Announcements

1. Famous interdisciplinary physics colloquium this week – “Tolerance and Intolerance in Protein Structure and Function”

2. Review of Thursday’s lecture – introduction to Newton’s laws

3. Today’s lecture –
   More examples of the application of Newton’s laws
   Friction forces
Newton’s 2nd Law:

\[ F = ma \]

Example:

\[ T - mg = ma \]
\[ T = mg + ma = mg(1 + a/g) \]

For example: \( mg = 500 \text{ N} \)

\[ a = 4.9 \text{ m/s}^2 \]

\[ T = 750 \text{ N} \]
From HW 5:

7. HW05 5.P.052. [53130] A 100 kg crate is pushed at constant speed up the frictionless 30.0° ramp shown in Fig. 5-53.

(a) What magnitude of horizontal force $F$ is required?

(b) What force is exerted by the ramp on the crate?

\[
F_{\text{net}} = (F \cos \theta - mg \sin \theta)\hat{i} + (N - F \sin \theta - mg \cos \theta)\hat{j}
\]
A worker drags a crate across a factory floor by pulling on a rope tied to the crate (Fig. 5-38). The worker exerts a force of 450 N on the rope, which is inclined at 38° to the horizontal, and the floor exerts a horizontal force of 125 N that opposes the motion.

![Diagram of a crate being pulled by a rope](image)

**Figure 5-38**

(a) Calculate the acceleration of the crate if its mass is 328 kg.

\[ \text{[0.1]} \quad \text{m/s}^2 \]

(b) Calculate the acceleration of the crate if its weight is 328 N.

\[ \text{[0.1]} \quad \text{m/s}^2 \]

\[
\mathbf{F}_{\text{net}} = (T \cos \theta - f)\mathbf{i} \\
+ (N - mg + T \sin \theta)\mathbf{j}
\]
radial: \( -T - mg = -\frac{mv^2}{r} \)

tangential: \( 0 = ma_\theta \)

radial: \( -T = -\frac{mv^2}{r} \)

tangential: \( mg = ma_\theta \)

radial: \( T - mg \cos \theta = \frac{mv^2}{r} \)

tangential: \( -mg \sin \theta = ma_\theta \)

radial: \( T - mg = \frac{mv^2}{r} \)

tangential: \( 0 = ma_\theta \)
Newton’s second law

\[ F = m \cdot a \]

Types of forces:

**Fundamental**
- Gravitational
- Electrical
- Magnetic
- Elementary particles

**Approximate**
- \( F = -mg \) (with a checkmark)

**Empirical**
- Friction
- Support
- Elastic
Friction forces

The term “friction” is used to describe the category of forces that oppose motion. One example is surface friction which acts on two touching solid objects. Another example is air friction. There are several reasonable models to quantify these phenomena.

Surface friction: \[ f = \begin{cases} -F_{\text{applied}} \\ \pm \mu N \end{cases} \]

- Normal force between surfaces
- Material-dependent coefficient

Air friction:
\[ D = \begin{cases} -Kv & \text{at low speed} \\ -K'v^2 & \text{at high speed} \end{cases} \]

\(K\) and \(K'\) are materials and shape dependent constants
Models of surface friction forces

Coefficients of friction $\mu_s$, $\mu_k$ depend on the surfaces; usually, $\mu_s > \mu_k$
Some estimates of static and kinetic friction:

<table>
<thead>
<tr>
<th>Material</th>
<th>$\mu_s$</th>
<th>$\mu_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubber on concrete</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Wood on wood</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Steel on steel with lubrication</td>
<td>0.09</td>
<td>0.05</td>
</tr>
<tr>
<td>Teflon on teflon</td>
<td>0.04</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Surface friction:

\( F - f_s = 0 \) if \( F < \mu_s n = \mu_s mg \)

if \( F > \mu_s n = \mu_s mg \), then \( F - f_k = ma \) (\( f_k = \mu_k mg \))

\[
a = \frac{F - \mu_k mg}{m}
\]
$f = \mu_k N$

$mg$

$N$

$v_0$
A worker drags a crate across a factory floor by pulling on a rope tied to the crate (Fig. 5-38). The worker exerts a force of 450 N on the rope, which is inclined at 38° to the horizontal, and the floor exerts a horizontal force of 125 N that opposes the motion.

\[
\mathbf{F}_{\text{net}} = (T \cos \theta - f)\hat{i} + (N - mg + T \sin \theta)\hat{j}
\]

\[
f = \mu_k N = \mu_k (mg - T \sin \theta)
\]

\[
a = \frac{\mathbf{F}_{\text{net}}}{m} = \frac{T(\cos \theta + \mu_k \sin \theta) - \mu_k mg}{m}
\]
just before block slips:

\[ f_{s,\text{max}} - mg \sin \theta = \mu_s n - mg \sin \theta = \mu_s mg \cos \theta - mg \sin \theta = 0 \]

\[ \Rightarrow \mu_s = \tan \theta \]
Suppose you place a rectangular box on an inclined surface as shown in the figure and you notice that the box slides down the incline at constant velocity. (Assume that the box and incline are near the surface of the Earth.) Which of the following statements might be true?

1. There is no net force acting on the box.
2. There is a net force acting on the box.
3. The coefficient of static friction for the sliding box is equal to $\tan \theta$.
4. The coefficient of kinetic friction for the sliding box is equal to $\tan \theta$.
Suppose that $\mu_s = 0.75$ which means that the block starts to slide when $\theta = 37^\circ$. What is $f$ when $\theta = 20^\circ$?

(a) $mg \sin \theta$  
(b) $\mu_s mg \sin \theta$  
(c) $mg \cos \theta$  
(d) $\mu_s mg \cos \theta$
Peer instruction question

Suppose that \( \mu_s = 0.75 \) which means that the block starts to slide when \( \theta = 37^\circ \). What is \( f \) when \( \theta = 40^\circ \)?

(a) \( mg \sin \theta \)  
(b) \( \mu_k \, mg \sin \theta \)  
(c)\( mg \cos \theta \)  
(d) \( \mu_k \, mg \cos \theta \)
radial: \(-f_s = -m \frac{v^2}{r}\)

extreme condition: \(f_{s,\text{max}} = \mu_s n = \mu_s mg \rightarrow v_{\text{max}} = \sqrt{\mu_s gr}\)
Banked curve (ignoring friction): \[ n \sin \theta = \frac{mv^2}{r} \]

Optimal banking angle: \[ \tan \theta = \frac{v^2}{rg} \]

Optimal banking angle:

\[ n \cos \theta - mg = 0 \]

\[ n = \frac{mg}{\cos \theta} \]
More practice with Newton’s laws:

along surface: $F - mg \sin\theta \pm f_s = 0$

perpendicular to surface: $n - mg \cos\theta = 0 \rightarrow n = mg \cos\theta$

Condition for pushing up the incline:

$F > mg (\sin\theta + \mu_s \cos\theta)$
More practice with Newton’s laws:

along surface: \( F \cos\theta - mg \sin\theta - f_s = 0 \)

perpendicular to surface: \( n - F\sin\theta - mg \cos\theta = 0 \)

Condition for pushing up the incline:

\[
F = \frac{mg (\sin\theta + \mu_s \cos\theta)}{\cos\theta - \mu_s \sin\theta}
\]
Peer instruction question

Suppose that the forces on a soap bubble falling through the air can be described by: \( F = -mg - mbv \) If at \( t=0 \) \( v(t)=0 \), what will be its velocity at a later time?

(a) \( v(t)=-gt \) (The bubble will accelerate under the effects of gravity and air friction (the “b” term) has a negligible effect.)

(b) \( v(t)=0 \) (Air friction will eventually stop the bubble from moving.)

(c) \( v(t)=(\text{Constant}) \) (The magnitude of the air friction force will increase with time, eventually balancing the force of gravity.)

(d) \( |v(t)|>gt \) (Air friction makes the bubble move faster than gravity alone.)
\[ \mathbf{F} = m \mathbf{a} = \frac{d\mathbf{v}}{dt} = m \frac{d^2 \mathbf{r}}{dt^2} \]

Examples (one dimension):

\[ F = F_0 \quad \text{(constant)} \quad \Rightarrow \quad x(t) = x_0 + v_0 t + \frac{1}{2} \frac{F_0}{m} t^2 \]

\[ F = F_0 \sin \omega t \quad \Rightarrow \quad x(t) = x_0 + v_0 t - \frac{F_0}{m \omega^2} \sin \omega t \]

\[ F = -kx \quad \Rightarrow \quad x(t) = x_0 \cos \sqrt{\frac{k}{m}} t \]

\[ F = mg - mbv \quad \Rightarrow \quad v(t) = -\frac{g}{b} + \frac{g}{b} e^{-bt} \]