Announcements

1. Physics lecture today 4 PM – Professor Brian Matthews from U. Oregon – "Tolerance and Intolerance in Protein Structure and Function"
2. Third exam on Chaps. 15-20 – Tuesday, Nov. 25th
   Bring:
   - Equation sheet (8.5″ x 11″)
   - Calculator
   - Clear head
3. Extra problem solving session ?
   Sunday night 10-11 PM ?
   Monday evening 6-7 PM ?
   Other ?

Advice for preparing for exam:

- Review lecture notes and text chapters
- Prepare equation sheet
- Work problems using equation sheet and calculator
  - From homework
  - From previous exams
  - From on line quizes

Topics:

- Simple harmonic motion & resonance
- Wave motion, standing waves, sound waves
- The physics of fluids
- Thermodynamics & the ideal gas law

Simple harmonic motion & resonance

Hooke's law: $F = -kx = ma = m\frac{d^2x}{dt^2}$

$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Solution :

$$x(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{k}{m}}$$

Simple pendulum: $\tau = mgL\sin\Theta = -I\alpha = \text{-}I\frac{d^2\Theta}{dt^2}$

$$\Rightarrow \frac{d^2\Theta}{dt^2} = -\frac{mgL}{I}\sin\Theta \approx -\frac{mgL}{I}\Theta$$

Solution (for small Θ) :

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{\mathrm{mgL}}{\mathrm{I}}}$$

The notion of resonance:

Suppose F=–kx+F_0 sin(Ωt)

According to Newton’s second law:

$$-kx + F_0\sin(\Omega t) = m\frac{d^2x}{dt^2}$$

Differential equation ("inhomogeneous") :

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x + \frac{F_0}{m}\sin(\Omega t)$$

A solution to this equation takes the form :

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) \equiv \frac{F_0/m}{\omega^2 - \Omega^2}\sin(\Omega t)$$

Physics of a “driven” harmonic oscillator:

$$-kx + F_0 \sin(\Omega t) = m\frac{d^2x}{dt^2}$$

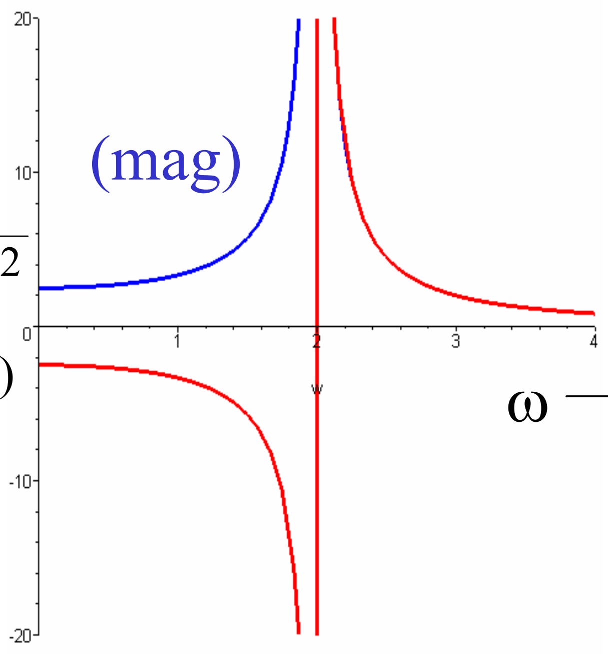
“driving” frequency

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) \equiv \frac{F_0/m}{\omega^2 - \Omega^2}\sin(\Omega t)$$

“natural” frequency

$$\frac{F_0/m}{\omega^2 - \Omega^2}$$

(Ω=2rad/s)



ω →

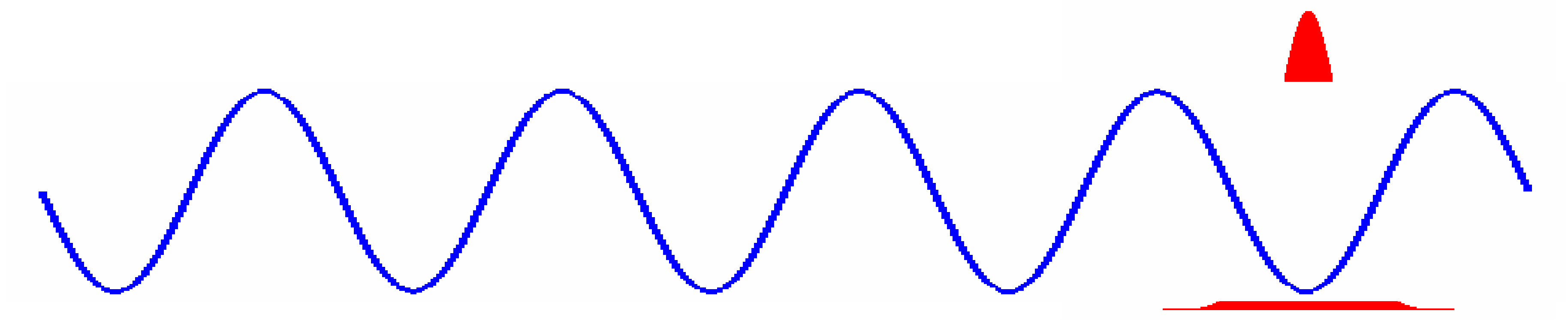
The phenomenon of wave motion

The wave equation

$$y(x,t)$$

position time

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$



The wave equation:

$$\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}}$$

where $v \equiv \sqrt{\frac{T}{\mu}}$ (for a string)

Solutions: $y(x,t) = f\,(x \pm vt)$

function of *any* shape

Moving "pulse": $y(x,t) = y_0 e^{-(x-vt)^2}$

"wave vector"

<u>not</u> spring constant!!!

Periodic wave: $y(x,t) = y_0 \sin\left(k\left(x - vt\right) + \varphi\right)$

$$y(x,t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \varphi\right) \qquad \frac{\lambda}{T} = v$$

Superposition of waves:

$$y_{right}(x,t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \varphi\right) \quad y_{left}(x,t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right) + \varphi\right)$$

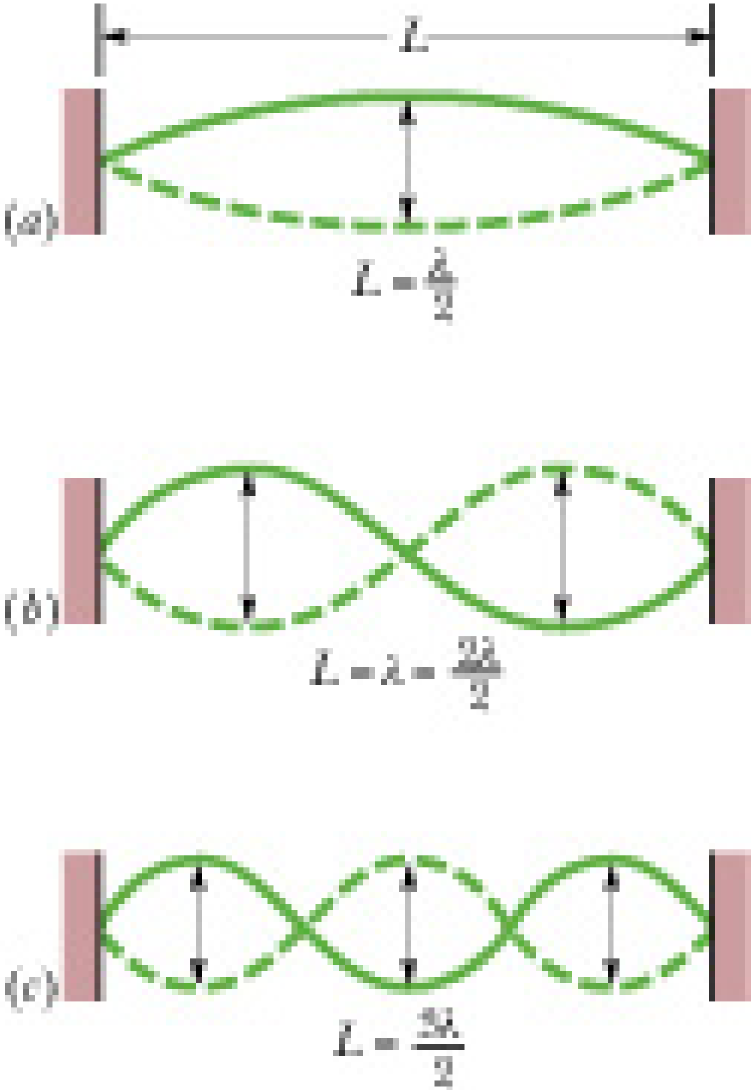
$$\sin A \pm \sin B = 2\sin\left[\tfrac{1}{2}(A \pm B)\right]\cos\left[\tfrac{1}{2}(A \mp B)\right]$$

"Standing" wave:

$$y_{right}(x,t) + \mathrm{y}_{left}(x,t) = 2y_0 \sin\left(\frac{2\pi x}{\lambda} + \varphi\right)\cos\left(\frac{2\pi t}{T}\right)$$

Constraints of standing waves: $(\varphi = 0)$

$$y_{right}(x,t) + y_{left}(x,t) = 2y_0 \sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi t}{T}\right)$$



for string :

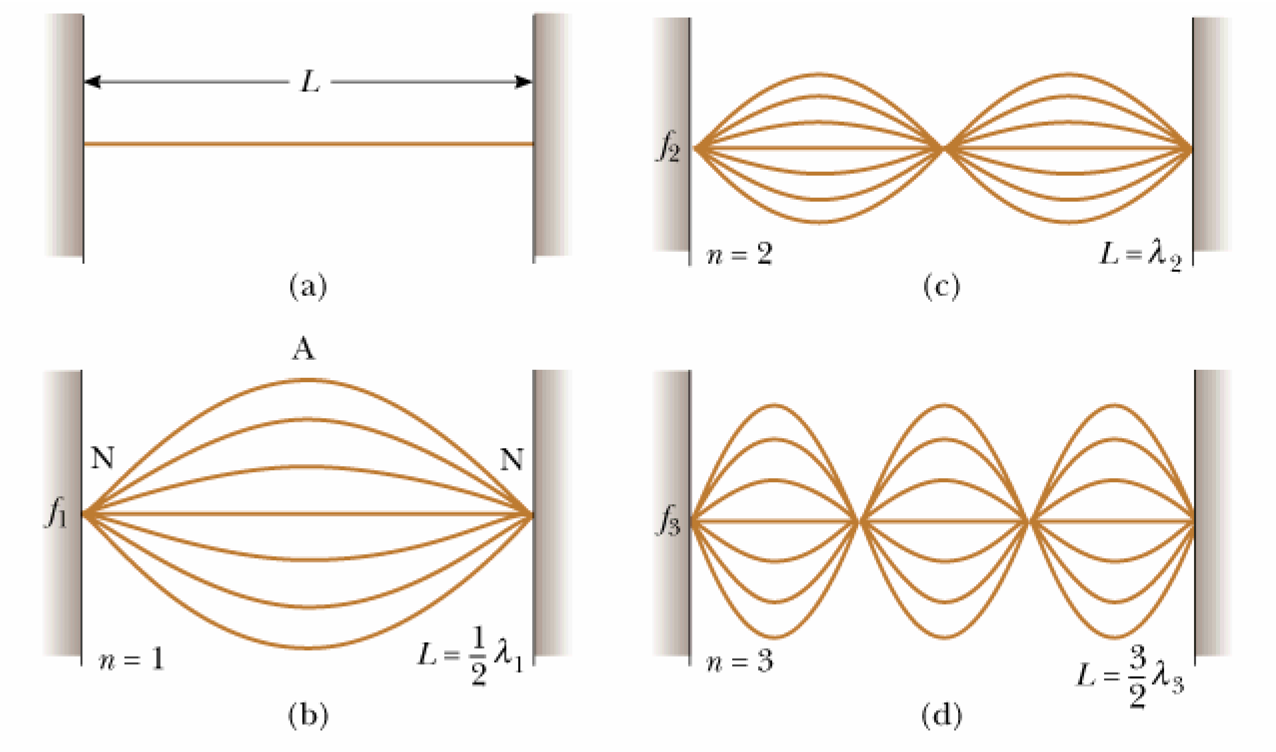
$$\lambda = \frac{2L}{n}; n = 1,2,3\cdots$$

$$\frac{\lambda}{T} = v \quad \Rightarrow T = \frac{2L}{nv}$$

$$f = \frac{1}{T} = \frac{nv}{2L}$$

The sound of music

String instruments (Guitar, violin, etc.)



$$\lambda_n = \frac{2L}{n}$$

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$y_{\text{standing}}(x,t) = 2y_0 \sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi t}{T}\right)$$

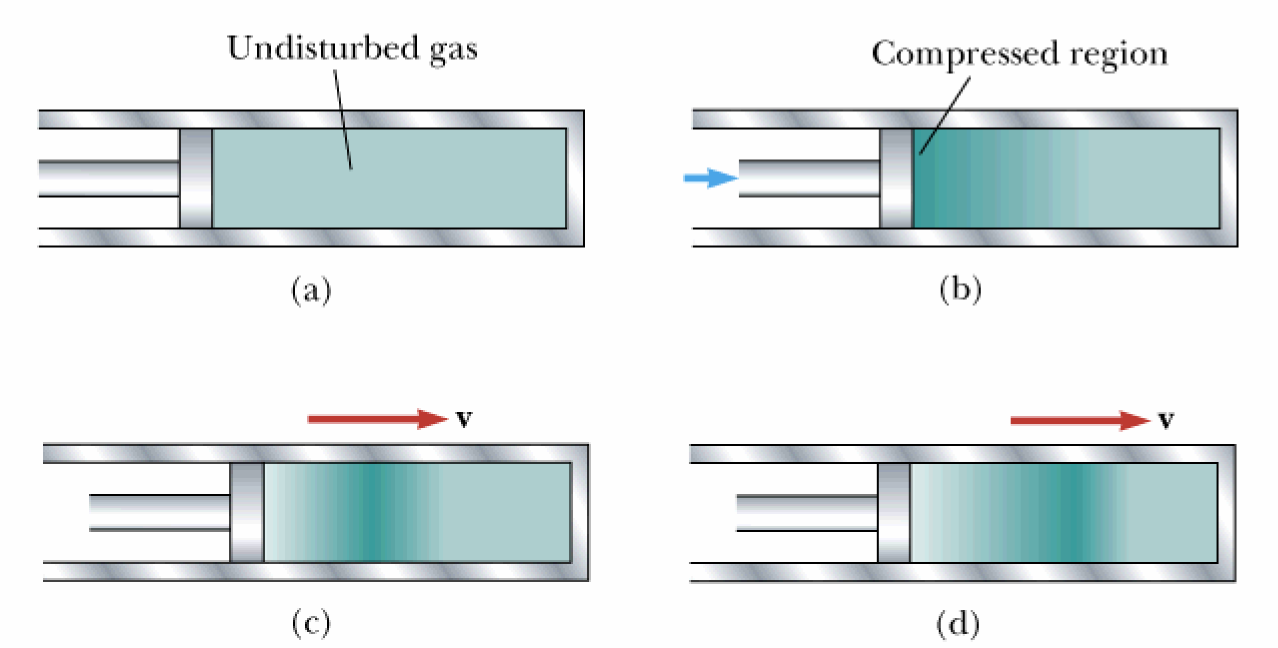
➔ Couples to sound in the air

Sound waves

Longitudinal waves propagating in a fluid or solid

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \qquad y(x,t) \text{ describes density or pressure variations}$$

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow \sqrt{\frac{B}{\rho}} \quad \left(\sqrt{\frac{\text{compressibility}}{\text{density}}} \right) = \sqrt{\frac{\gamma P}{\rho}} \approx 343 \text{ m/s}$$



Periodic sound wave

In terms of pressure:

$$P(x,t) = P_0 + \Delta P_{\max} \sin\left(\frac{2\pi x}{\lambda} \pm \frac{2\pi t}{T}\right)$$

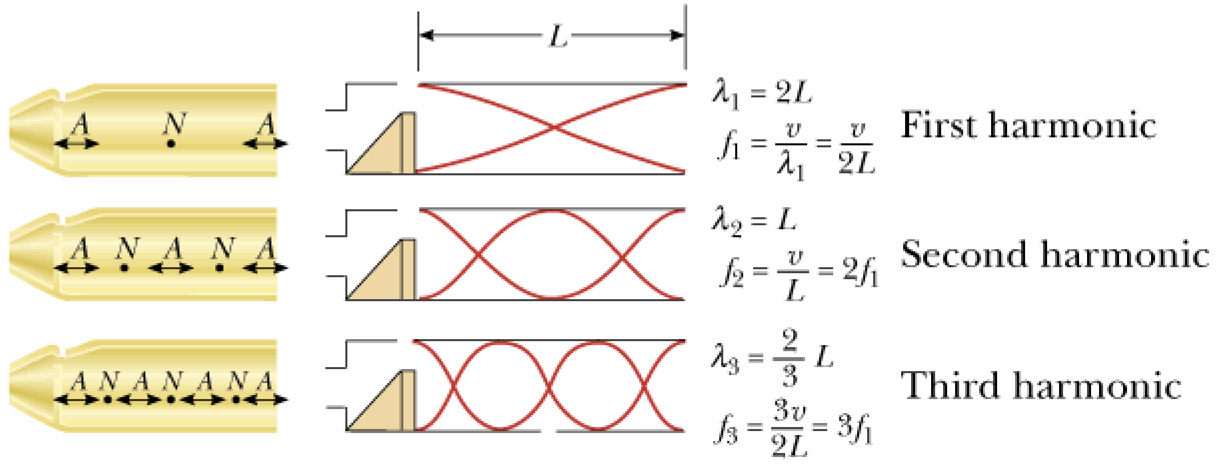
Sound intensity: (energy/(unit time · unit area))

$$I \equiv \frac{(\Delta P_{\max})^2}{2\rho v}$$

Decibel scale:

$$\beta \equiv 10 \log\left(\frac{I}{I_0}\right) \qquad I_0 = 10^{-12}\,\mathrm{W/m^2}$$

“Wind” instruments (standing waves in air)

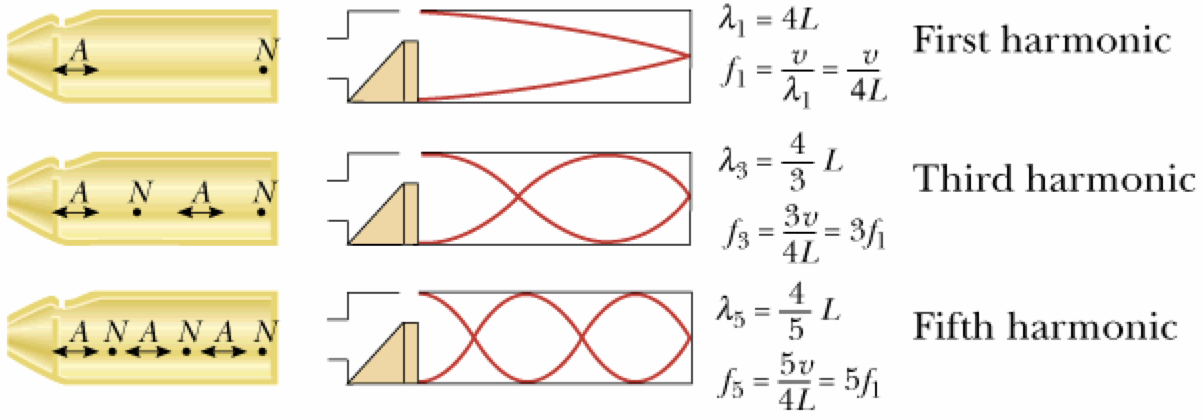


(a) Open at both ends

$$\lambda_n = \frac{2L}{n}; n = 1,2,3,\ldots$$

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}$$

$$v = \sqrt{\frac{\gamma P}{\rho}} \approx 343 \text{ m/s}$$



(b) Closed at one end, open at the other

$$\lambda_n = \frac{4L}{n}; n = 1,3,5,\ldots$$

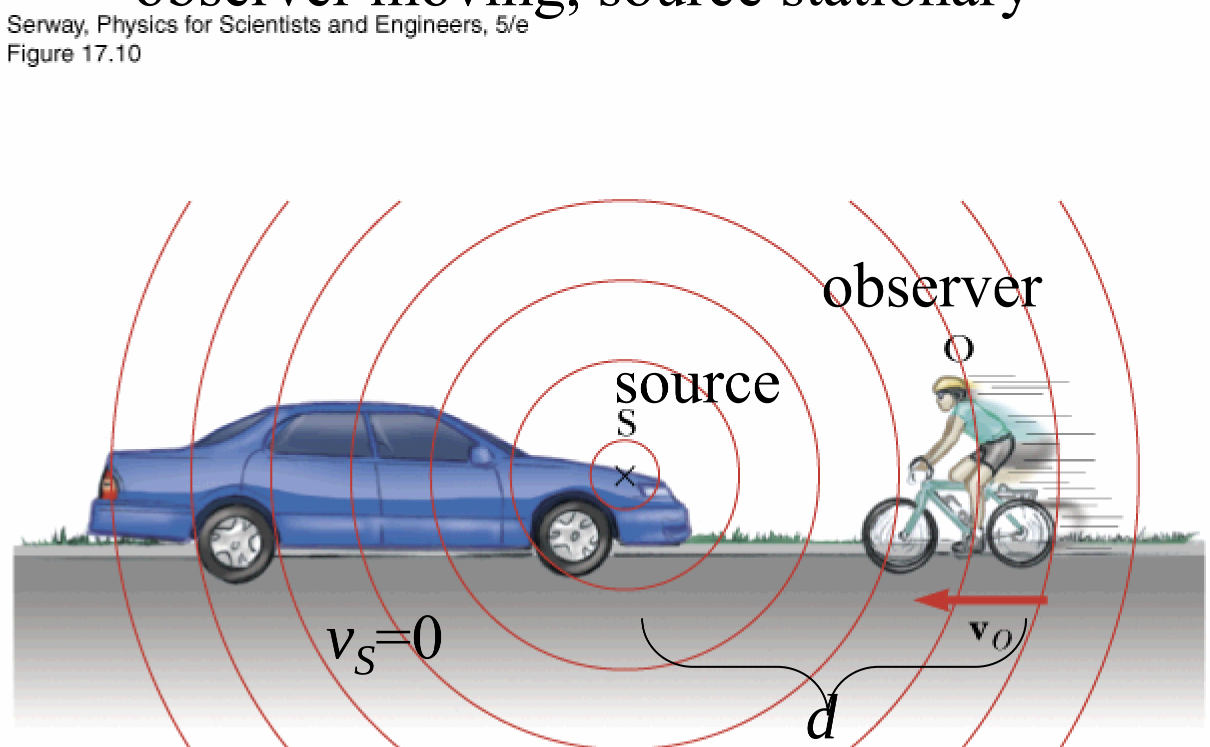
$$f_n = \frac{v}{\lambda_n} = \frac{nv}{4L}$$

$$v = \sqrt{\frac{\gamma P}{\rho}} \approx 343 \text{ m/s}$$

The “Doppler” effect

v=sound velocity

observer moving, source stationary

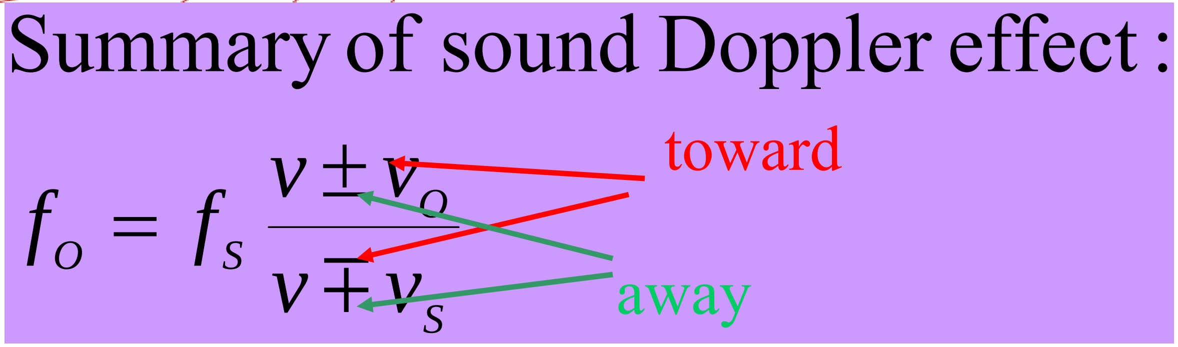


$$vt_1 = d - v_O t_1$$

$$v(t_2 - T) = d - v_O t_2$$

$$t_2 - t_1 = \frac{1}{f_O} = T\frac{v}{v + v_O}$$

$$\boxed{f_O = f_S \frac{v + v_O}{v}}$$



Summary of sound Doppler effect :

$$f_O = f_S \frac{v \pm v_O}{v \mp v_S}$$

toward

away

The physics of fluids.

- Fluids include liquids (usually “incompressible) and gases (highly “compressible”).

- **Fluids obey Newton’s equations of motion**, but because they move within their containers, the application of Newton’s laws to fluids introduces some new forms.

  - Pressure: P=force/area 1 $(N/m^2) = 1$ Pascal
  - Density: ρ =mass/volume $1\ kg/m^3 = 0.001$ gm/ml

Note: In this chapter P≡pressure (NOT MOMENTUM)

Density: ρ =mass/volume

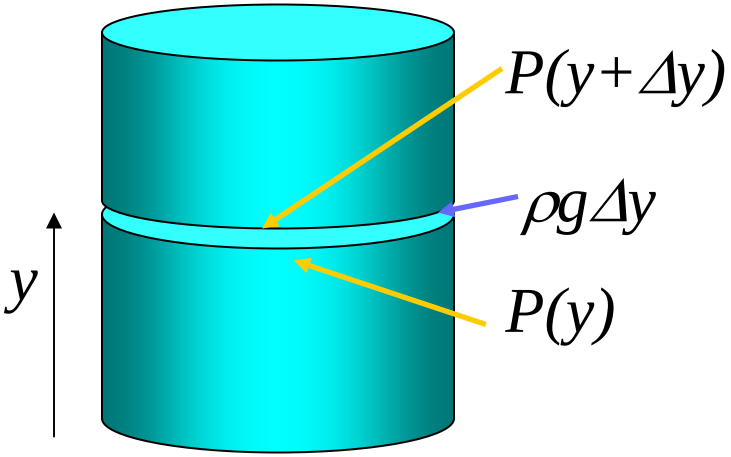
Effects of the weight of a fluid:

$$F(y) = F(y+\Delta y) + mg$$

$$\frac{F(y)}{A} = \frac{F(y+\Delta y)}{A} + \frac{mg}{A}$$

$$P(y) = P(y+\Delta y) + \rho g \Delta y$$

$$\lim_{\Delta y \to 0} \frac{P(y+\Delta y) - P(y)}{\Delta y} = \frac{dP}{dy}$$



$$\Rightarrow \frac{dP}{dy} = -\rho g$$

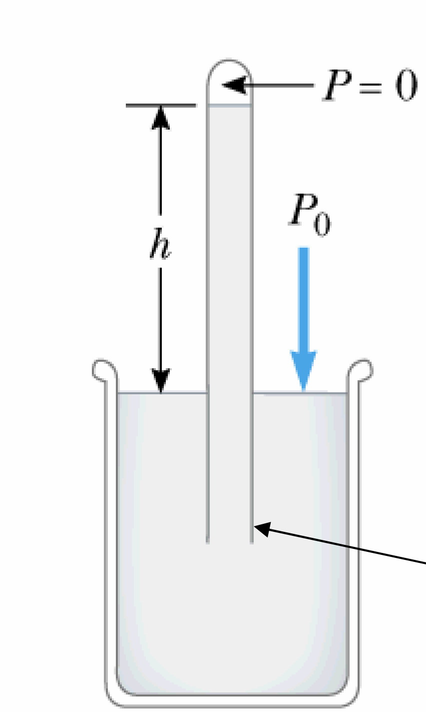
Note: In this formulation +y is defined to be in the up direction.

For an “incompressible” fluid (such as mercury):

$$\rho = 13.585 \times 10^3 \text{ kg/m}^3 \text{ (constant)}$$

$$\frac{dP}{dy} = -\rho g \quad \Rightarrow P = P_0 - \rho g (y - y_0)$$

Example:



$$h = y - y_0 = \frac{P_0}{\rho g}$$

$$= \frac{1.013 \times 10^5 \text{ Pa}}{13.595 \times 10^3 \text{ kg/m}^3 \cdot 9.8 \text{ m/s}^2}$$

$$= 0.76 \text{ m}$$

$$\rho = 13.595 \times 10^3 \text{ kg/m}^3$$

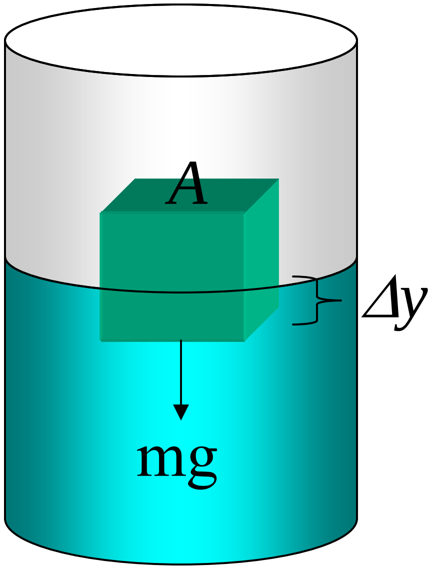
Buoyant force for fluid acting on a solid:

$$F_B=\rho_{\text{fluid}}V_{\text{displaced}}g$$

$$P(y) = P(y+\Delta y) + \rho_{\text{fluid}} g\Delta y$$

Buoyant force: $F_B = F_{bottom} - F_{top}$

$$F_B = \{P(y) - P(y+\Delta y)\}A = \rho_{\text{fluid}} g\Delta y A = \rho_{\text{fluid}} g V_{submerged}$$



$$F_B \quad - \quad mg = 0$$

$$\rho_{\text{fluid}}V_{\text{submerged}}g - \rho_{\text{solid}}V_{\text{solid}}g = 0$$

$$\frac{V_{submerged}}{V_{solid}} = \frac{\rho_{solid}}{\rho_{fluid}}$$

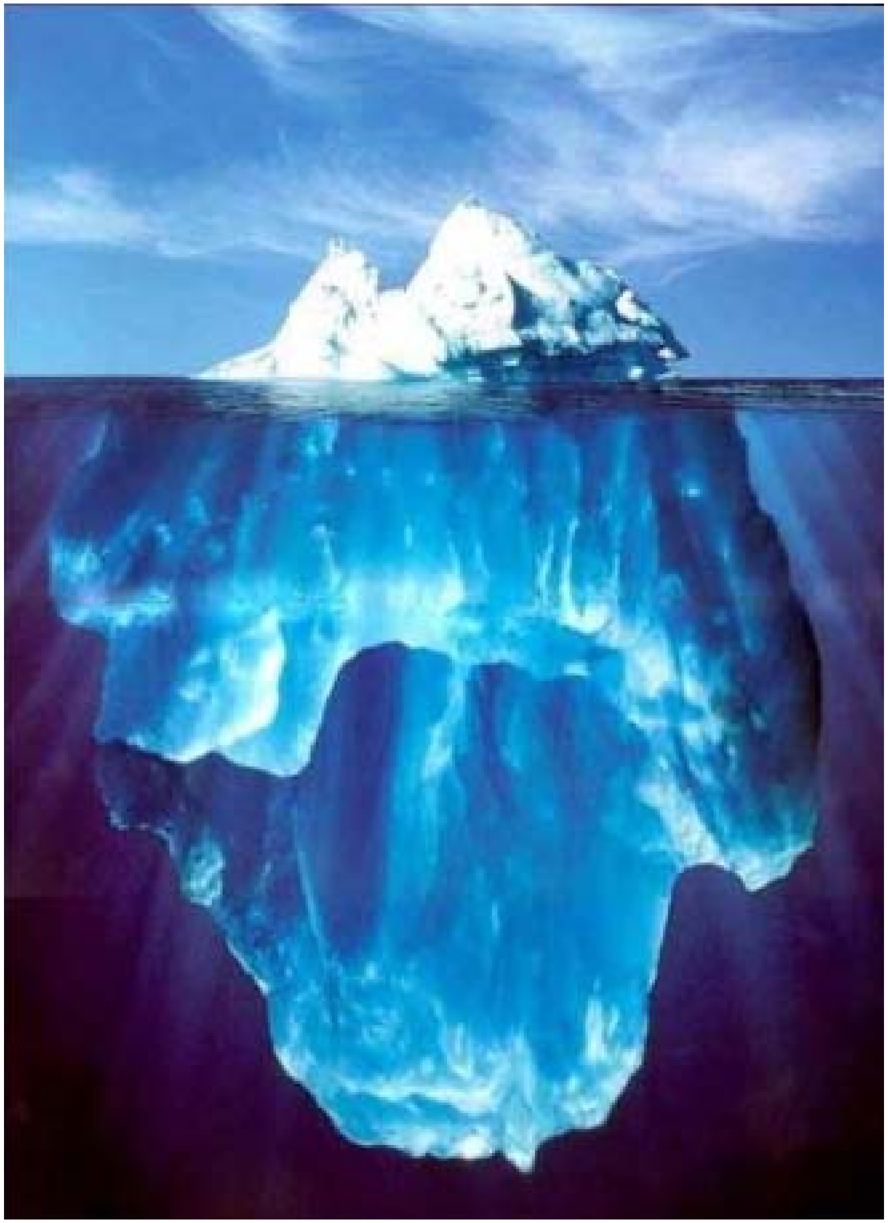
Bouyant forces:

the tip of the iceburg

$$F_B = m_{ice} g$$

$$\rho_{water} g V_{submerged} = \rho_{ice} g V_{total}$$

$$\frac{V_{submerged}}{V_{total}} = \frac{\rho_{ice}}{\rho_{water}} \approx \frac{0.917}{1.024} = 93.9\%$$



Source:http://bb-bird.com/iceburg.html

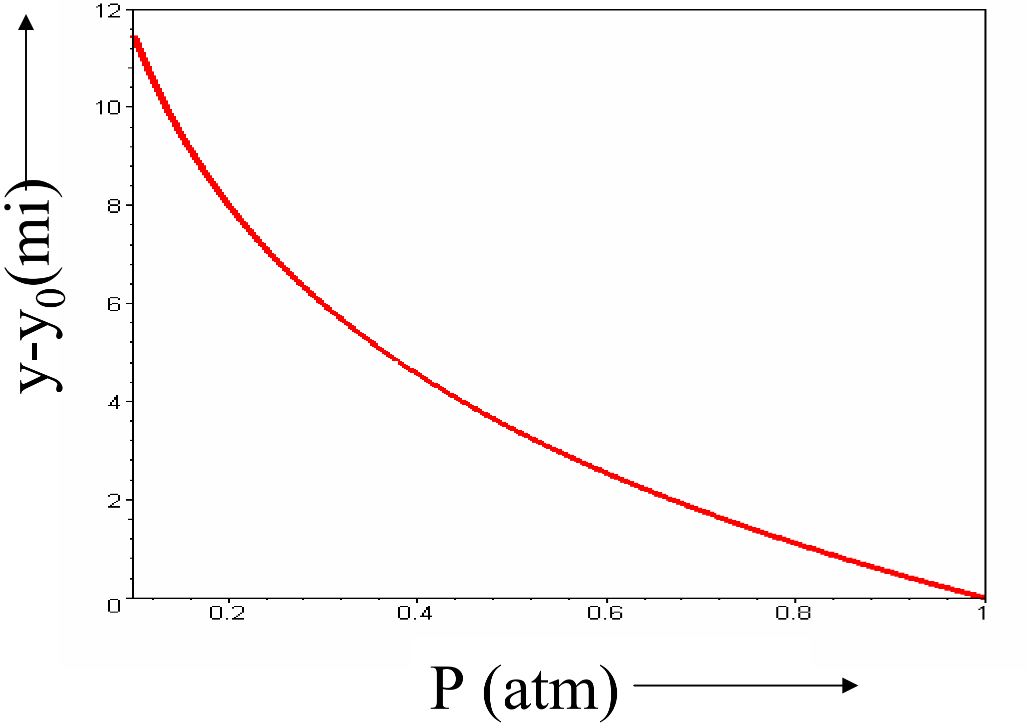
Effects of the weight of a "compressible" fluid on pressure.

$$\frac{dP}{dy} = -\rho g$$

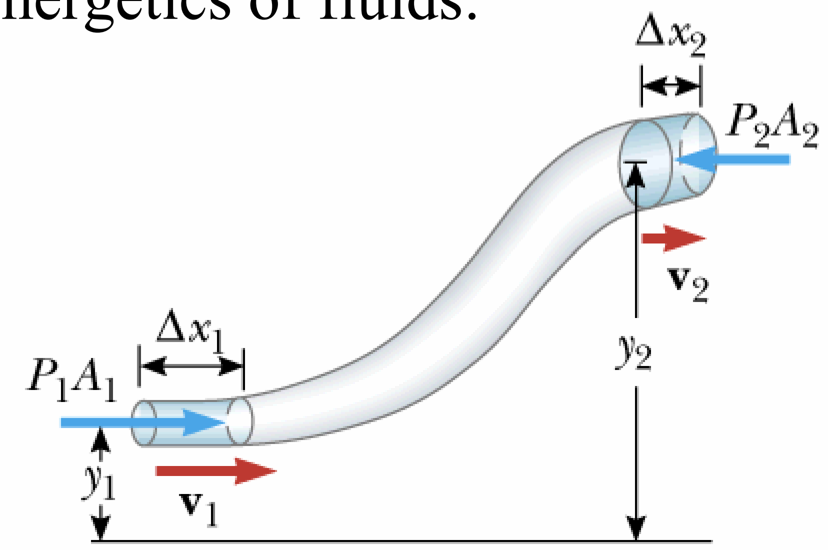
For a gas, $\rho = P\frac{\rho_0}{P_0}$

$$\frac{dP}{dy} = -P\left(\frac{\rho_0 g}{P_0}\right)$$

Solution : $P(y) = P_0 e^{-\frac{\rho_0 g}{P_0}(y-y_0)} \approx P_0 e^{-\frac{y-y_0}{8000m}} \approx P_0 e^{-\frac{y-y_0}{5mi}}$



Energetics of fluids:



Harcourt, Inc.

$\Delta x_1 = v_1 \Delta t$

$\Delta x_2 = v_2 \Delta t$

$A_1 \Delta x_1 = A_2 \Delta x_2$

$m = \rho A_1 \Delta x_1$

$K_2 + U_2 = K_1 + U_1 + W_{12}$

$\frac{1}{2} m v_2^2 + mgh_2 = \frac{1}{2} m v_1^2 + mgh_1 + (P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2)$

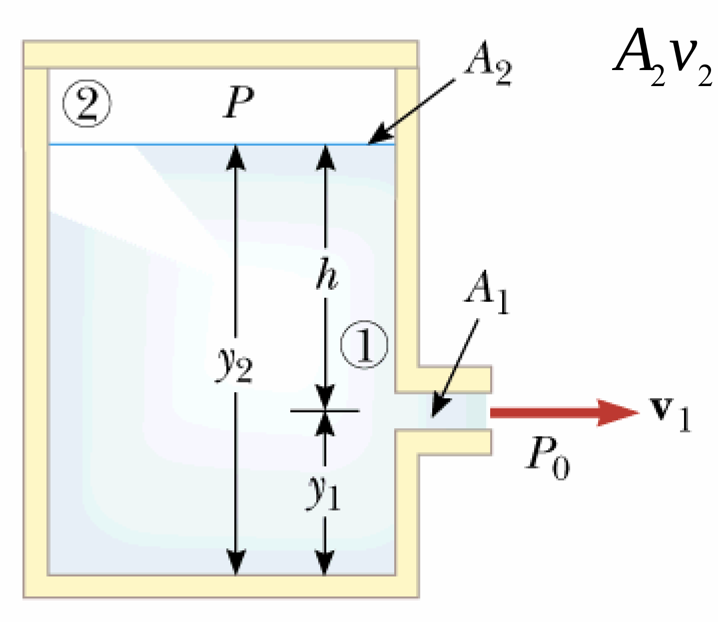
$\Rightarrow P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 = P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1$

Bernoulli's equation:

$$P_2 + ½\,\rho v_2^2 + \rho g h_2 = P_1 + ½\,\rho v_1^2 + \rho g h_1$$

Example: Suppose we know A_1, A_2, ρ, P, y_1, y_2

$$P + ½\,\rho v_2^2 + \rho g y_2 = P_0 + ½\,\rho v_1^2 + \rho g y_1$$



$$A_2 v_2 = A_1 v_1$$

$$v_1 = \sqrt{\frac{2(gh - (P_0 - P)/\rho)}{1 - \left({}^{A_1}\!/\!{}_{A_2}\right)^2}}$$

Thermodynamic statement of conservation of energy –

First Law of Thermodynamics

$$\boxed{\Delta E_{int} = Q - W}$$

Work done by system

Heat added to system

“Internal” energy of system

How is temperature related to E_{int}?

Consider an ideal gas

- ➔ Analytic expressions for physical variables
- ➔ Approximates several real situations

Ideal Gas Law: $\mathbf{P\,V = n\,R\,T}$

pressure (Pa)

volume (m^3)

number of moles

gas constant (8.31 J/(mole ·K))

temperature (K)

Review of results from ideal gas analysis in terms of the specific heat ratio $\gamma \equiv C_P/C_V$:

$$\Delta E_{\text{int}} = \frac{n}{\gamma\text{-}1} R\Delta T = nC_V \Delta T$$

$$C_V = \frac{R}{\gamma\text{-}1} \qquad\qquad C_P = \frac{\gamma R}{\gamma\text{-}1}$$

$$\text{Note}: \; \gamma = \frac{C_P}{C_V}$$