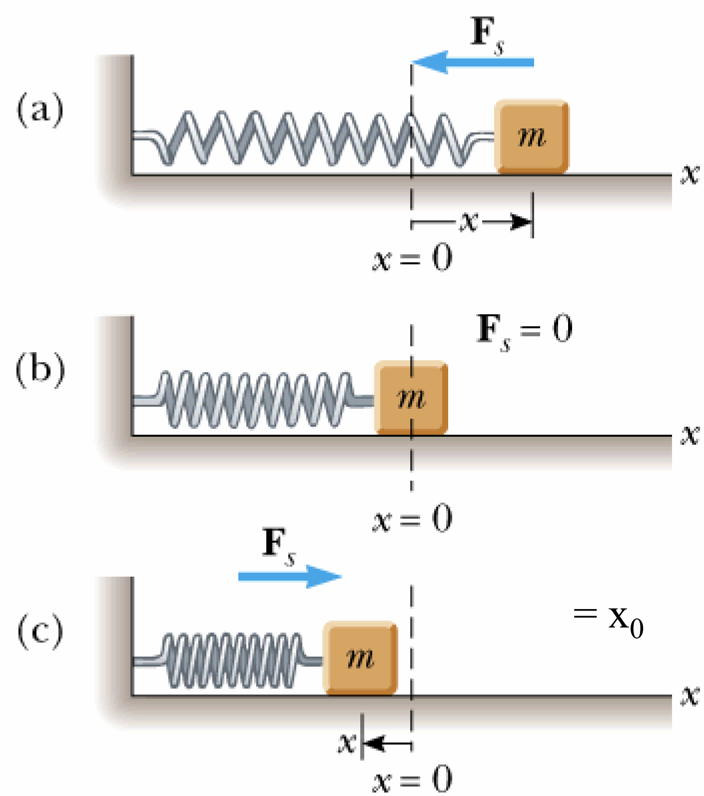
(Note: no physics seminar this week due to Project Pumpkin)

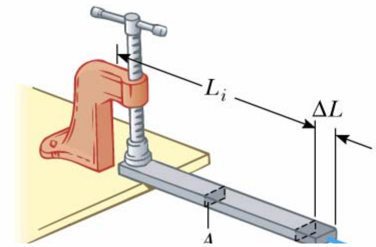
Behavior of materials:

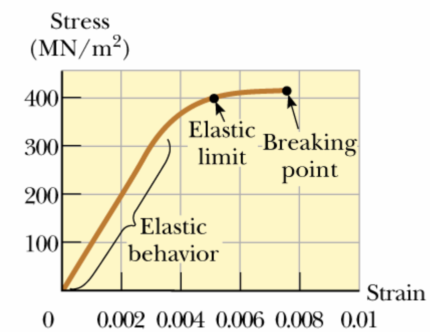
Hooke's law

$$F_s = -k(x-x_0)$$

Serway, Physics for Scientists and Engineers, 5/e
Figure 13.1







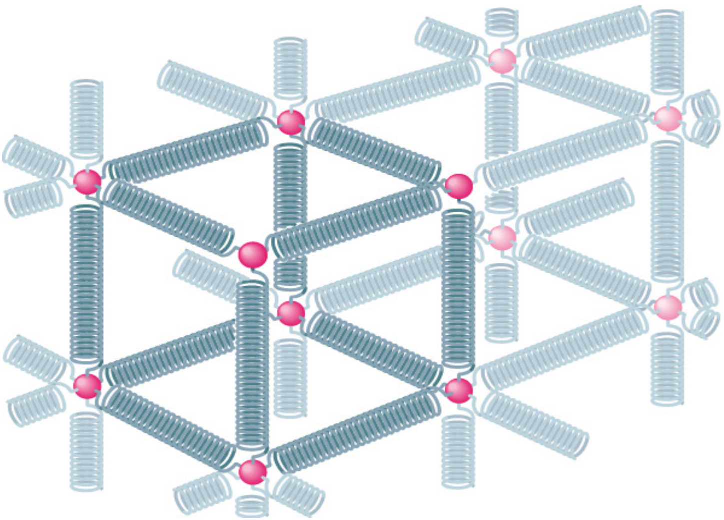
Young's modulus

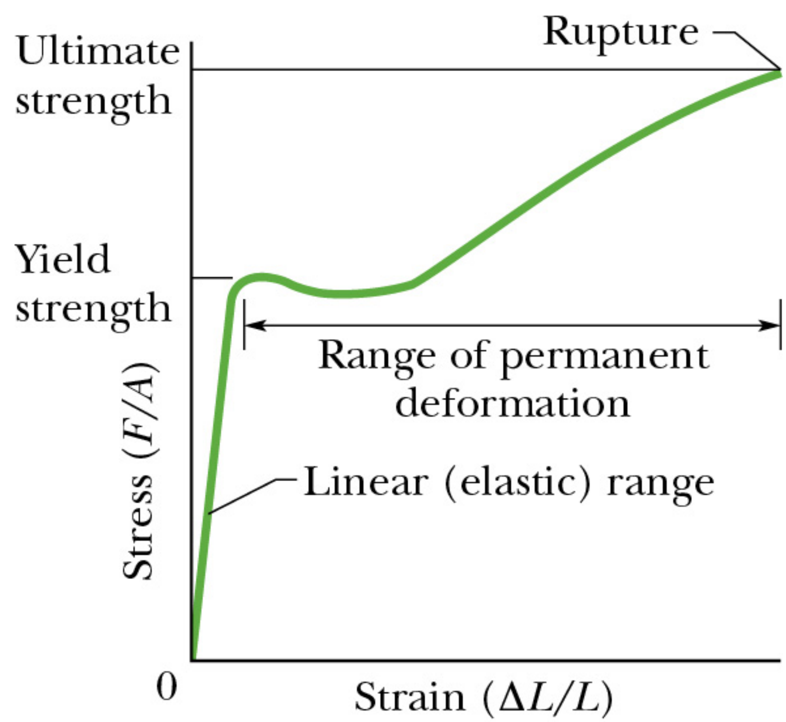
$$E = \frac{F_{applied} / A}{\Delta L / L}$$

$$F_{material} = -F_{applied}$$

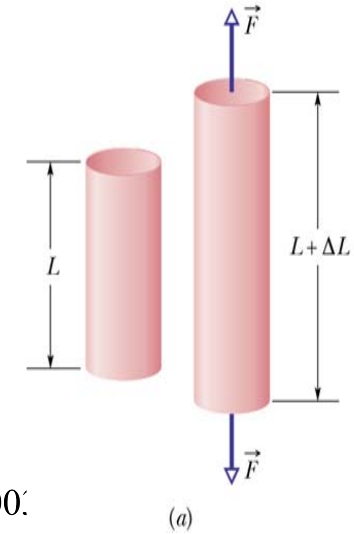
$$\Rightarrow F_{material} = -\left(\frac{EA}{L}\right)\Delta L$$

Microscopic picture of material with springs representing bonds between atoms





Measurement of elastic response:

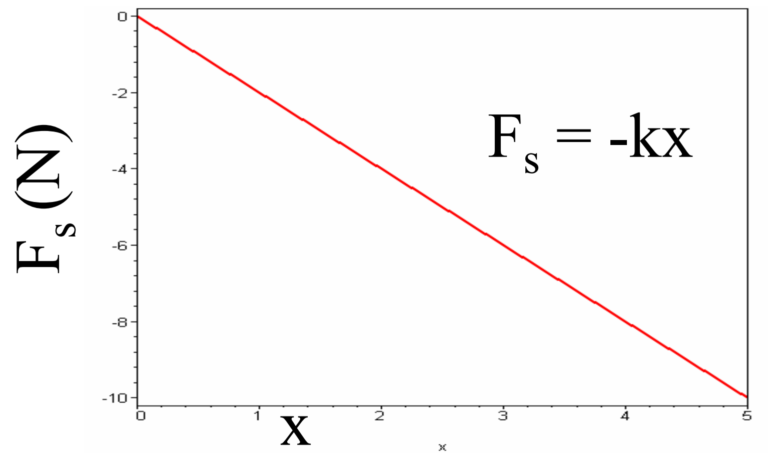


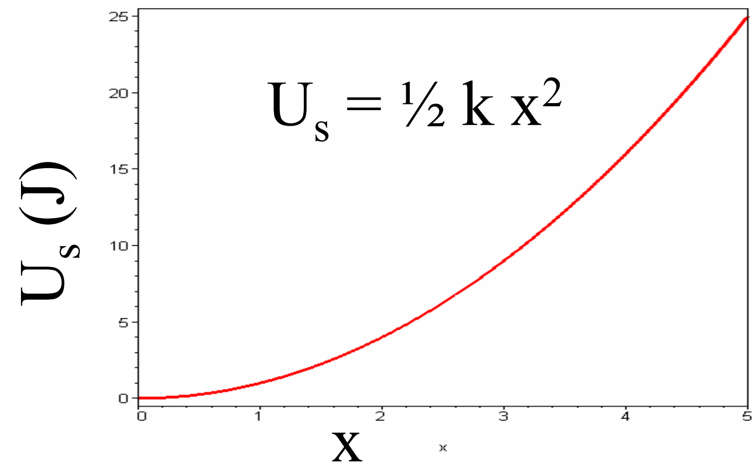
$$E = \frac{F_{applied} / A}{\Delta L / L}$$

$$F_{material} = -F_{applied}$$

$$\Rightarrow F_{material} = -\overbrace{\left(\frac{EA}{L}\right)}^{k}\Delta L$$

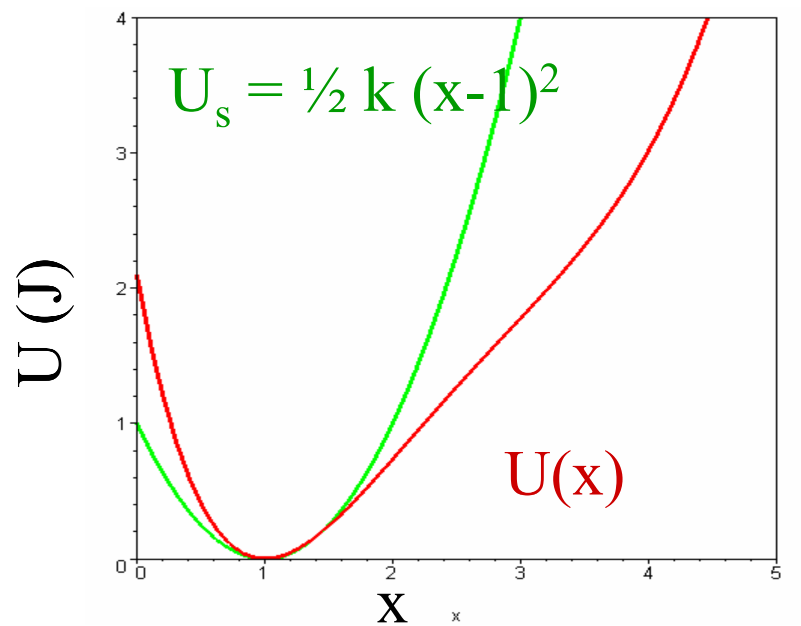
Potential energy associated with Hooke's law:





General potential energy curve:

$$k = \frac{d^2U}{dx^2}(x=1)$$



Motion associated with Hooke's law forces

Newton's second law:

F = -k x = m a

$$F = -kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$ ➔ "second-order" linear differential equation

How to solve a second order linear differential equation:

Earlier example – constant force F_0 ➔ acceleration a_0

$$\frac{d^2x}{dt^2} = \frac{F_0}{m} \equiv a_0$$

$x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2$

2 constants (initial values)

Hooke's law motion:

$$F = -kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Forms of solution:

$$\left.\begin{array}{l} x(t) = A\cos(\omega t + \varphi) \\ x(t) = A\cos(\omega t) + B\sin(\omega t) \end{array}\right\}$$ where: $\omega \equiv \sqrt{\frac{k}{m}}$

2 constants (initial values)

Verification: (Class exercise – write out steps of this "proof")

Differential relations:

$$\frac{d\sin(\omega t+\varphi)}{dt}=\omega\cos(\omega t+\varphi)$$

$$\frac{d\cos(\omega t+\varphi)}{dt}=-\omega\sin(\omega t+\varphi)$$

Therefore: $$\frac{d^2A\cos(\omega t+\varphi)}{dt^2}=-\omega^2\ A\cos(\omega t+\varphi)$$

$$\Rightarrow x(t)=A\cos(\omega t+\varphi) \qquad \textit{satisfies}$$

$$\frac{d^2x}{dt^2}=-\frac{k}{m}x \qquad \textit{provided} \qquad \textit{that} \qquad \omega^2=\frac{k}{m}$$

“Simple harmonic motion” in practice

A block with a mass of 0.2 kg is connected to a light spring for which the force constant is 5 N/m and is free to oscillate on a horizontal, frictionless surface. The block is displaced 0.05 m from equilibrium and released from rest. Find its subsequent motion.

$\omega = \sqrt{k/m} = \sqrt{5/0.2}\text{rad/s} = 5\text{rad/s}$

$x(t) = A\cos(\omega t+\phi)$ $\quad\quad$ $x(0) = A\cos(\phi) = 0.05$ m

$v(t) = -A\omega\sin(\omega t+\phi)$ $\quad\quad$ $v(0) = -A\omega\sin(\phi) = 0$ m/s

➔ $\phi = 0$ and $A = 0.05$ m

Online Quiz for Lecture 15
Simple harmonic motion -- Oct. 30, 2003

- Suppose you have a spring with unknown spring constant k.

1. When you suspend a 50 N weight vertically from the spring, the spring stretches by 0.03 m. What can you infer is the spring constant (in units of N/m).
   (a) 0.03 (b) 1.5 (c) 50 (d) 1667 (e) None of these.
2. If you displace the weight and the spring from its new equilibrium point, what will be the frequency (in cycles per second) of the oscillations?
   (a) 2.88 (b) 5.77 (c) 18.07 (d) 166.7 (e) None of these.
3. Does it change the oscillation frequency if the weight moves horizontally versus vertically?

Peer instruction question:

A certain mass m on a spring oscillates with a characteristic frequency of 2 cycles per second. Which of the following changes to the mass would increase the frequency to 4 cycles per second?

(a) 2m (b) 4m (c) m/2 (d) m/4

Simple harmonic motion:

$$F = -kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Conveniently evaluated in *radians*

$$x(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{k}{m}}$$

Constants

Note that:

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi)$$

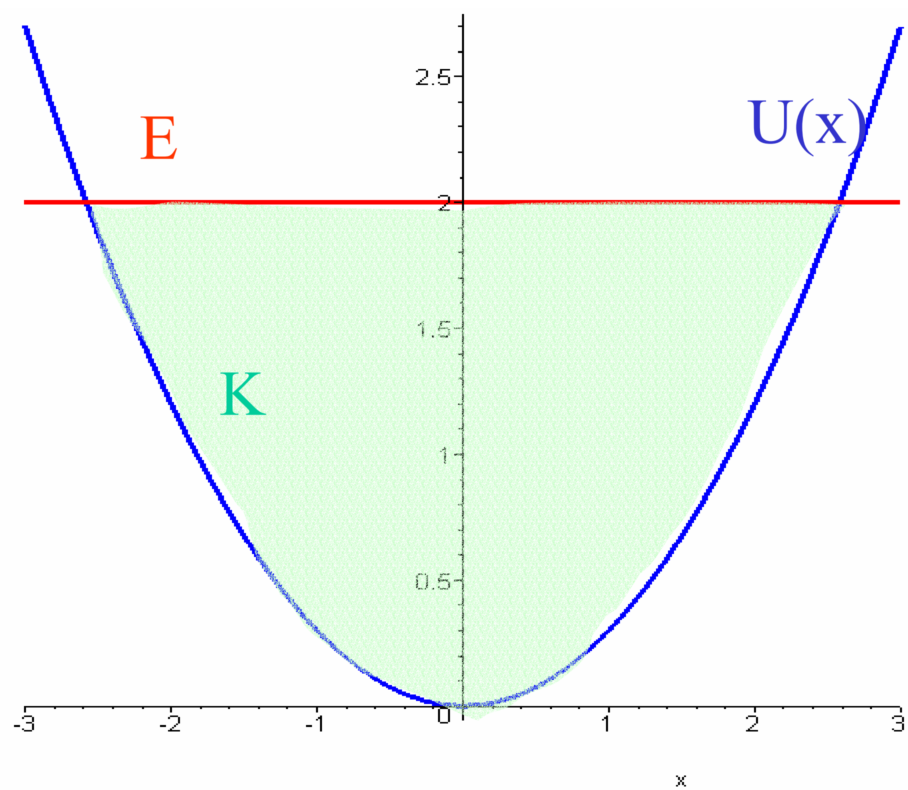
$$a(t) = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \varphi)$$

Mechanical energy associated with simple harmonic motion

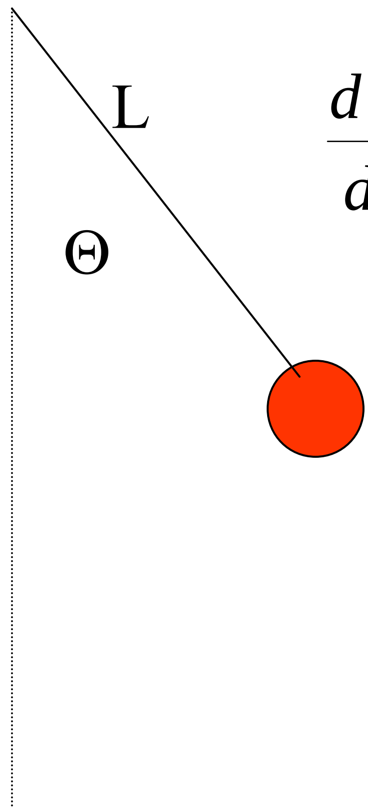
$$E = K + U = ½\, m\, v^2 + ½\, k\, x^2$$

$$E = \tfrac{1}{2} m\{A\omega \sin(\omega t + \varphi)\}^2 + \tfrac{1}{2} k\{A\cos(\omega t + \varphi)\}^2$$

$$= \tfrac{1}{2} m\omega^2 A^2 = \tfrac{1}{2} kA^2$$



Simple harmonic motion for a pendulum:



$$\tau = mgL\sin\Theta = -I\alpha = -I\frac{d^2\Theta}{dt^2}$$

$$\frac{d^2\Theta}{dt^2} = -\frac{mgL}{I}\sin\Theta = -\frac{g}{L}\sin\Theta \quad (\text{since } I = mL^2)$$

Approximation for small Θ:

$$\sin\Theta \approx \Theta$$

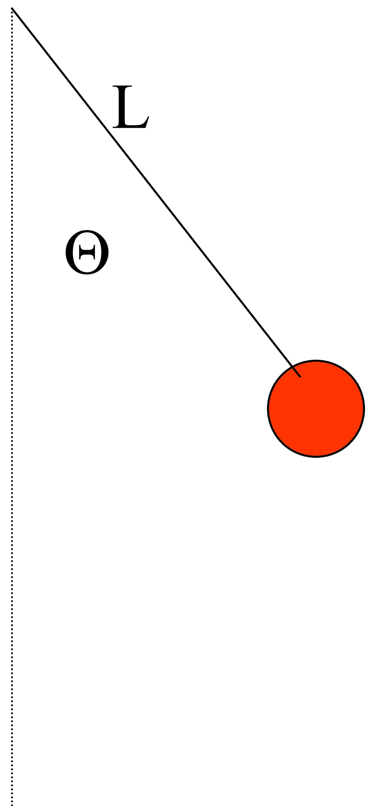
$$\Rightarrow \frac{d^2\Theta}{dt^2} = -\frac{g}{L}\Theta$$

Solution :

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{g}{L}}$$

Pendulum example:

Suppose L=2m, what is the period of the pendulum?



$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\text{m/s}^2}{2\text{m}}} = 2.2135 \text{ rad/s} = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = 2.84 \text{ s}$$

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{g}{L}}$$

The notion of resonance:

Suppose $F=-kx+F_0 \sin(\Omega t)$

According to Newton:

$$-kx + F_0 \sin(\Omega t) = m\frac{d^2x}{dt^2}$$

Differential equation ("inhomogeneous"):

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x + \frac{F_0}{m}\sin(\Omega t)$$

Solution:

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) \equiv \frac{F_0/m}{\omega^2 - \Omega^2}\sin(\Omega t)$$

Physics of a "driven" harmonic oscillator:

$$-kx + F_0 \sin(\Omega t) = m\frac{d^2x}{dt^2}$$

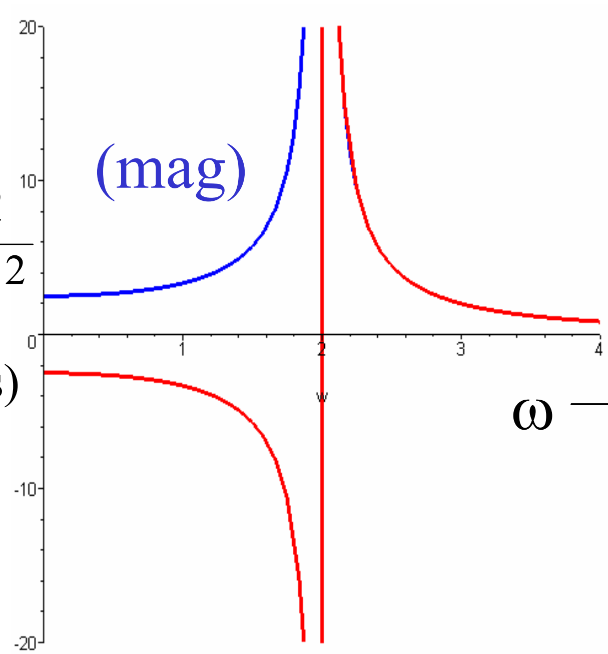
"driving" frequency

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) \equiv \frac{F_0/m}{\omega^2 - \Omega^2}\sin(\Omega t)$$

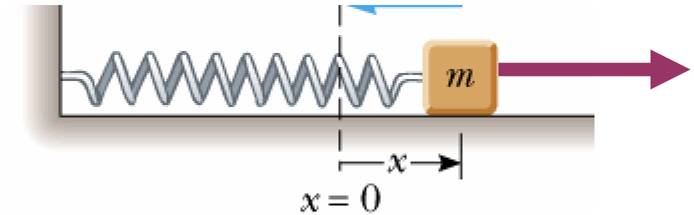
"natural" frequency

$$\frac{F_0/m}{\omega^2 - \Omega^2}$$

(Ω=2rad/s)



ω →



F(t)=1 N sin (3t)

Examples:

Suppose a mass m=0.2 kg is attached to a spring with k=1.81N/m and an oscillating driving force as shown above. Find the steady-state displacement x(t).

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) = \frac{1/0.2}{1.81/0.2 - 3^2}\sin(3t)\text{ m} = 100\sin(3t)\text{ m}$$

Note: If k=1.90 N/m then:

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) = \frac{1/0.2}{1.90/0.2 - 3^2}\sin(3t)\text{ m} = 10\sin(3t)\text{ m}$$