Announcements

1. 3rd exam
   Redo due Thursday, Dec. 4th
   Presentations for Exam 3 (or 1 or 2) ???
   if ?? when?? how many??
2. Final exam, Wednesday, Dec. 10th
   Extra problem solving sessions??
   if ?? when?? how many??
3. Physics colloquium Thursday, Dec. 4th at 4 PM
   Professor Scott Wollenwebber, WFU School of Medicine -- "Positron Emission Tomography: From Basic Physics to Functional Images"
4. Today's lecture – analysis of thermodynamic processes
   Efficiency of a thermodynamic process
   Carnot, Otto, Diesel processes
   Notion of entropy

Review of thermodynamic ideas

"First law" of thermodynamics: $\Delta E_{int} = Q - W$

For an ideal gas: $PV = nRT$

$$\Delta E_{int} = \frac{n}{\gamma\text{-}1} R\Delta T = nC_V \Delta T \quad ; \; C_V = \frac{R}{\gamma\text{-}1}, \quad \text{where: } \gamma \equiv \frac{C_P}{C_V}$$

Special cases: Isovolumetric (V=constant) ➔ W = 0

Isobaric (P=constant) ➔ $C_P = \dfrac{\gamma R}{\gamma - 1}$

Isothermal process (T=constant) ➔ $\Delta E_{int} = 0$

$$W = \int_{V_i}^{V_f} P dV = nRT \ln\left(\frac{V_f}{V_i}\right) = P_i V_i \ln\left(\frac{V_f}{V_i}\right)$$

Adiabatic process (Q = 0)

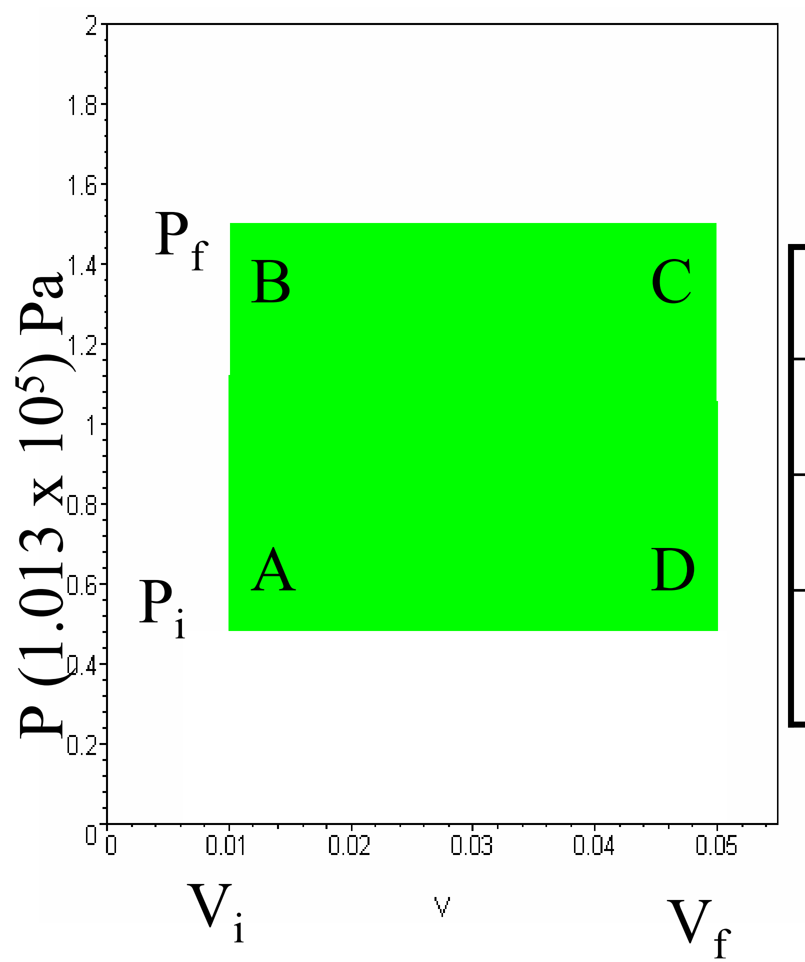
$$P_i V_i^{\gamma} = P_f V_f^{\gamma} \qquad T_i V_i^{\gamma\text{-}1} = T_f V_f^{\gamma\text{-}1}$$

Extra credit:

Show that the work done by an ideal gas which has an initial pressure P_i and initial volume V_i when it expands *adiabatically* to a volume V_f is given by:

$$W = \int_{V_i}^{V_f} P dV = \frac{P_i V_i}{\gamma - 1}\left(1 - \left(\frac{V_i}{V_f}\right)^{\gamma-1}\right)$$

Examples process by an ideal gas:

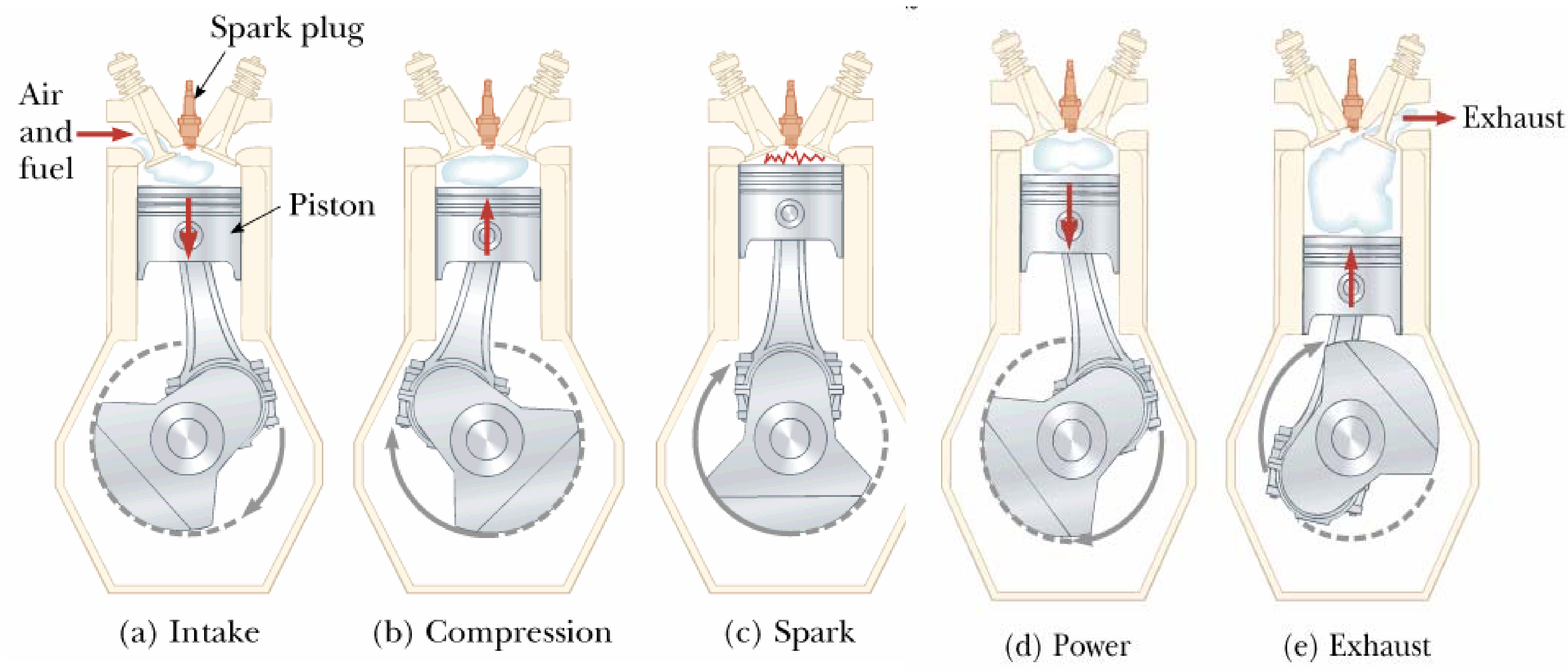


	A→B	B→C	C→D	D→A
Q	$\frac{V_i(P_f - P_i)}{\gamma - 1}$	$\frac{\gamma P_f(V_f - V_i)}{\gamma - 1}$	$\frac{-V_f(P_f - P_i)}{\gamma - 1}$	$\frac{-\gamma P_i(V_f - V_i)}{\gamma - 1}$
W	0	$P_f(V_f - V_i)$	0	$-P_i(V_f - V_i)$
ΔE_{int}	$\frac{V_i(P_f - P_i)}{\gamma - 1}$	$\frac{P_f(V_f - V_i)}{\gamma - 1}$	$\frac{-V_f(P_f - P_i)}{\gamma - 1}$	$\frac{-P_i(V_f - V_i)}{\gamma - 1}$

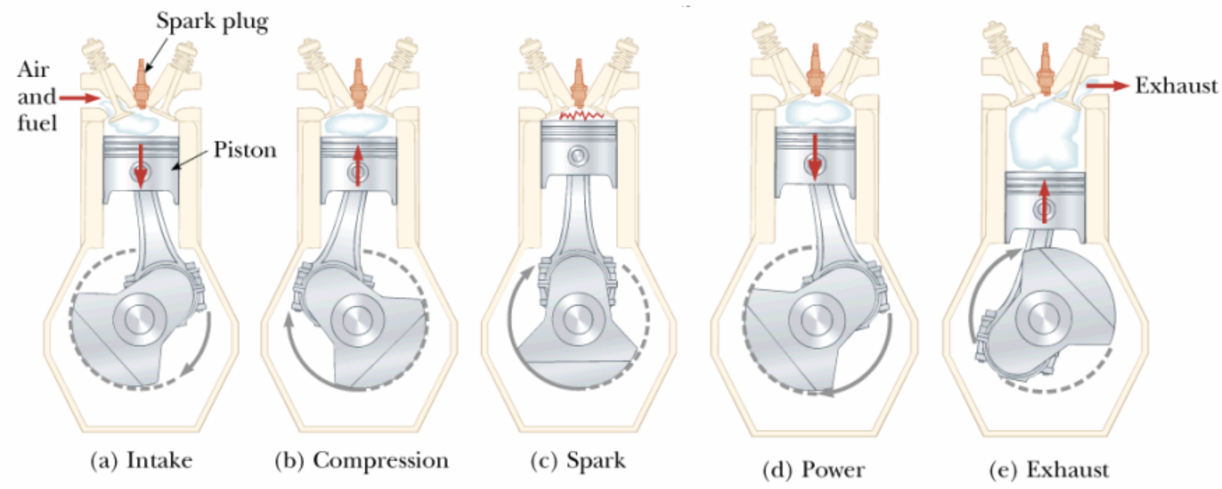
Efficiency as an engine:

$$e = W_{net/} Q_{input}$$

Otto cycle:


Spark plug
Air and fuel
Piston
Exhaust
(a) Intake
(b) Compression
(c) Spark
(d) Power
(e) Exhaust

Otto cycle:


Spark plug
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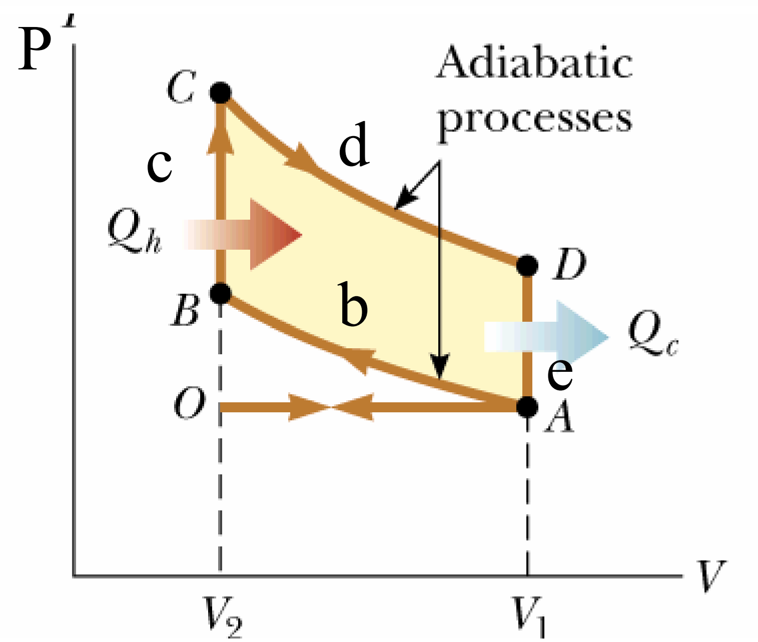
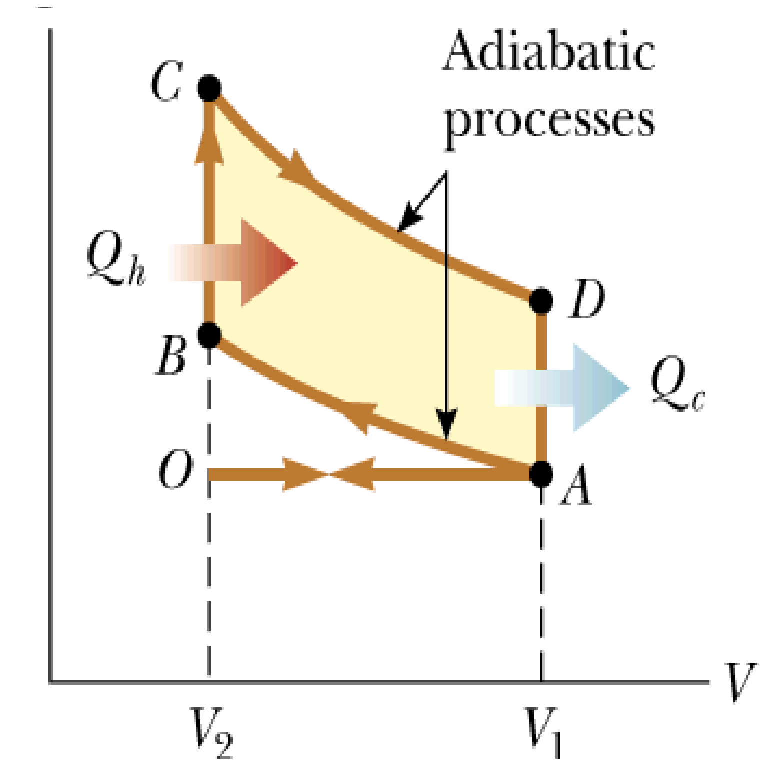

P
C
c
d
Adiabatic
processes
Q_h
D
B
b
Q_c
e
O
A
V
V_2
V_1

P Otto cycle



$Q_{AB}=0$

$$Q_{BC} = \frac{V_2(P_C - P_B)}{\gamma - 1}$$

$Q_{CD}=0$

$$Q_{DA} = \frac{-V_1(P_D - P_A)}{\gamma - 1}$$

$$P_A V_1^\gamma = P_B V_2^\gamma;$$

$$P_D V_1^\gamma = P_C V_2^\gamma$$

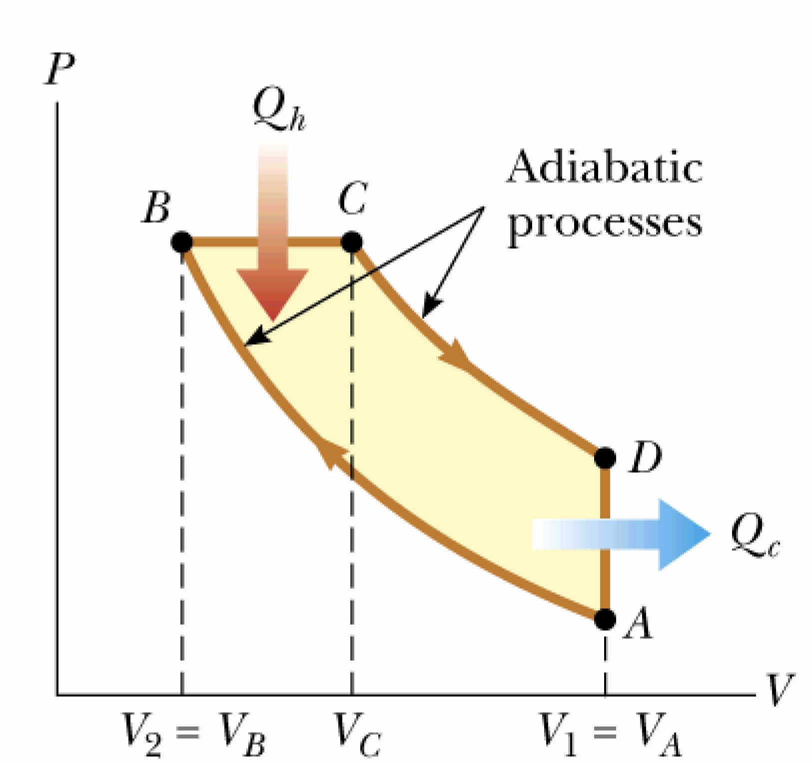
$$e = \frac{Q_{BC} + Q_{DA}}{Q_{BC}} = 1 + \frac{Q_{DA}}{Q_{BC}} = 1 - \frac{V_1(P_D - P_A)}{V_2(P_C - P_B)}$$

$$\Rightarrow e = 1 - \frac{1}{(V_1/V_2)^{\gamma-1}}$$

Example: r=5, γ=1.4

e=0.475

Diesel cycle



$Q_{AB}=0$

$$Q_{BC} = \frac{\gamma P_B (V_C - V_B)}{\gamma - 1}$$

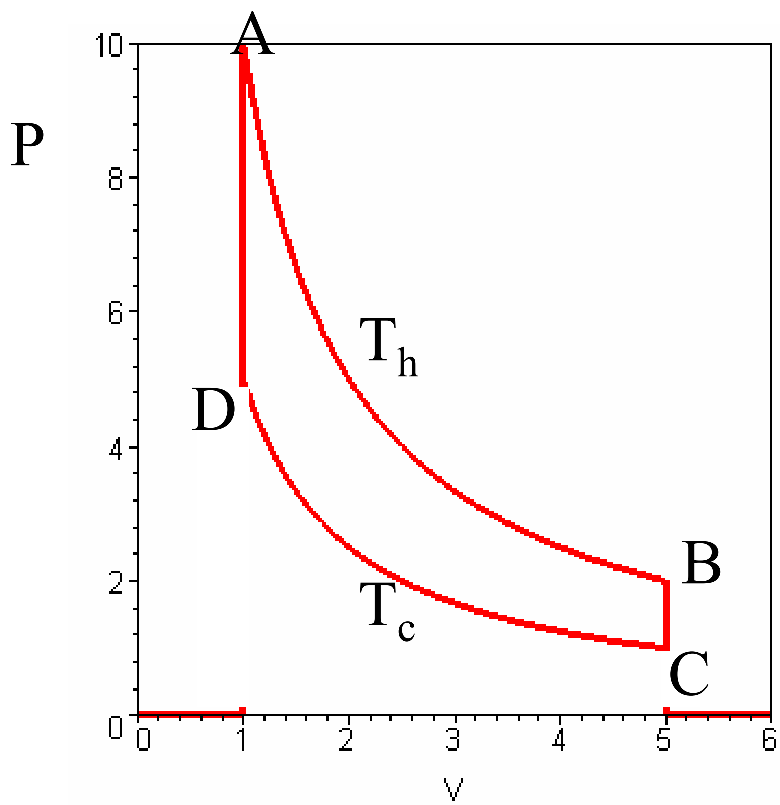
$Q_{CD}=0$

$$Q_{DA} = \frac{-V_D (P_D - P_A)}{\gamma - 1}$$

$$e = 1 - \frac{1}{\gamma}\left(\frac{\left[\frac{1}{V_D/V_C}\right]^{\gamma} - \left[\frac{1}{V_A/V_B}\right]^{\gamma}}{\left[\frac{1}{V_D/V_C}\right] - \left[\frac{1}{V_A/V_B}\right]} \right)$$

Example: $V_D/V_C=5$, $V_A/V_B=15$, $\gamma=1.4$ $\quad e=0.558$

Stirling engine



$$e = \frac{W_{AB} + W_{CD}}{Q_{AB} + Q_{DA}}$$

	A→B	B→C	C→D	D→A
Q	$nRT_h \ln\left(\frac{V_B}{V_A}\right)$	$-\frac{nR(T_h - T_c)}{\gamma - 1}$	$-nRT_c \ln\left(\frac{V_C}{V_D}\right)$	$\frac{nR(T_h - T_c)}{\gamma - 1}$
W	$nRT_h \ln\left(\frac{V_B}{V_A}\right)$	0	$-nRT_c \ln\left(\frac{V_C}{V_D}\right)$	0
ΔE_{int}	0	$-\frac{nR(T_h - T_c)}{\gamma - 1}$	0	$\frac{nR(T_h - T_c)}{\gamma - 1}$

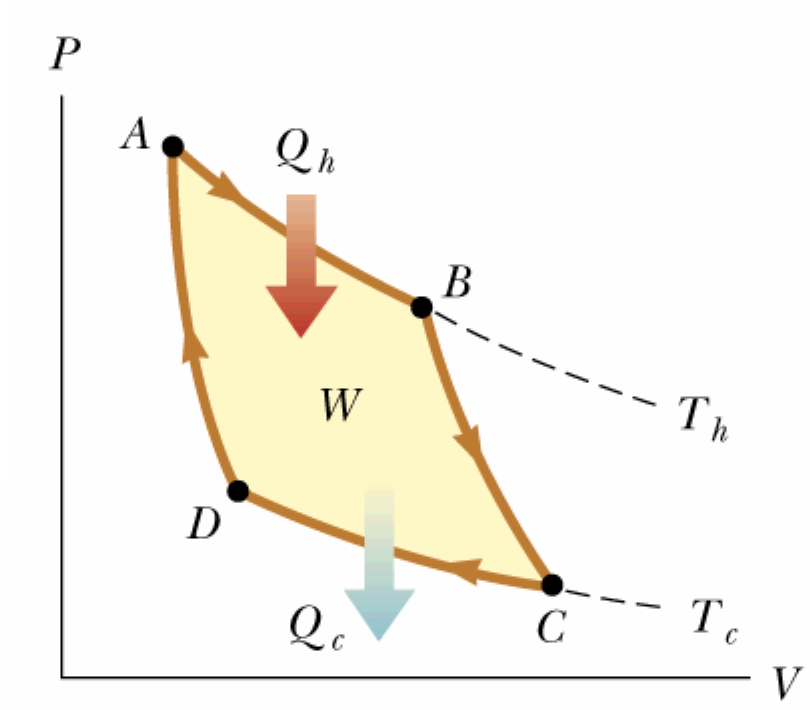
Example:

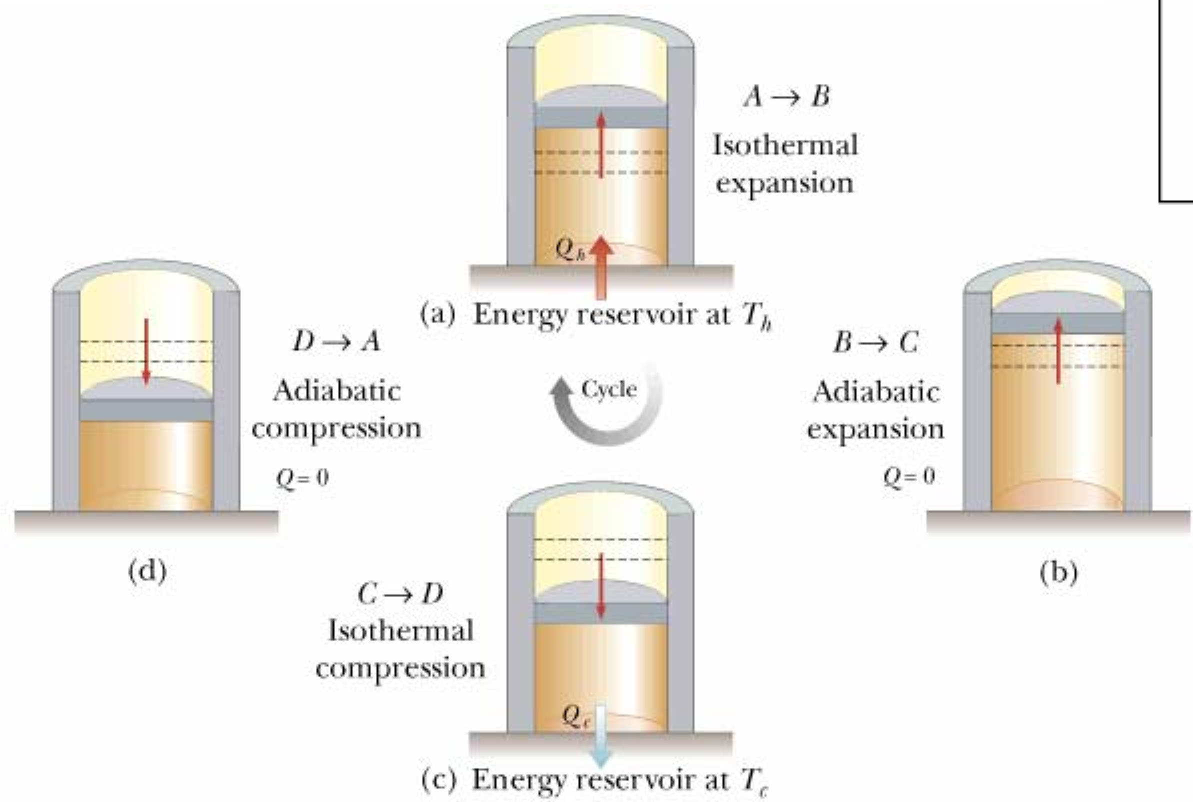
$T_h = 3T_c \quad V_B = V_C = 5V_A = 5V_D \quad \gamma = 1.3$

$$e = \frac{nR(T_h - T_c)\ln(V_B / V_A)}{nRT_h \ln(V_B / V_A) + \frac{nR(T_h - T_c)}{\gamma - 1}} = 50.6\%$$

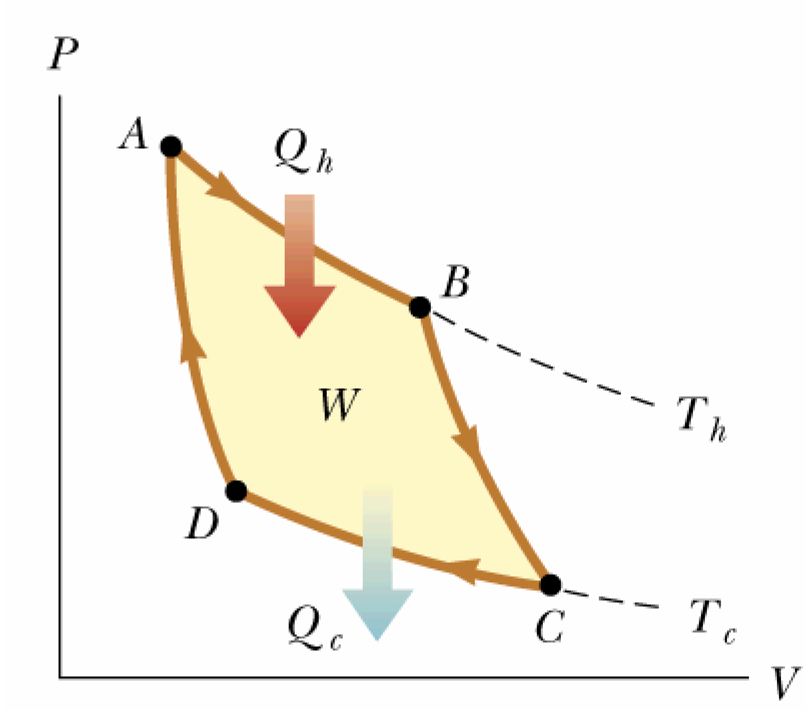
$$e_{\text{Carnot}} = 66.7\%$$

Carnot process





Carnot cycle



	A→B	B→C	C→D	D→A
Q	$nRT_h \ln\left(\frac{V_B}{V_A}\right)$	0	$-nRT_c \ln\left(\frac{V_C}{V_D}\right)$	0
W	$nRT_h \ln\left(\frac{V_B}{V_A}\right)$	$\frac{nR(T_h - T_c)}{\gamma - 1}$	$-nRT_c \ln\left(\frac{V_C}{V_D}\right)$	$-\frac{nR(T_h - T_c)}{\gamma - 1}$
ΔE_{int}	0	$-\frac{nR(T_h - T_c)}{\gamma - 1}$	0	$\frac{nR(T_h - T_c)}{\gamma - 1}$

$$e = \frac{Q_{AB} + Q_{CD}}{Q_{AB}}$$

$$= 1 - \frac{T_c}{T_h} \frac{\ln(V_D / V_C)}{\ln(V_B / V_A)}$$

$$= 1 - \frac{T_c}{T_h}$$

Carnot cycle

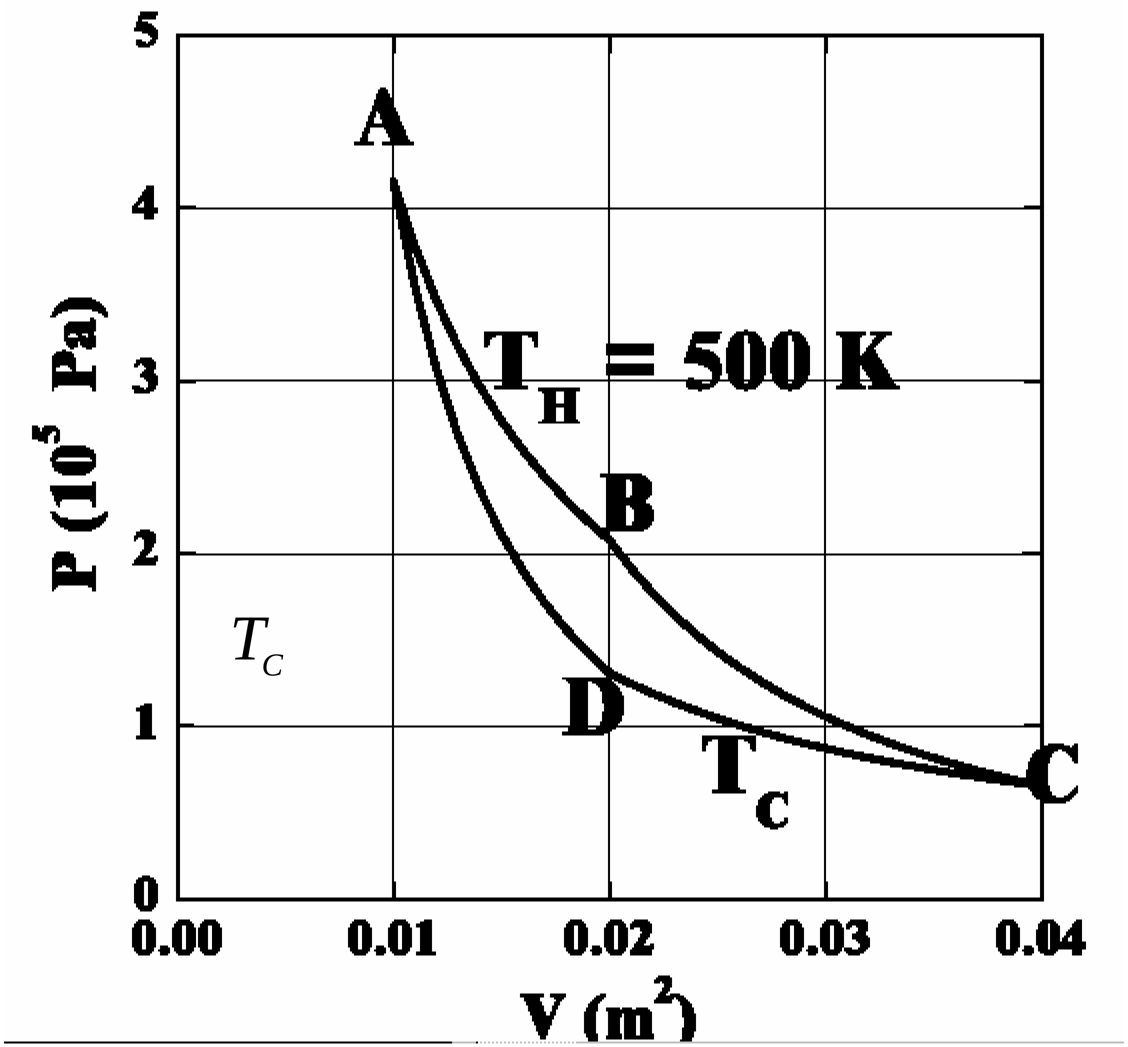
$$Q_{AB} = nRT_H \ln\left(\frac{V_B}{V_A}\right)$$

$Q_{BC}=0$

$$Q_{CD} = -nRT_C \ln\left(\frac{V_C}{V_D}\right)$$

$Q_{DA}=0$

$$e = \frac{Q_{AB}+Q_{CD}}{Q_{AB}} = 1 - \frac{T_C}{T_H}$$



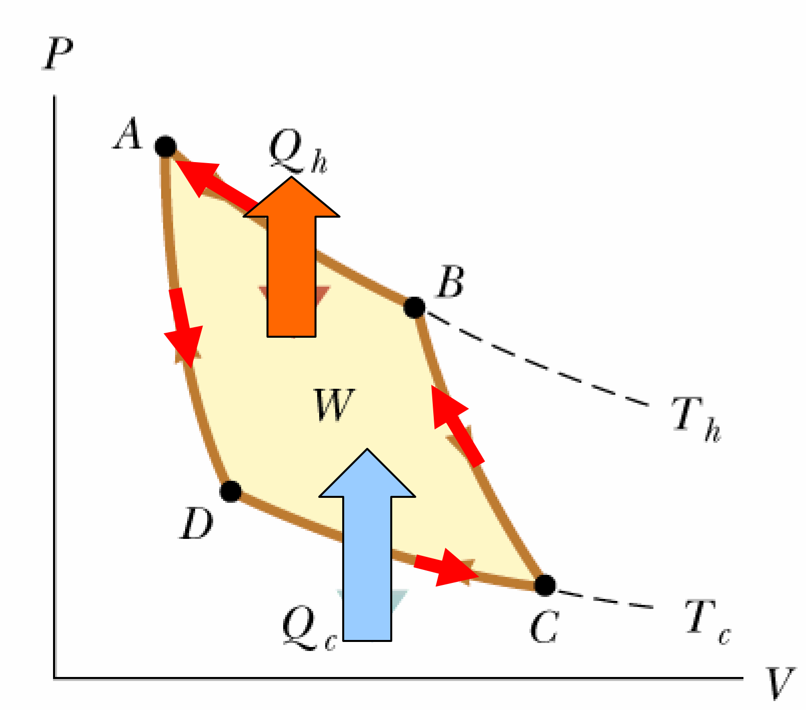
Examples

Efficiency of a Carnot engine operating between the temperatures of T_c=0°C and T_h=100°C:

$$e = 1 - \frac{273.15}{373.15} = 26.8\%$$

➔ For a Carnot engine, it is clear that we cannot achieve e=100%; not possible to completely transform heat into work. It is possible to show that the Carnot engine is the most efficient that one can construct between the two operating temperatures T_c and T_h.

Carnot cycle for cooling and heating



"coefficient of performance"

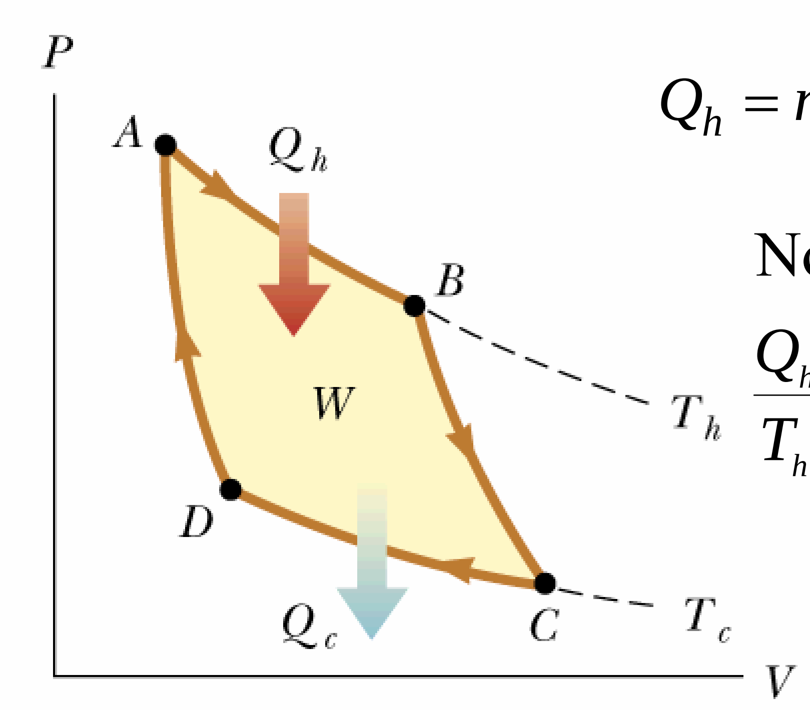
$COP_{heating}=|Q_h/W|=T_h/(T_h-T_c)$

$COP_{cooling}=|Q_c/W|=T_c/(T_h-T_c)$

Example: Suppose that on a cold winter day, a heat pump has a compressor which brings outdoor air at T_c=-3°C into a room at T_h=22°C. What is the COP?

COP=295.15/25=11.8

More about Carnot cycle



$$Q_h = nRT_h \ln\left(\frac{V_B}{V_A}\right) \qquad Q_c = -nRT_c \ln\left(\frac{V_B}{V_A}\right)$$

Notice that :

$$\frac{Q_h}{T_h} + \frac{Q_c}{T_c} = \frac{|Q_h|}{T_h} - \frac{|Q_c|}{T_c} = 0$$

$$S_{AB} = nR \ln\left(\frac{V_B}{V_A}\right)$$

$$S_{BC} = 0$$

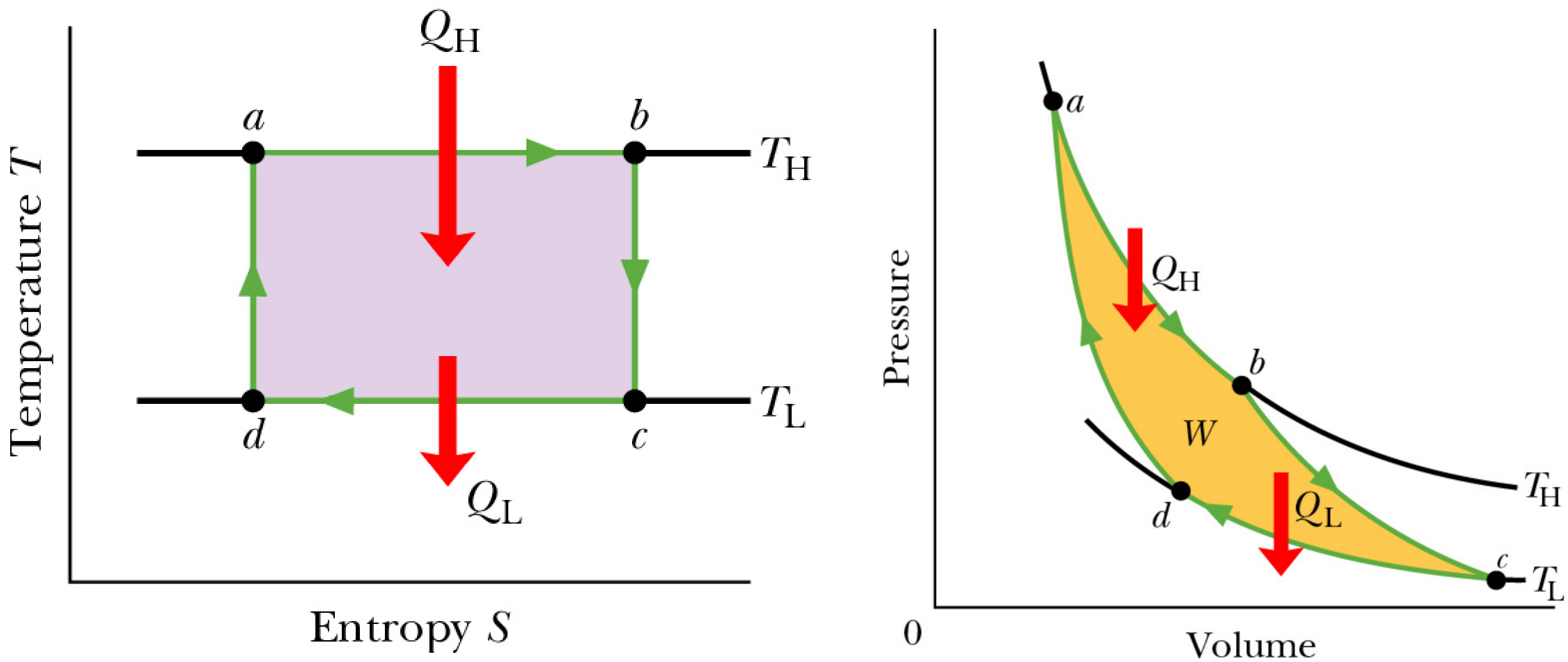
$$S_{CD} = -nR \ln\left(\frac{V_B}{V_A}\right)$$

$$S_{DA} = 0$$

Define entropy :

$$S_{AB} = \int_A^B \frac{dQ}{T}$$

Carnot cycle shown in a T-S diagram:



Other examples of entropy calculations:

Ideal gas:

Isovolumetric process:

$$dQ = nC_V dT = \frac{nR}{\gamma - 1} dT$$

$$S = \frac{nR}{\gamma - 1}\int_A^B \frac{dT}{T} = \frac{nR}{\gamma - 1}\ln\left(\frac{T_B}{T_A}\right)$$

Isobaric process:

$$dQ = nC_P dT = \frac{\gamma nR}{\gamma - 1} dT$$

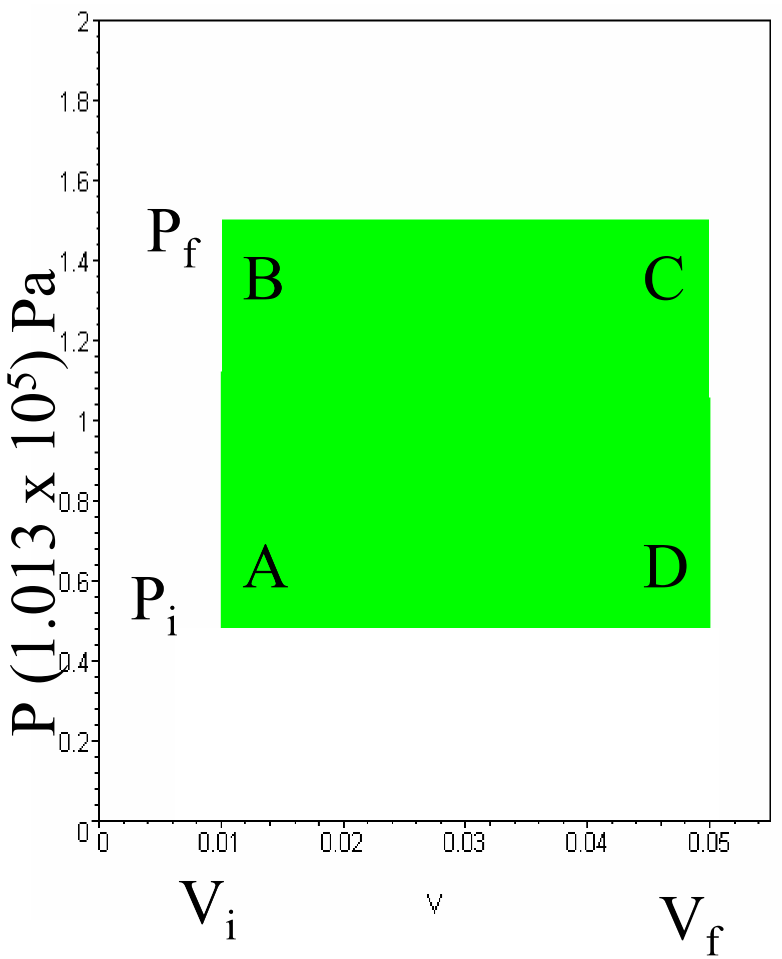
$$S = \frac{\gamma nR}{\gamma - 1}\int_A^B \frac{dT}{T} = \frac{\gamma nR}{\gamma - 1}\ln\left(\frac{T_B}{T_A}\right)$$

Melting of solid having mass *m* and latent heat *L* at melting temperature T_M:

$$S = \int_0^m \frac{L\,dm}{T} = \frac{Lm}{T_M}$$

Peer instruction question:

Consider the "square cycle" shown below. What can you say about the entropy change in each cycle:



(A) $S_{ABCDA} = 0$

(B) $S_{ABCDA} > 0$

(C) $S_{ABCDA} < 0$

Online Quiz for Lecture 21
Entropy -- Dec. 2, 2003

Suppose that you have 1 kg of ice at temperature 273.16 K. What would be the change in entropy if the ice were completely melted to water at 273.16 K?
(a) 1.219 J/K (b) 1219 J/K (c) 333000 J/K (d) 2256000 J/K

4. HRW6 21.P.023. [52279] A Carnot engine operates between 226°C and 126°C, absorbing 6.30×10^4 J per cycle at the higher temperature.

(a) What is the efficiency of the engine?

[.1] ____ %

(b) How much work per cycle is this engine capable of performing?

[.1] ____ J

6. HRW6 21.P.046. [52282] An inventor claims to have invented four engines, each of which operates between constant-temperature reservoirs at 400 and 300 K. Data on each engine, per cycle of operation, are:

engine A, $Q_H = 200$ J, $Q_L = -175$ J, and $W = 40$ J;
engine B, $Q_H = 500$ J, $Q_L = -200$ J, and $W = 400$ J;
engine C, $Q_H = 600$ J, $Q_L = -200$ J, and $W = 400$ J;
engine D, $Q_H = 100$ J, $Q_L = -90$ J, and $W = 10$ J.

Of the first and second laws of thermodynamics, which (if either) does each engine violate?

engine A
- both
- neither
- second law
- first law [.1]

engine B
- second law
- both
- first law
- neither [.1]

engine C
- neither
- second law
- first law
- both [.1]

engine D
- both
- first law
- second law
- neither [.1]

5. HRW6 21.P.029. [52280] One mole of an ideal monatomic gas is taken through the cycle shown in Fig. 21-24. Assume that $p = 2p_0$, $V = 2V_0$, $p_0 = 1.03 \times 10^5$ Pa, and $V_0 = 0.0227$ m^3.

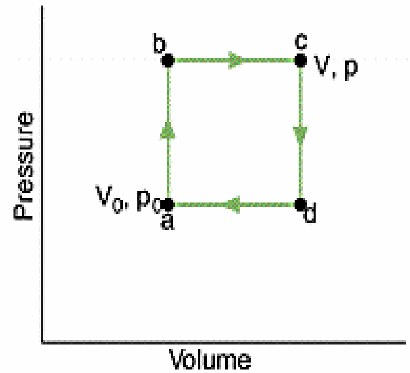


Figure 21-24.

(a) Calculate the work done during the cycle.

[.1] [2340] J

(b) Calculate the energy added during stroke *abc*.

[.1] [15200] J

(c) Calculate the efficiency of the cycle.

[.1] [15.4]%

(d) What is the efficiency of an ideal engine operating between the highest and lowest temperatures that occur in the cycle?

[.1] [75]%

How does this compare to the efficiency calculated in (c)?