Announcements

1. **Sign up for presentations now available –**

   Fri. 11/07 at 4 PM, Sun. 11/09 at 3 PM, Mon. 11/10 at 4 PM

2. **Physics colloquium today at 3 PM in Olin101**

   Professor Ceder from MIT – "The physics of transition metal oxides in rechargeable lithium batteries"

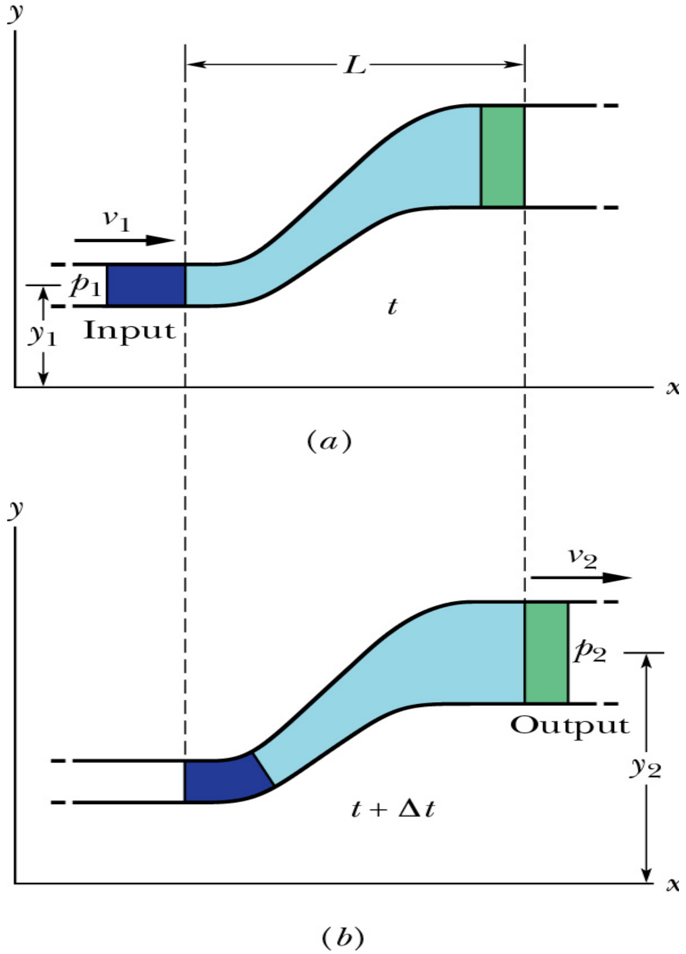
3. **Today's lecture –**

   A few comments about the physics of fluids

   The physics of Wave motion (Chap. 17)

Summary of results concerning the physics of fluids --

Bernoulli's equation: $P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 = P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1$



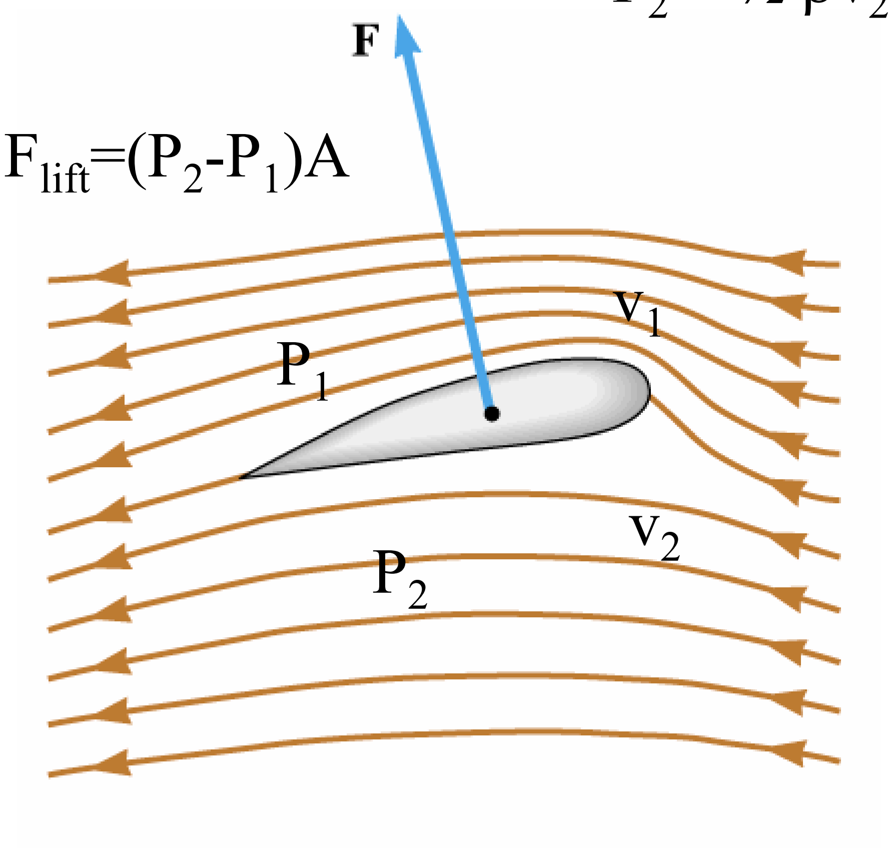
Applies to incompressible fluids or fluids in steamline flow.

Assumes no friction or turbulent flow.

Can also be used to analyze static fluids ($v_i = 0$).

Streamline flow of air around an airplane wing:

$$P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 = P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1$$



$$h_1 \approx h_2$$

$$v_1 > v_2$$

$$P_2 = P_1 + \tfrac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

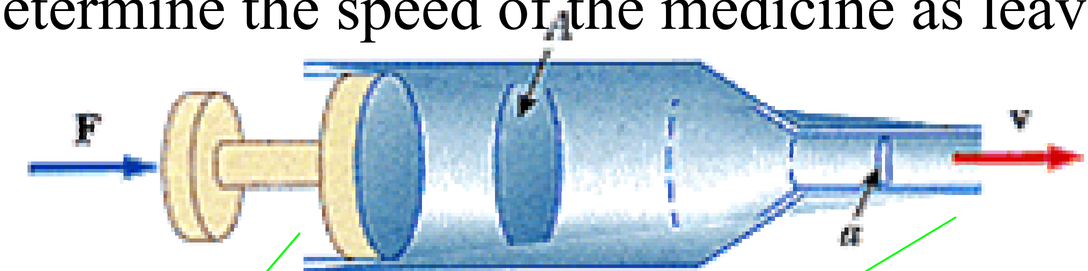
Example:

$v_1 = 270$ m/s, $v_2 = 260$ m/s

$\rho = 0.6$ kg/m^3, $A = 40$ m^2

$\rightarrow F_{lift} = 63{,}600$ N

A hypodermic syringe contains a medicine with the density of water. The barrel of the syringe has a cross-sectional area $A=2.5\mathrm{x}10^{-5}\mathrm{m}^2$, and the needle has a cross-sectional area $a= 1.0\mathrm{x}10^{-8}\mathrm{m}^2$. In the absence of a force on the plunger, the pressure everywhere is 1 atm. A force F of magnitude 2 N acts on the plunger, making the medicine squirt horizontally from the needle. Determine the speed of the medicine as leave the needle's tip.

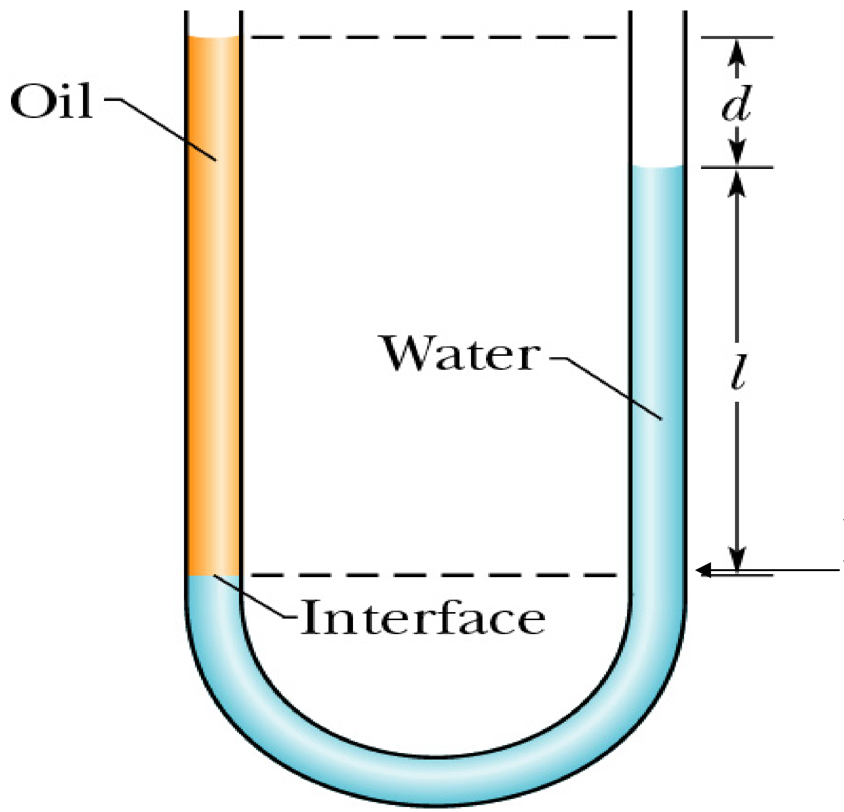


$$P_0 + \frac{F}{A} + \frac{1}{2}\rho V^2 = P_0 + \frac{1}{2}\rho v^2 \qquad AV = av$$

$$v = \sqrt{\frac{2F}{A\rho\left(1-\left(a/A\right)^2\right)}} = 12.6\mathrm{m/s}$$

Another example; v=0

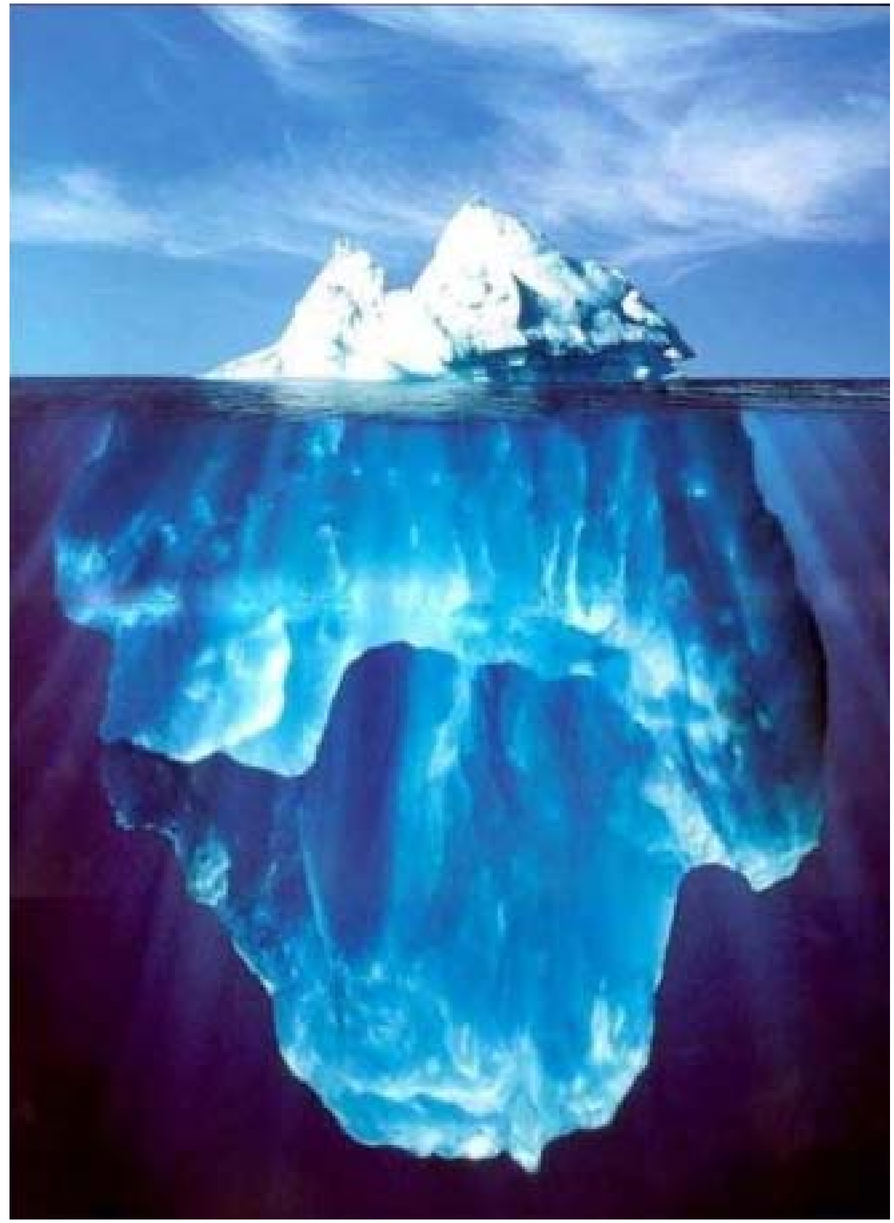
$$P_0 + \rho_{oil} g (l + d) = P_0 + \rho_{water} g l + \rho_{air} g d \qquad \rho_{oil} \approx \rho_{water} \frac{l}{l+d}$$



Potential energy reference

Bouyant forces:

the tip of the iceburg



$$F_B = m_{ice} g$$

$$\rho_{water} g V_{submerged} = \rho_{ice} g V_{total}$$

$$\frac{V_{submerged}}{V_{total}} = \frac{\rho_{ice}}{\rho_{water}} \approx \frac{0.917}{1.024} = 93.9\%$$

Source:http://bb-bird.com/iceburg.html

The phenomenon of wave motion

The wave equation

$$y(x,t)$$

Wave variable — position — time

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

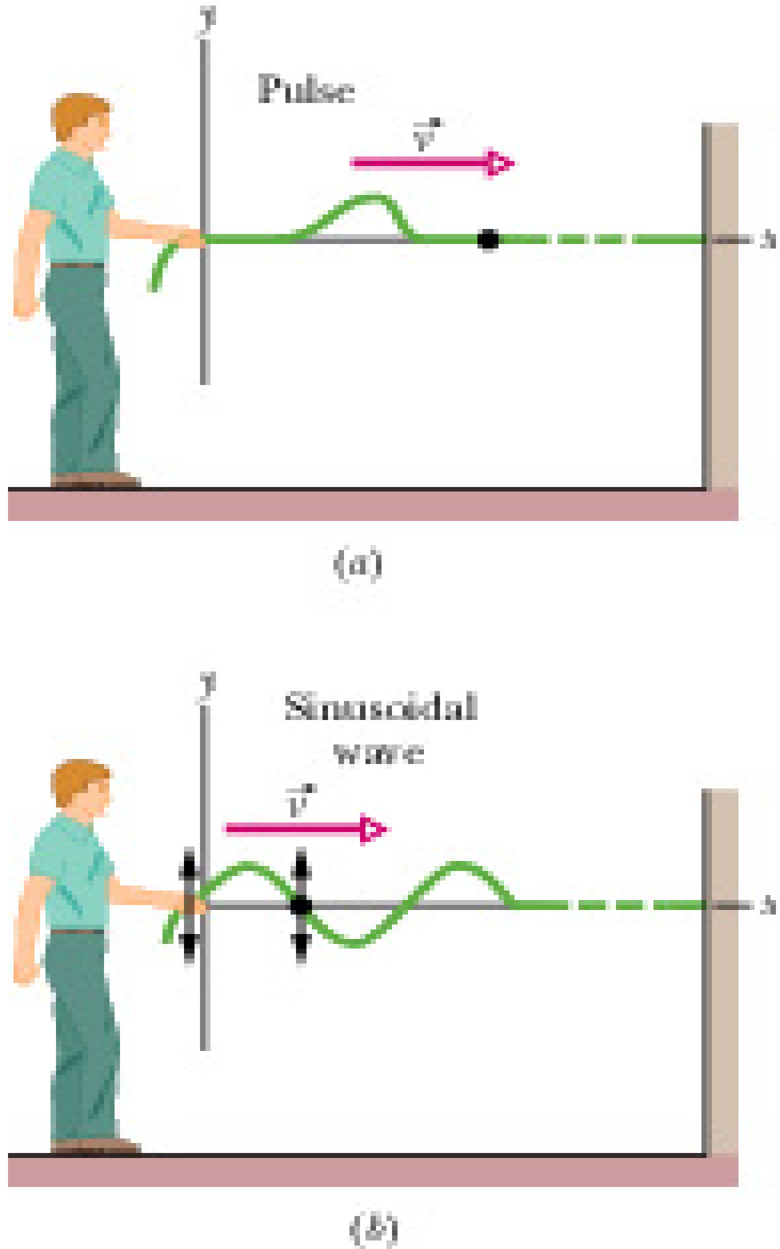
- What does the wave equation mean?
- Examples
- Mathematical solutions of wave equation and descriptions of waves

Example: Water waves

Needs more sophistocated analysis:

Source: http://www.eng.vt.edu/fluids/msc/gallery/gall.htm

Waves on a string:



Typical values for v:

$3x10^8$ m/s light waves

~1000 m/s wave on a string

331 m/s sound in air

Peer instruction question

Which of the following properties of a wave are characteristic of the medium in which the wave is traveling?

(A) Its frequency

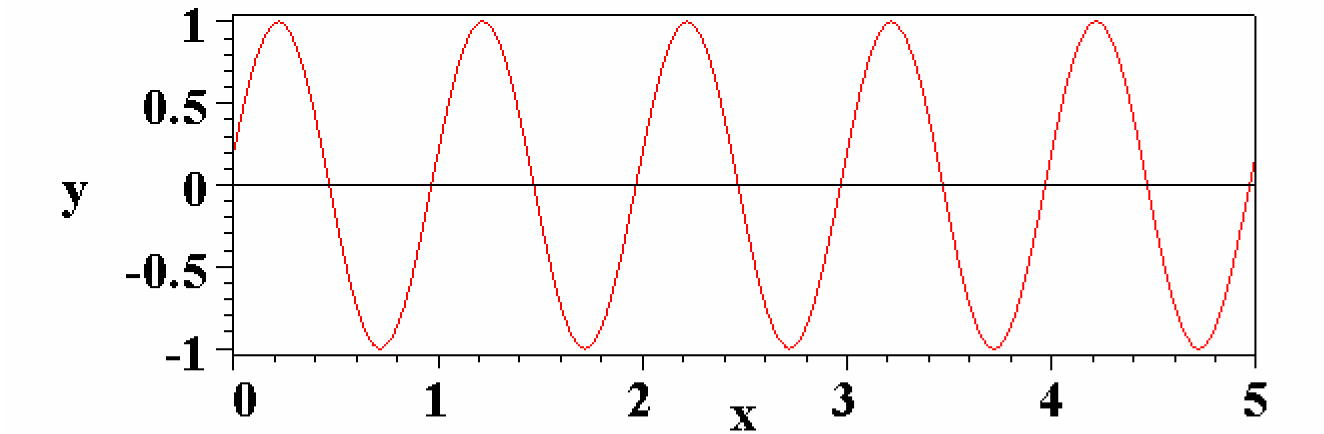
(B) Its wavelength

(C) Its velocity

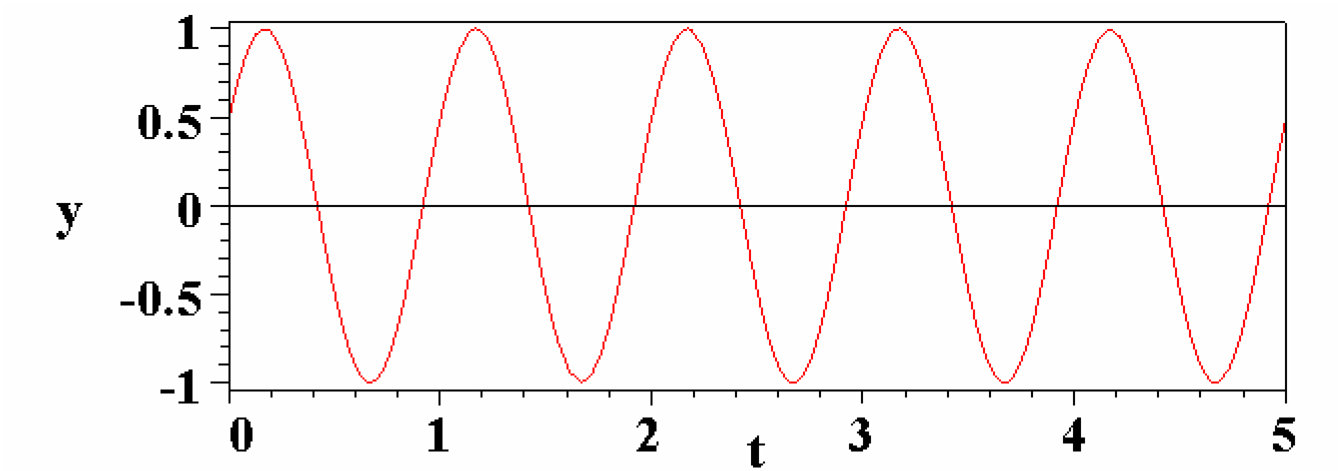
(D) All of the above

Mechanical waves occur in continuous media. They are characterized by a value (y) which changes in both time (t) and position (x).

Example -- periodic wave

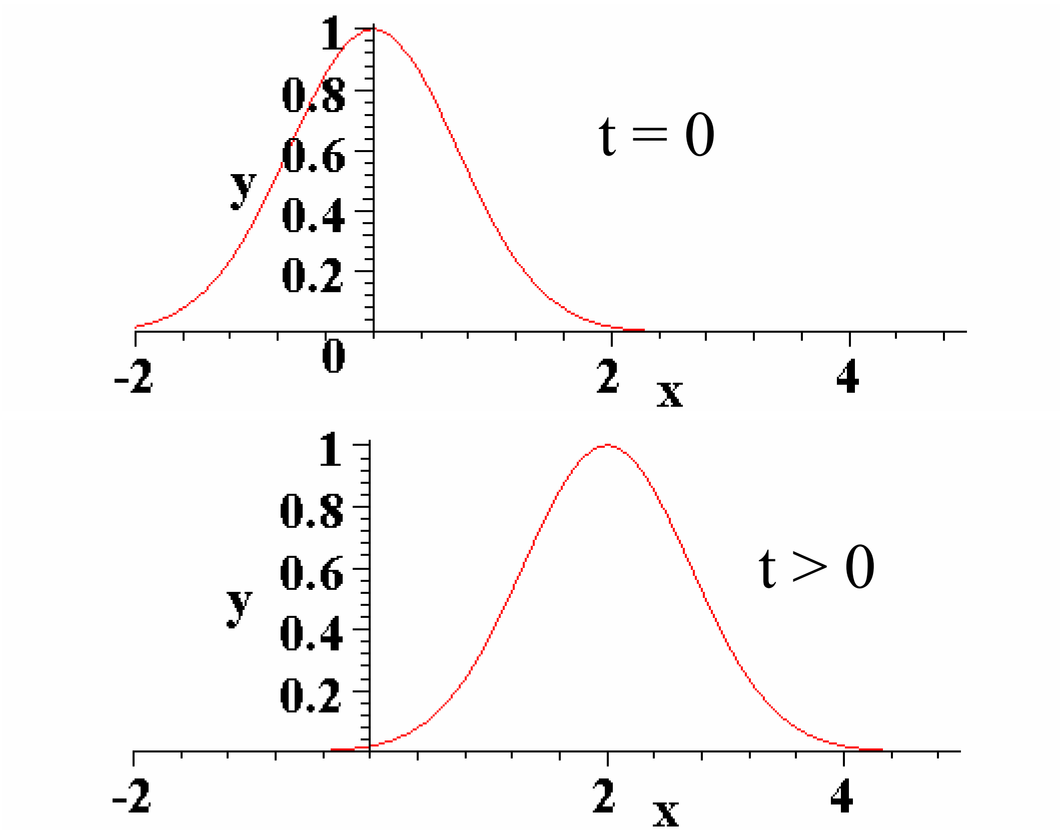


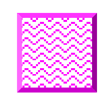
$y(t_0,x)$



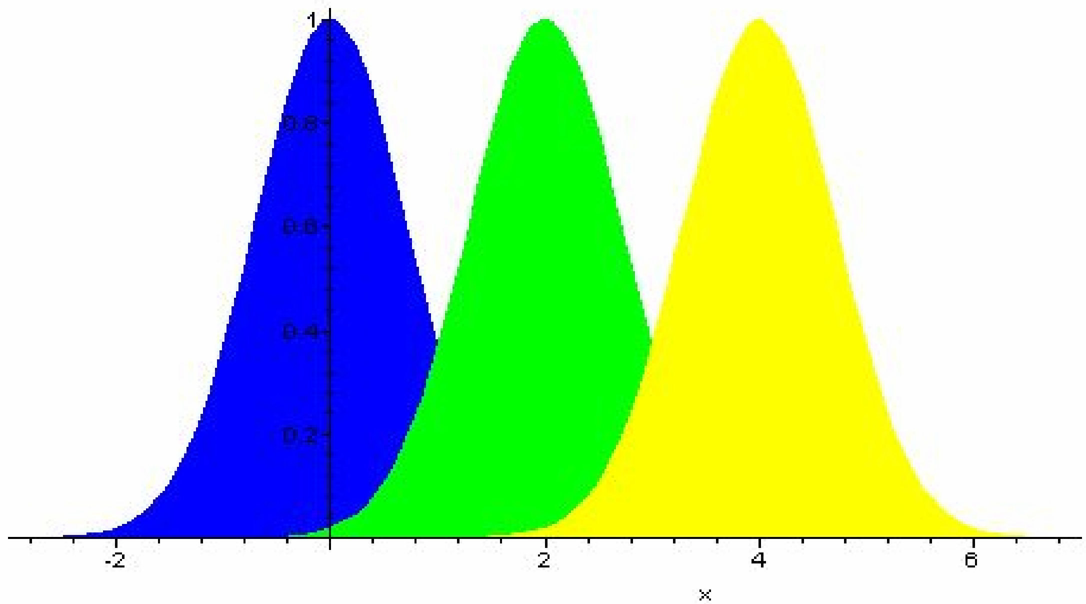
$y(t,x_0)$

General traveling wave –





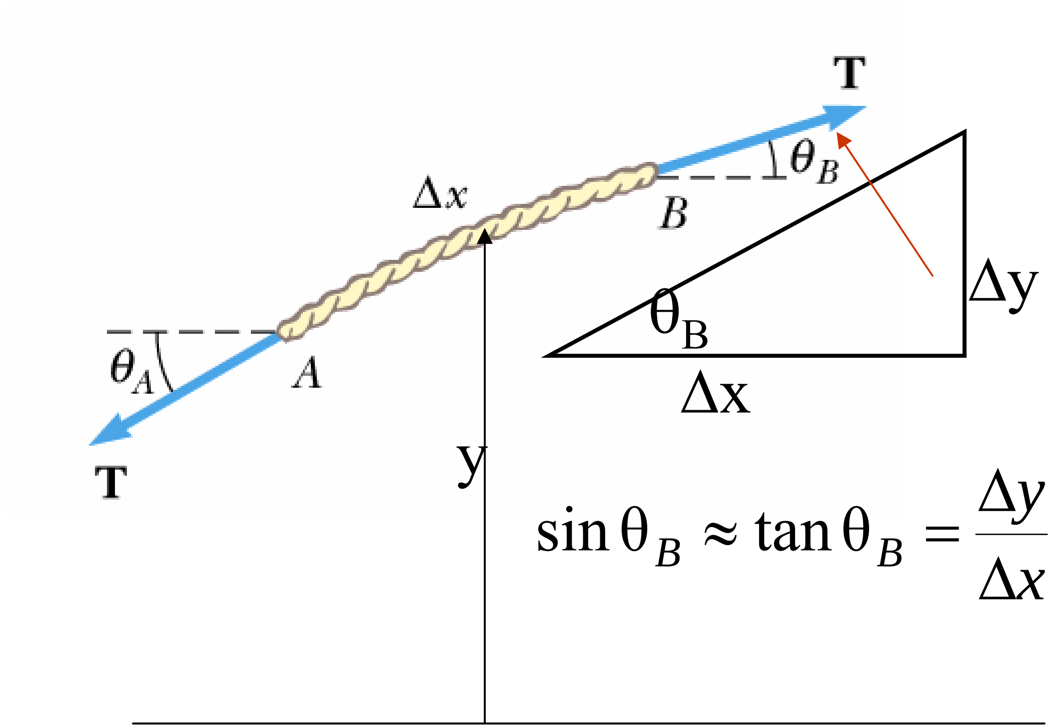
Online Quiz for Lecture 17
Wave motion -- Nov. 6, 2003



The figure above shows three snap shots of a transverse wave plotted versus distance (x in meters). The snapshots were taken at t=0s (blue), t=1s (green), and t=2s (yellow). What is the velocity of the wave? (a) 1 m/s (b) 2 m/s (c) 3 m/s (d) 4 m/s (e) 6 m/s

Basic physics behind wave motion --

example: transverse wave on a string with tension T and mass per unit length μ



$$\sin\theta_B \approx \tan\theta_B = \left.\frac{\Delta y}{\Delta x}\right|_B$$

$$m\frac{d^2y}{dt^2} = T\sin\theta_B - T\sin\theta_A$$

$$m \approx \mu\Delta x$$

$$\Rightarrow \mu\Delta x\frac{d^2y}{dt^2} \approx T\left(\left.\frac{\Delta y}{\Delta x}\right|_B - \left.\frac{\Delta y}{\Delta x}\right|_A\right)$$

$$\lim_{\Delta x\to 0}\frac{1}{\Delta x}\left(\left.\frac{\Delta y}{\Delta x}\right|_B - \left.\frac{\Delta y}{\Delta x}\right|_A\right) = \frac{\partial^2 y}{\partial x^2}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2}$$

The wave equation:

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

where $v \equiv \sqrt{\frac{T}{\mu}}$ (for a string)

Solutions: $y(x,t) = f\,(x \pm vt)$ ← function of *any* shape

Note: Let $u \equiv x \pm vt$

$$\frac{\partial f(u)}{\partial x} = \frac{\partial f(u)}{\partial u}\frac{\partial u}{\partial x}$$

$$\frac{\partial^2 f(u)}{\partial x^2} = \frac{\partial^2 f(u)}{\partial u^2}\left(\frac{\partial u}{\partial x}\right)^2 = \frac{\partial^2 f(u)}{\partial u^2}$$

$$\frac{\partial^2 f(u)}{\partial t^2} = \frac{\partial^2 f(u)}{\partial u^2}\left(\frac{\partial u}{\partial t}\right)^2 = \frac{\partial^2 f(u)}{\partial u^2}v^2$$

Examples of solutions to the wave equation:

Moving "pulse": $y(x,t) = y_0 e^{-(x-vt)^2}$

phase factor

Periodic wave: $y(x,t) = y_0 \sin\left(k(x-vt) + \varphi\right)$

"wave vector"
not spring constant!!!

$$k = \frac{2\pi}{\lambda}$$

$$kv = \frac{2\pi}{T} = 2\pi f = \omega$$

$$y(x,t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \varphi\right) \qquad \frac{\lambda}{T} = v$$

Periodic traveling waves:

$$y(x,t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \varphi\right) \qquad \frac{\lambda}{T} = v$$

phase (radians)

velocity (m/s)

period (s); $T = 1/f$

wave length (m)

Amplitude

Combinations of waves ("superposition")

Note that :

$$\sin A \pm \sin B = 2\sin\left[\tfrac{1}{2}(A \pm B)\right]\cos\left[\tfrac{1}{2}(A \mp B)\right]$$

$$\sin A \pm \sin B = 2\sin\left[\tfrac{1}{2}(A \pm B)\right]\cos\left[\tfrac{1}{2}(A \mp B)\right]$$

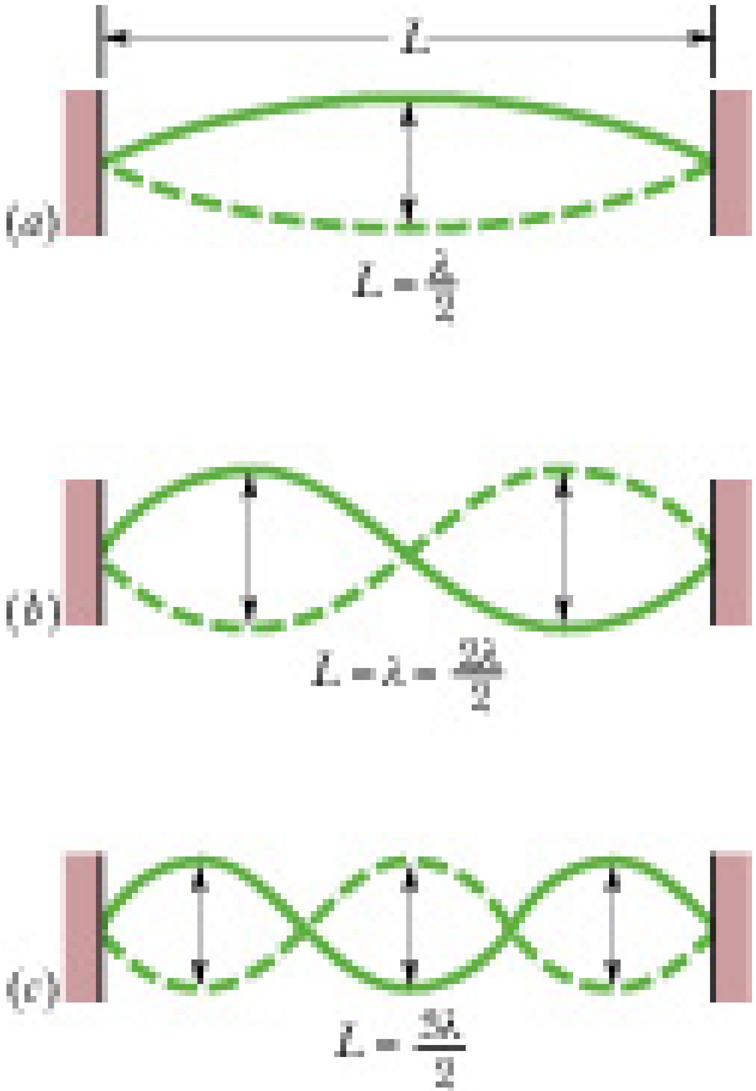
$$y_{right}(x,t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \varphi\right) \qquad y_{left}(x,t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right) + \varphi\right)$$

"Standing" wave:

$$y_{right}(x,t) + \mathrm{y}_{left}(x,t) = 2y_0 \sin\left(\frac{2\pi x}{\lambda} + \varphi\right)\cos\left(\frac{2\pi t}{T}\right)$$

Constraints of standing waves: ($\varphi = 0$)

$$y_{right}(x,t) + y_{left}(x,t) = 2y_0 \sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi t}{T}\right)$$



for string :

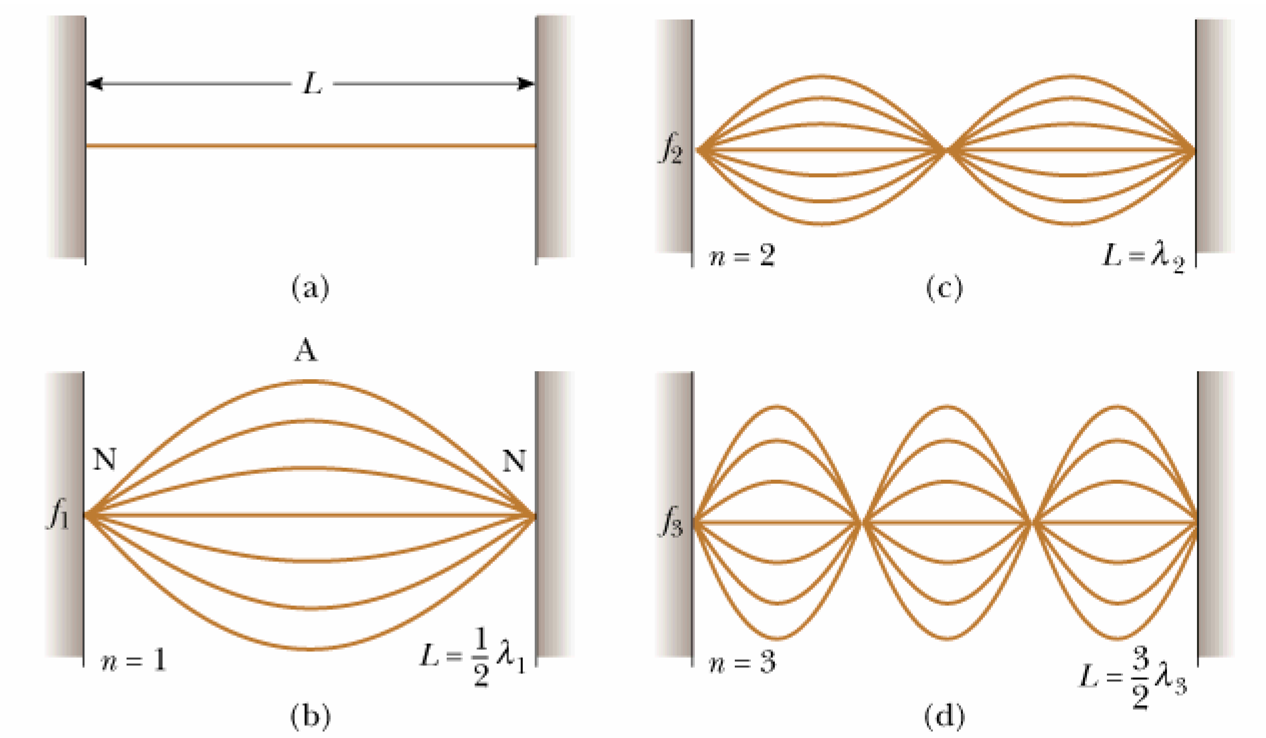
$$\lambda = \frac{2L}{n}; n = 1,2,3\cdots$$

$$\frac{\lambda}{T} = v \quad \Rightarrow T = \frac{2L}{nv}$$

$$f = \frac{1}{T} = \frac{nv}{2L}$$

The sound of music

String instruments (Guitar, violin, etc.)



$$\lambda_n = \frac{2L}{n}$$

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$$

$$v = \sqrt{\frac{T}{\mu}}$$

(no sound yet.....) $y_{\text{standing}}(x,t) = 2y_0 \sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi t}{T}\right)$

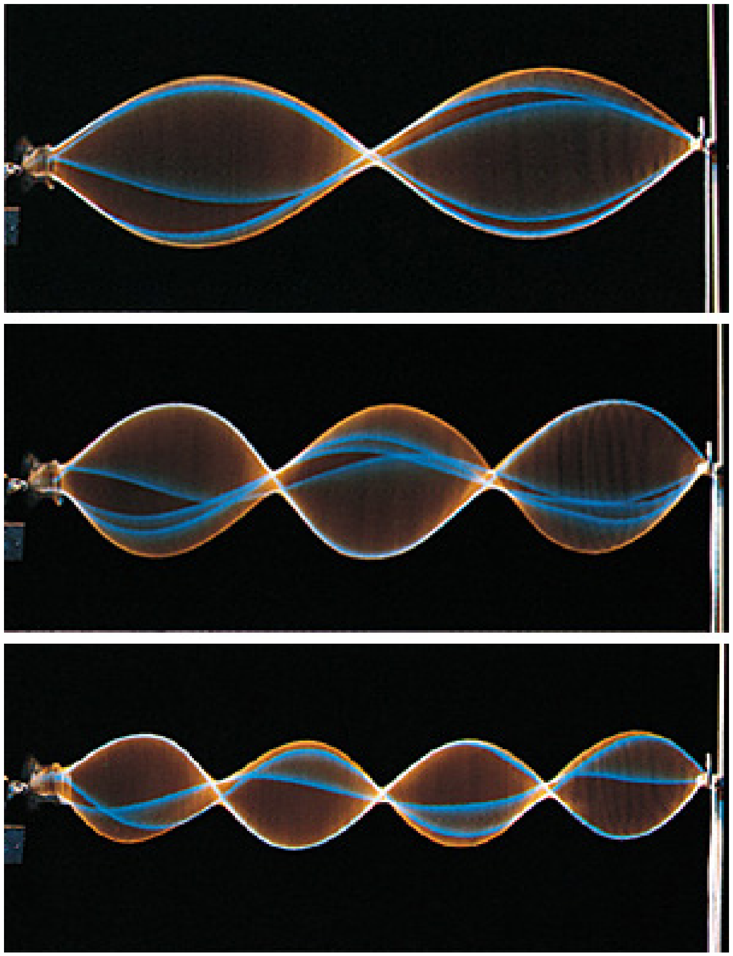


Fig. 17-20 Stroboscopic photographs reveal (imperfect) standing wave patterns on a string being made to oscillate by a vibrator at the left end. The patterns occur at certain frequencies of oscillation.

$$n = 2 \quad \lambda = L$$

$$n = 3 \quad \lambda = \tfrac{2}{3} L$$

$$n = 4 \quad \lambda = \tfrac{2}{4} L$$

(top) Richard Megna / Fundamental Photographs
(center) Richard Megna / Fundamental Photographs
(bottom) Richard Megna / Fundamental Photographs

coupling to air

Peer instruction question

Suppose you pluck the "A" guitar string whose fundamental frequency is f=440 cycles/s. The string is 0.5 m long so the wavelength of the standing wave on the string is λ=1m. What is the velocity of the wave on string?

(A) 1/220 m/s (B) 1/440 m/s (C) 220 m/s (D) 440 m/s

If you increased the tension of the string, what would happen?

3. HRW6 17.P.019. [52089] A sinusoidal transverse wave is traveling along a string toward decreasing x. Figure 17-29 shows a plot of the displacement as a function of position at time $t = 0$. The string tension is 2.7 N, and its linear density is 19 g/m.

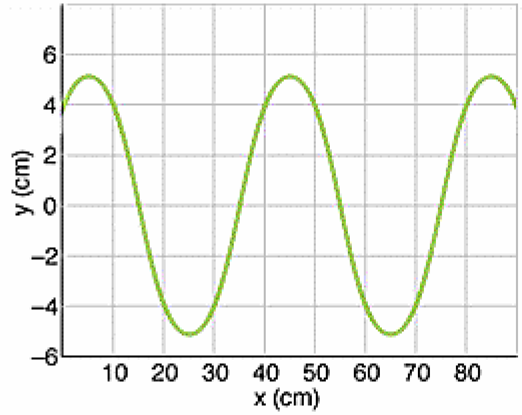


Figure 17-29.

(a) Find the amplitude.

[0.0714286] ______ cm

(b) Find the wavelength.

[0.0714286] ______ m

(c) Find the wave speed.

[0.0714286] ______ m/s

(d) Find the period of the wave.

[0.0714286] ______ s

(e) Find the maximum speed of a particle in the string.

[0.0714286] ______ m/s

(f) Choose the equation describing the traveling wave. (Assume t is in seconds.)