# Announcements

1. **Web page has been updated –**

   **HW assignments posted through HW 8**

   **Practice exams posted**

2. **Last week – introduced notions of displacements, velocity, and acceleration in one dimension and introduced the general notion of vectors.**

3. **Today's lecture – displacement, velocity, acceleration vectors; trajectory motion**

# Review of vectors:

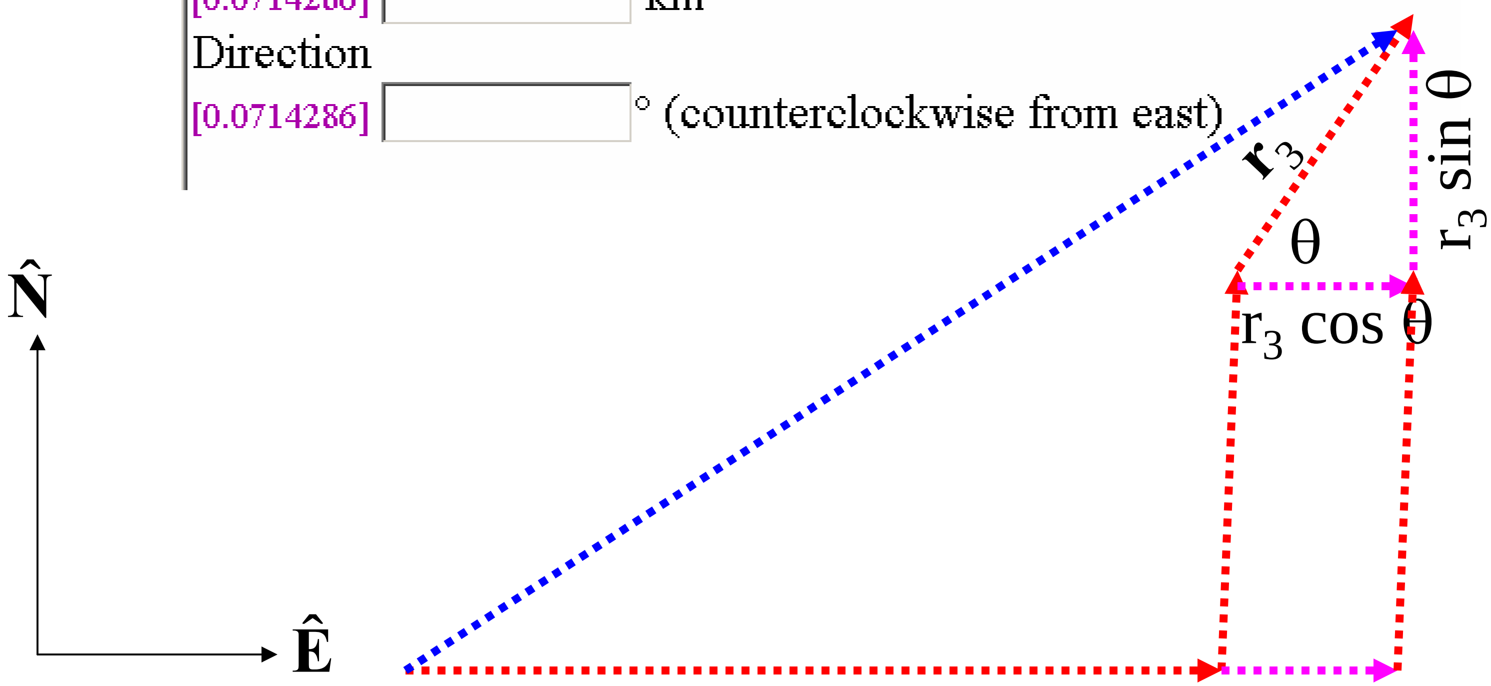
3. HRW6 3.P.010. [56506] A car is driven east for a distance of 51 km, then north for 25 km, and then in a direction 29° east of north for 20 km. Draw the vector diagram and determine the total displacement of the car from its starting point.

Magnitude

[0.0714286] ______ km

Direction

[0.0714286] ______ ° (counterclockwise from east)



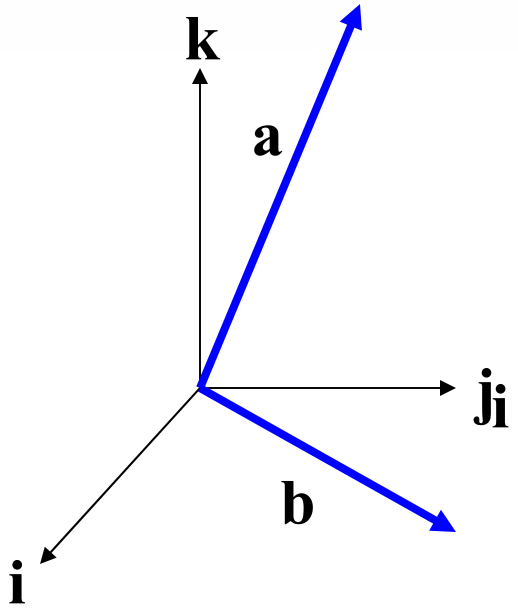
5. HRW6 3.P.031. [52635] Use the definition of scalar product, **a** · **b** = *ab* cos θ, and the fact that **a** · **b** = $a_x b_x + a_y b_y + a_z b_z$ (see Problem 46) to calculate the angle between the two vectors given by **a** = 2.0 **i** + 4.0 **j** + 2.0 **k** and **b** = 5.0 **i** + 3.0 **j** + 6.0 **k**.

[0.0714286] ____ °

$$\mathbf{a}\cdot\mathbf{b} = ab\cos\theta$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$b = \sqrt{b_x^2 + b_y^2 + b_z^2}$$



## Two-dimensional motion

**Displacement:** $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}$

**Velocity:** $\mathbf{v}(t) = \mathbf{v_x}(t)\,\mathbf{i} + \mathbf{v_y}(t)\,\mathbf{j}$ $\quad \mathbf{v}_x = \frac{dx}{dt} \quad \mathbf{v}_y = \frac{dy}{dt}$

**Acceleration:** $\mathbf{a}(t) = \mathbf{a_x}(t)\,\mathbf{i} + \mathbf{a_y}(t)\,\mathbf{j}$ $\quad \mathbf{a}_x = \frac{dv_x}{dt} \quad \mathbf{a}_y = \frac{dv_y}{dt}$

## Special case – Projectile motion

$\mathbf{a} = -g\,\hat{\mathbf{y}}$ (constant; g~9.8 m/s$^2$)

$v_x(t) = v_{xi}$

$x(t) = x_i + v_{xi}\, t$

$v_y(t) = v_{yi} - g\, t$

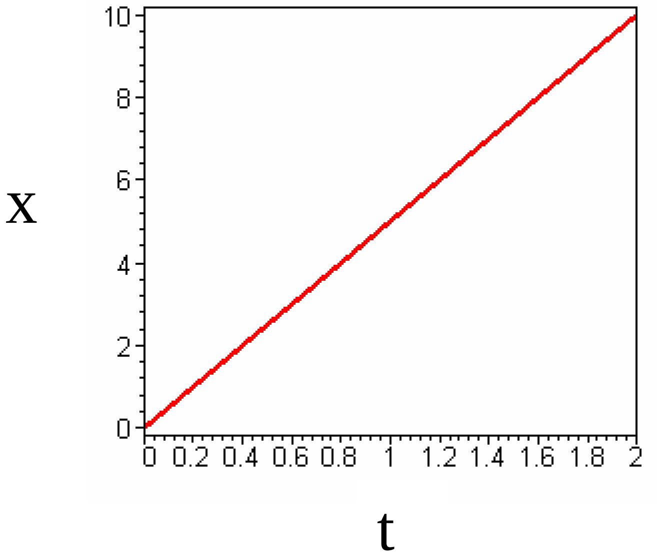
$y(t) = y_i + v_{yi}\, t - \frac{1}{2} g\, t^2$

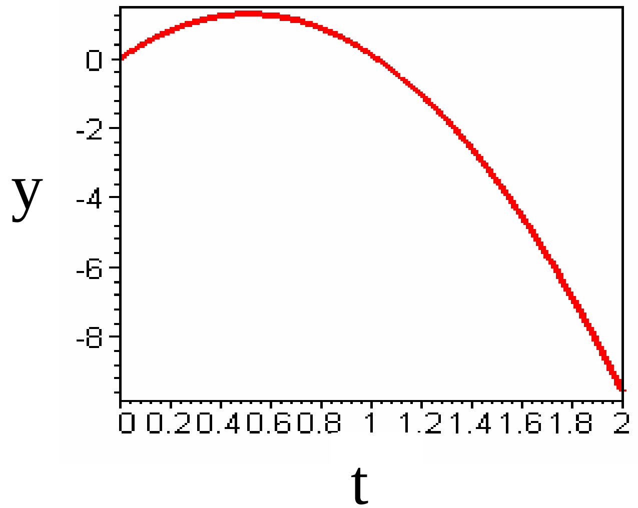
"parametric equations"

**Trajectory:**

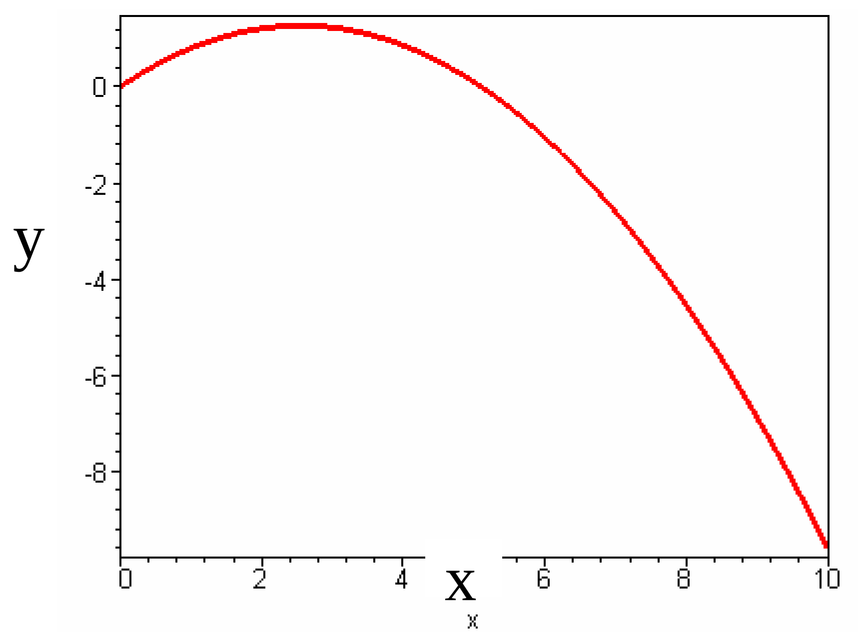
$$y(t) = y_i + v_{yi}\left(\frac{x(t) - x_i}{v_{xi}}\right) - \frac{1}{2} g \left(\frac{x(t) - x_i}{v_{xi}}\right)^2$$

Graphs of parametric equations:

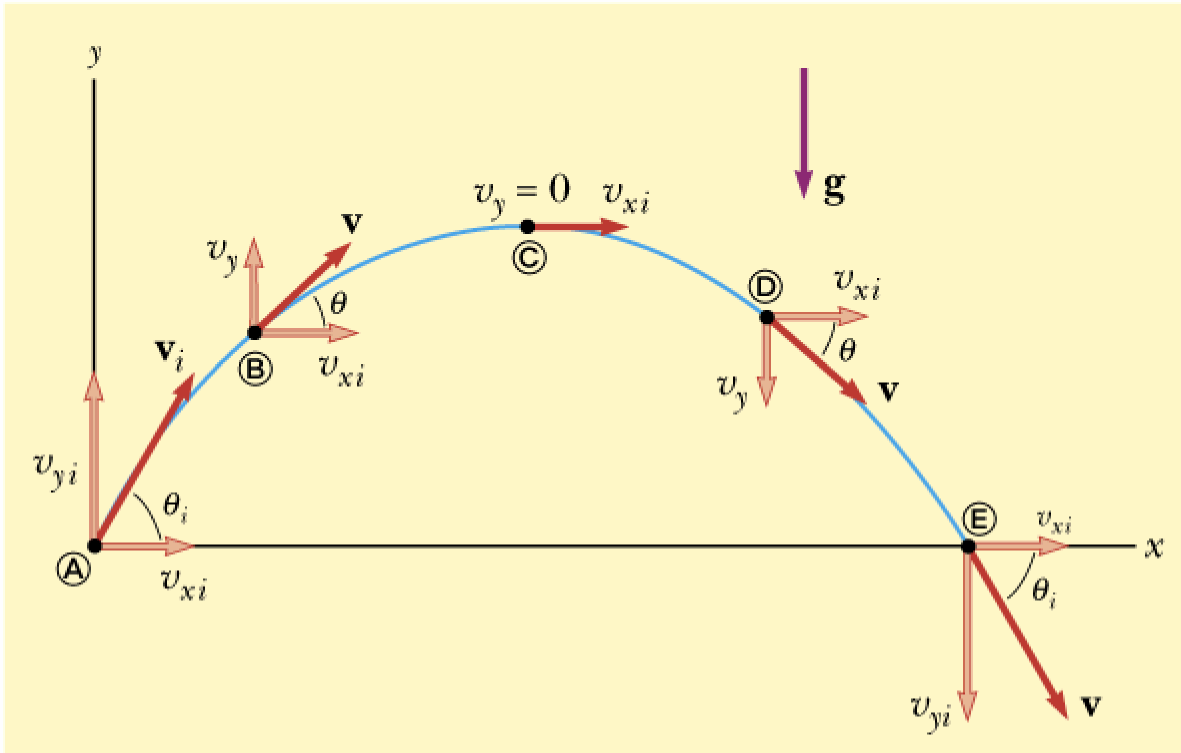



Graph of trajectory:



Serway, Physics for Scientists and Engineers, 5/e
Figure 4.6






$$y(t) = y_i + \tan\theta_i\left(x(t) - x_i\right) - \frac{1}{2}g\left(\frac{x(t) - x_i}{|\,\mathbf{v}_i\,|\cos\theta_i}\right)^2$$

$(v_{xi})$

# Other results derived from projectile equations

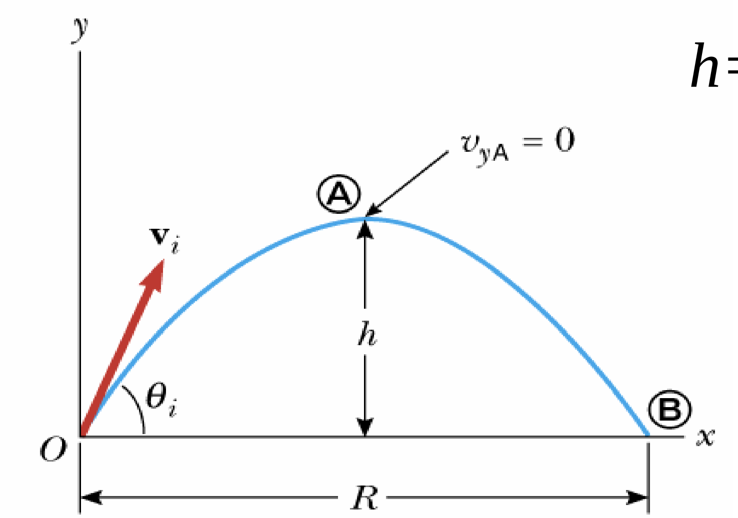
$v_x(t) = v_{xi}$

$x(t) = x_i + v_{xi}\, t$

$v_y(t) = v_{yi} - g\, t$

$y(t) = y_i + v_{yi}\, t - ½\, g\, t^2$

$$y(t) = y_i + \tan\theta_i \left(x(t) - x_i\right) - \frac{1}{2} g \left(\frac{x(t) - x_i}{|\mathbf{v}_i| \cos\theta_i}\right)^2$$




$h = y(t_{top});\quad t_{top} = v_{yi}/g$

$=> h = yi + ½\, v_{yi}^2/g$

$=> R = x(2t_{top}) - x_i = 2v_{xi}v_{yi}/g = v_i^2 \sin(2\theta)/g$

# On line quiz for lecture 4

A small airplane is traveling horizontally in the East direction at a constant velocity of 200 m/s at a height of 600 m, when it passes over your head. At the same time, a ball (which had been stuck to the plane) falls from the bottom of the plane. Suppose you are watching all of this happen and the air friction is negligible (not really a good approximation, but this makes the problem easier to analyze). Also assume that the ground to the east of you is completely flat (no hills, etc.).

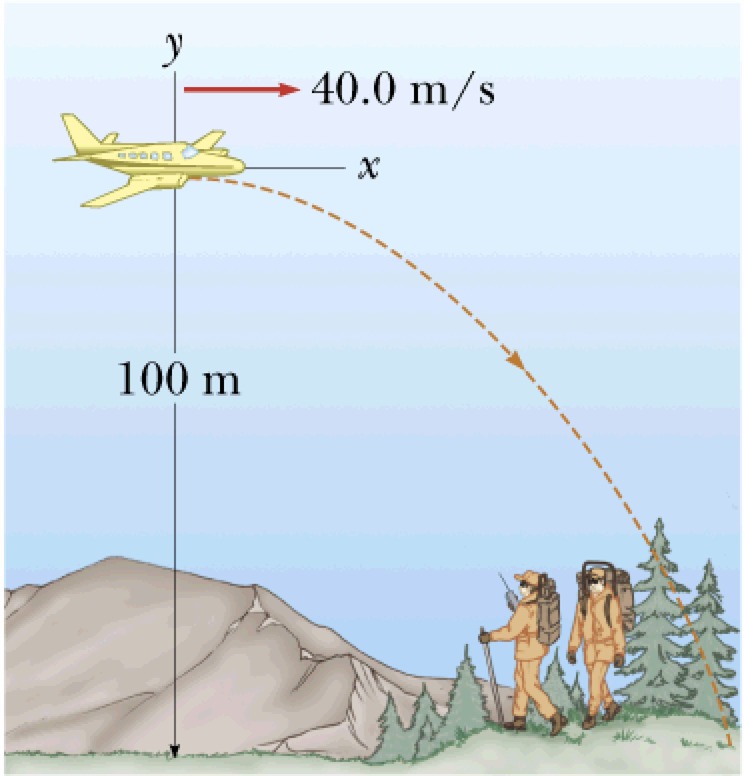
1. Is the ball likely to hit you? (a) yes (b) no

2. What is the magnitude of the horizontal velocity (in m/s) when the ball hits the ground? (a) 2 m/s (b) 20 m/s (c) 200 m/s (d) 2000 m/s

3. What is the magnitude of the vertical velocity when the ball hits the ground? (a) 0 m/s (b) 77 m/s (c) 108 m/s (d) 11760 m/s

4. Where will the ball land on the ground? (a) On your head? (b) Several hundred meters to the East of you? (c) Several thousand meters to the East of you? (d) Several hundred meters to the West of you?

Serway, Physics for Scientists and Engineers, 5/e
Figure 4.13

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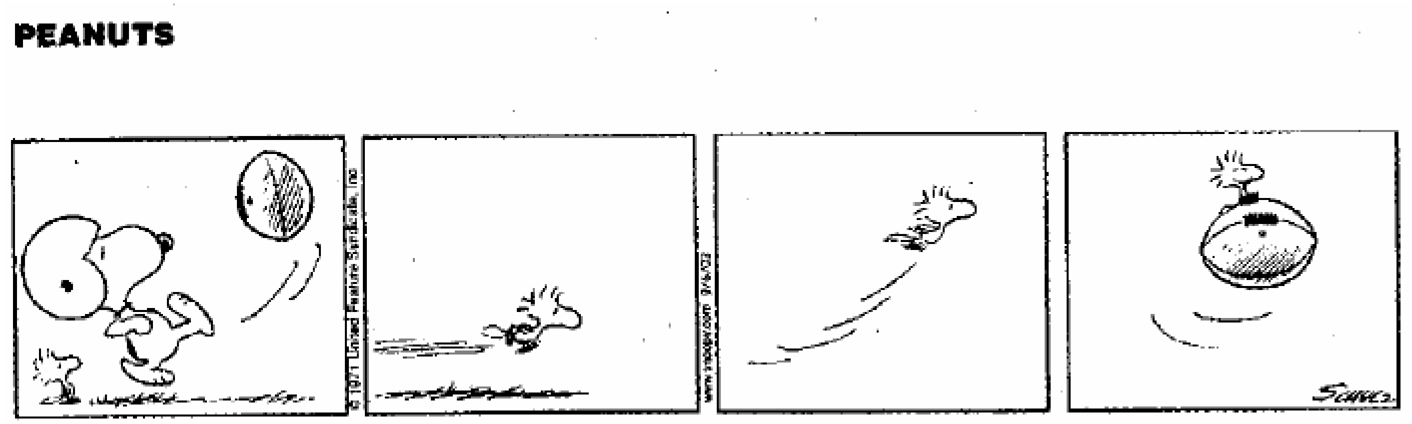
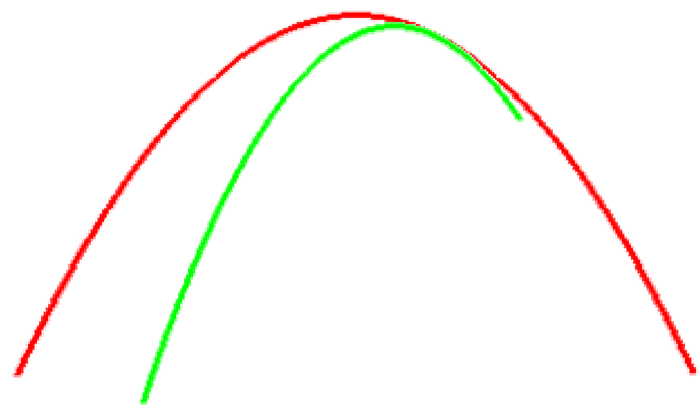
## Peer instruction questions

1. Suppose you hit a golf ball on earth at an initial speed of 50m/s at a $45^o$ angle on Earth so that it travels a distance $R_E$ before hitting the (level) ground. If you were to do the same thing on the moon which has a gravitational acceleration g/6 what distance $R_M$ would it travel? (a) $R_E/36$ (b)$R_E/6$ (c) $6R_E$ (d) $36\ R_E$

2. Suppose that you drop a ball from a height of 100 m above the surface of the earth and it takes a time $t_E$ to fall to the earth's ground. If you were to do the same thing 100 m above the surface of the moon, what is the time $t_M$ that it takes to fall to moon's ground? (a) $t_E/6$ (b) $t_E/\sqrt{6}$ (c) $\sqrt{6}\ t_E$ (d) $6t_E$

From: Monday's WS Journal: (courtesy of Charles Schulz)



$$\mathbf{r}_{ball}(t) = \mathbf{v}^{0}_{ball} t - \frac{1}{2} g \hat{\mathbf{y}} t^{2}$$

$$\mathbf{r}_{Woodstock}(t) = \mathbf{r}^{0}_{Woodstock} + \mathbf{v}^{0}_{Woodstock}[t - t_{Start}] - \frac{1}{2} g \hat{\mathbf{y}} [t - t_{Start}]^{2}$$

# Motion in two dimensions

Summary of equations describing trajectory motion:

**Parametric equations:**

$v_x(t) = v_{xi}$

$x(t) = x_i + v_{xi}\, t$

$v_y(t) = v_{yi} - g\, t$

$y(t) = y_i + v_{yi}\, t - \frac{1}{2}\, g\, t^2$

**Spatial trajectory:**

$$y(t) = y_i + v_{yi}\left(\frac{x(t) - x_i}{v_{xi}}\right) - \frac{1}{2} g \left(\frac{x(t) - x_i}{v_{xi}}\right)^2$$

September 7, 2002

**PHY 113 – "Derivation" of projectile motion equations**

In class, we have stressed the notion that it is important to understand the relationships between the basic equations of physics and the relations that can be derived from them. While these dreaded derivations are not generally a favorite lecture topic, they are very important for understanding the meaning and the proper use of the equations. These notes show an example of a simple derivation for projectile motion. While this derivation is not as formal nor as rigorous as a mathematical "proof", it is adequate for these purposes.

The basic equations for our starting point, are the equations which define the relationships between position, velocity, and acceleration. For this purpose, we will assume that we can describe these quantities in the $x-y$ plane:

$$\mathbf{r}(t) = x(t)\hat{\mathbf{x}} + y(t)\hat{\mathbf{y}}, \qquad (1)$$

$$\mathbf{v}(t) = v_x(t)\hat{\mathbf{x}} + v_y(t)\hat{\mathbf{y}}, \qquad (2)$$

$$\mathbf{a}(t) = a_x(t)\hat{\mathbf{x}} + a_y(t)\hat{\mathbf{y}}. \qquad (3)$$

The velocity is the rate of change of position:

$$\mathbf{v}(t) = \frac{d\mathbf{r}(t)}{dt}, \qquad (4)$$

and the acceleration is the rate of change of velocity:

$$\mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt}. \qquad (5)$$

Additional relationships can be obtained by taking the antiderivatives. The equations (1-5) are general definitions.

Now, let us consider a special case – where the acceleration is constant in space and time and is given by

$$\mathbf{a}(t) = -g\hat{\mathbf{y}}, \qquad (6)$$

with $g = 9.8\text{m/s}^2$, representing the acceleration of gravity in the $-\hat{\mathbf{y}}$ direction near the surface of the earth. By evaluating the relationships (1-5) for this case, we find:

$$v_x(t) = v_{xi} \qquad v_y(t) = v_{yi} - gt \qquad (7)$$

and

$$x(t) = x_i + v_{xi}t \qquad y(t) = y_i + v_{yi}t - \frac{1}{2}gt^2, \qquad (8)$$

where $x_i, y_i, v_{xi}, v_{yi}$ denote initial ($t=0$) positions and velocities.

The equations (6-8) describes how an object moves in a plane. If we know the constants $x_i, y_i, v_{xi}, v_{yi}$, we can determine its position and velocity at any time $t \geq 0$. Now our task is to use these relationships (1-8) to find the direct relationship between the $x$ and $y$. This will enable us to spatially trace out the path of the object as it moves on its trajectory. We can do this by manipulating Eq. (8) with the following steps:

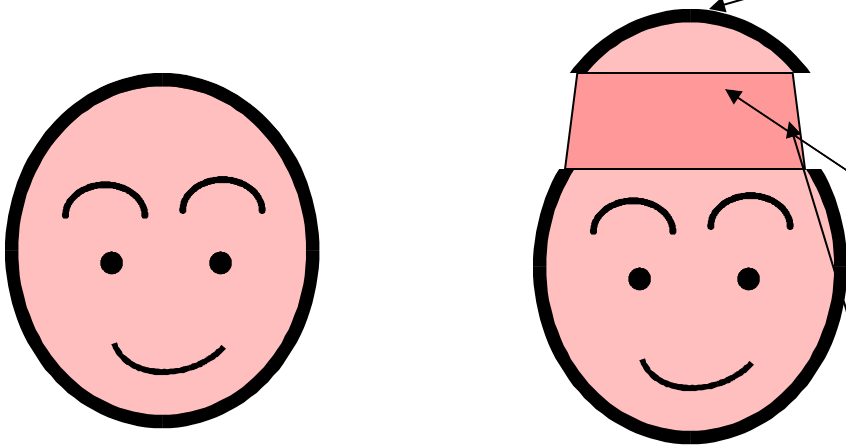
1. Solve the first equation for $t$:

$$x(t) = x_i + v_{xi}t \;\Rightarrow\; t = \frac{x(t) - x_i}{v_{xi}}. \qquad (9)$$

2. Replace $t$ in the second equation with the expression above:

$$y(t) = y_i + v_{yi}\left(\frac{x(t) - x_i}{v_{xi}}\right) - \frac{1}{2}g\left(\frac{x(t) - x_i}{v_{xi}}\right)^2. \qquad (10)$$

This result, shows that for every horizontal position of the object $x(t)$, the vertical position $y(t)$ traces out a parabolic path.



$$\mathbf{v}_x = \frac{dx}{dt}$$

$$\mathbf{a}_x = \frac{dv_x}{dt}$$

$v_x(t) = v_{xi}$

$x(t) = x_i + v_{xi}\, t$

$v_y(t) = v_{yi} - g\, t$

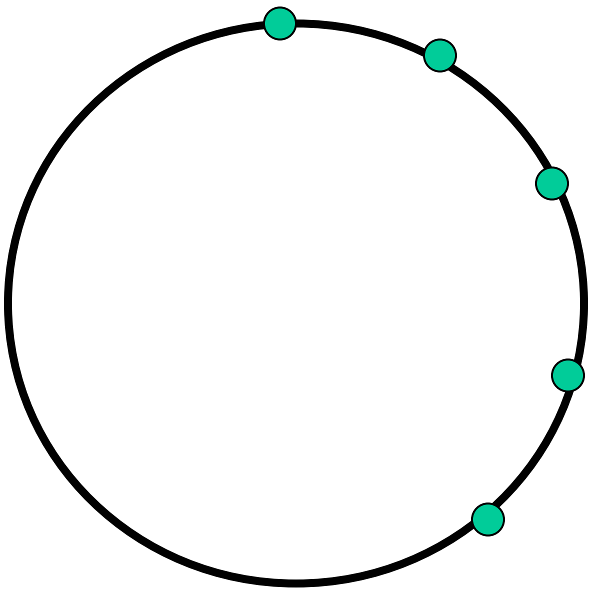
$y(t) = y_i + v_{yi}\, t - ½\, g\, t^2$

$$y(t) = y_i + v_{yi}\left(\frac{x(t) - x_i}{v_{xi}}\right) - \frac{1}{2}\, g \left(\frac{x(t) - x_i}{v_{xi}}\right)^2$$

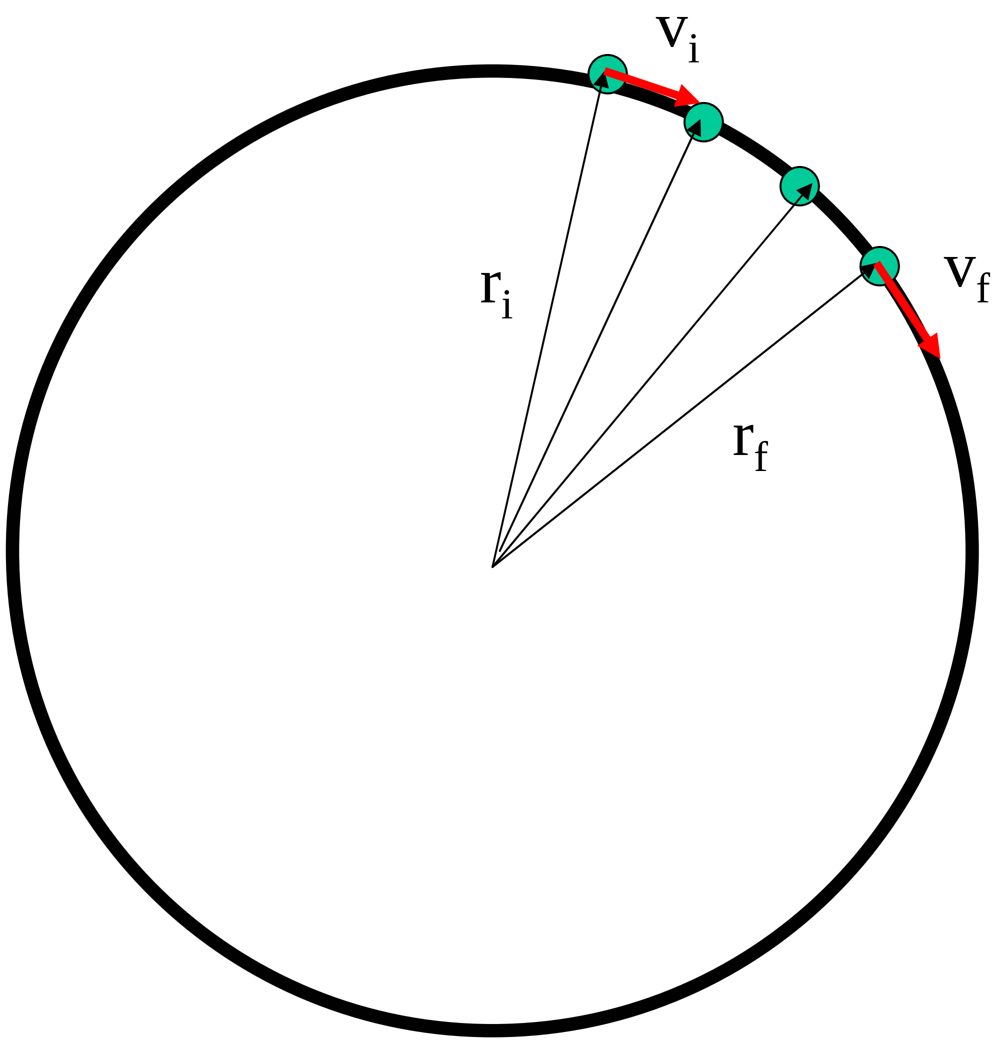
Advice:

1. Keep basic concepts and equations at the top of your head.
2. Construct an equation sheet of commonly used equations for consultation in problem solving (homework and exams).

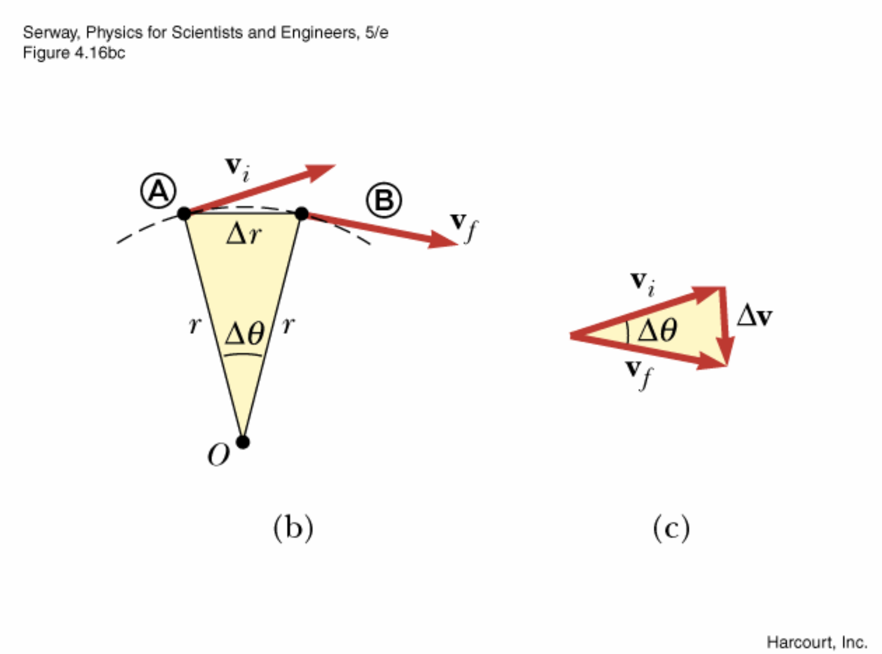
# Uniform circular motion

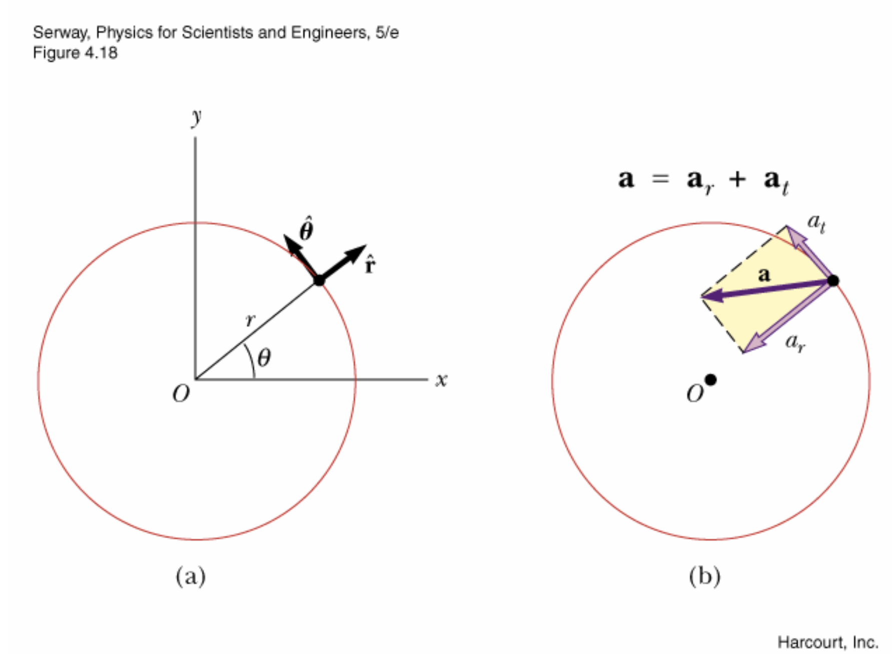


Is the object accelerating? (A) yes (B) no



$v_i$
$r_i$
$v_f$
$r_f$




if $v_i = v_f$ , then: $\mathbf{a} = -\frac{v^2}{r}\hat{\mathbf{r}}$

For example, suppose $v$=5m/s, $r$=0.5m; a=–50$\hat{\mathbf{r}}$ m/s$^2$.

## Peer instruction questions

Think about riding on a merry-go-round. Suppose that you start the merry-go-round rotating and then jump on. After you jump on, the merry-go-round continues to rotate, gradually slowing down. The questions below pertain to you standing on the rotating merry-go-round at a point near the edge.

1. What is the direction of your velocity?

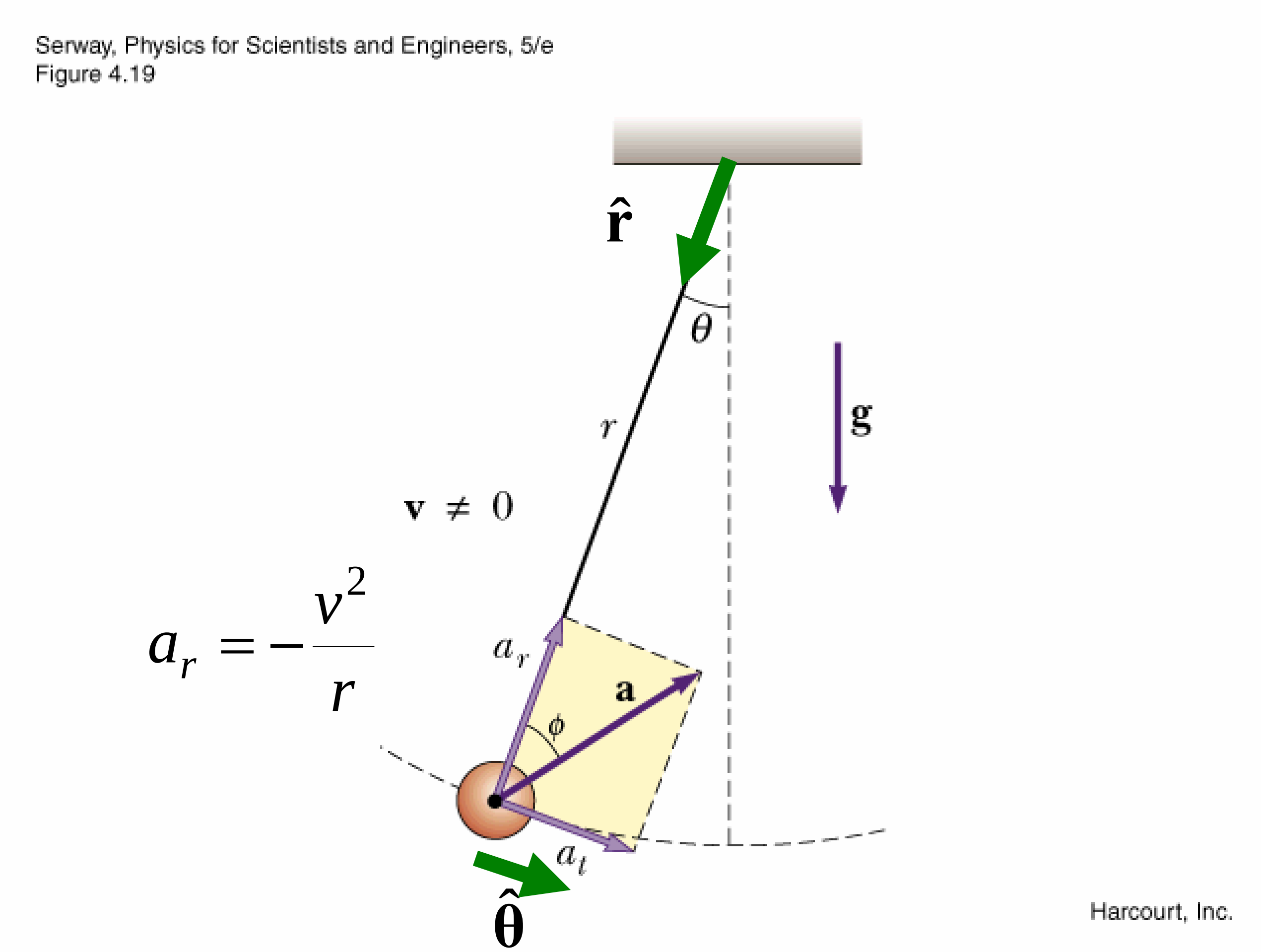
   (a) tangential (b) radial (c) up (d) down

2. Do you have a tangential component of acceleration? (a) yes (b) no

3. Do you have a radial component of acceleration? (a) yes (b) no

4. Do you have a vertical component of acceleration? (a) yes (b) no

# Addition of accelerations:




$$a_r = -\frac{v^2}{r}$$

$$a_\theta = g \sin\theta$$