Announcements

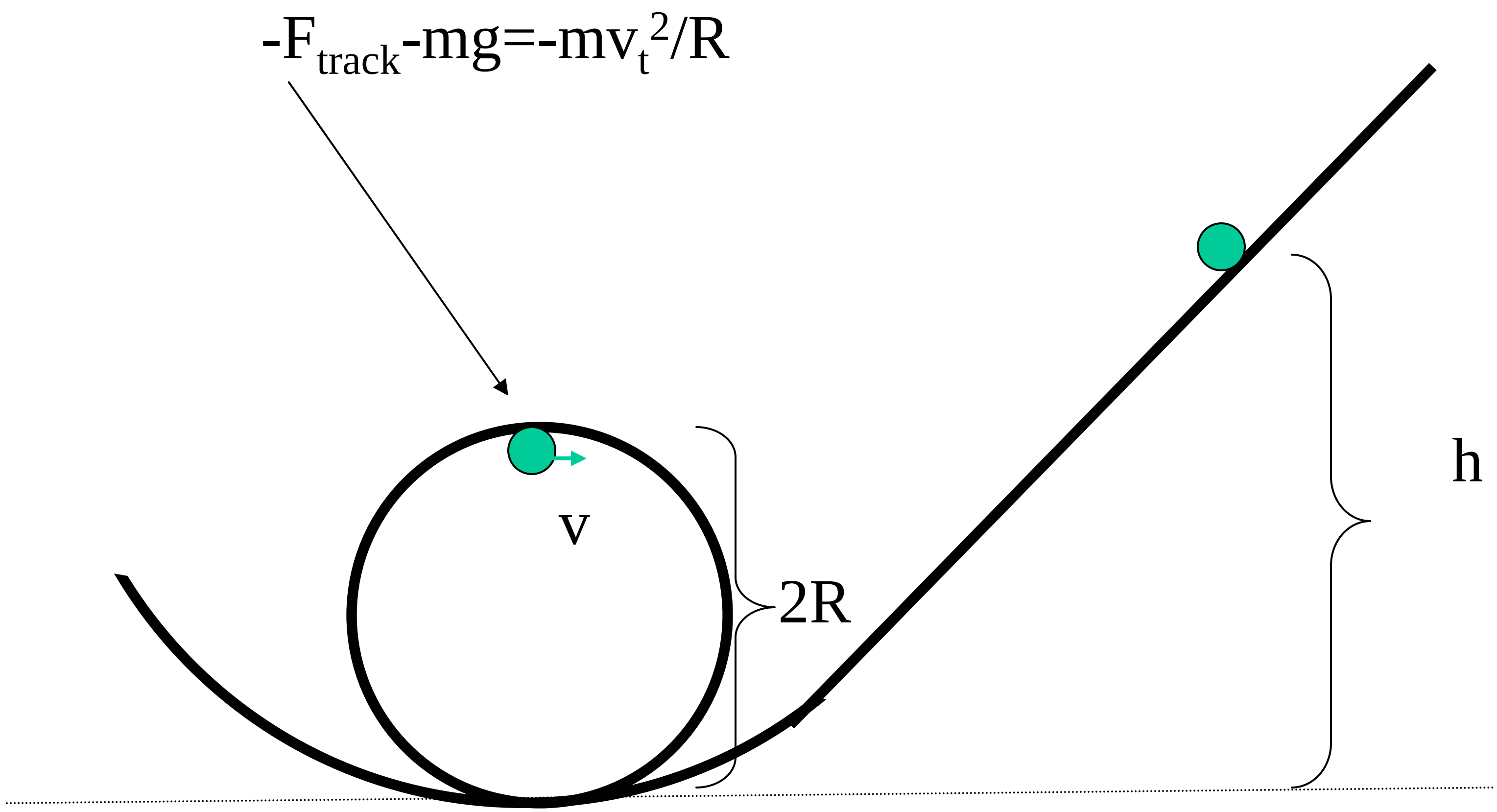
1. **Thursday, Sept. 25th, 4 PM – Physics Colloquium by Professor Steven Vogel, James B. Duke Professor of Biology at Duke U. – will discuss the physics of muscles**
2. **Tuesday, Sept. 30th, 9:30 AM – First exam –**
   a) **Covering Chapters 1-8 of HRW**
   b) **May bring 1 equation sheet, your calculator to exam**
   c) **If you have kept up with your HW, you may drop your lowest exam grade**
3. **Today – review work-kinetic energy theorem**
   – discuss power
   – introduce notion of potential energy

Work: $W_{i \to f} = \int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F} \cdot d\mathbf{r}$

Kinetic energy: $K = \frac{1}{2} m v^2$

Work – kinetic energy relation:

$$W_{net} = K_f - K_i$$

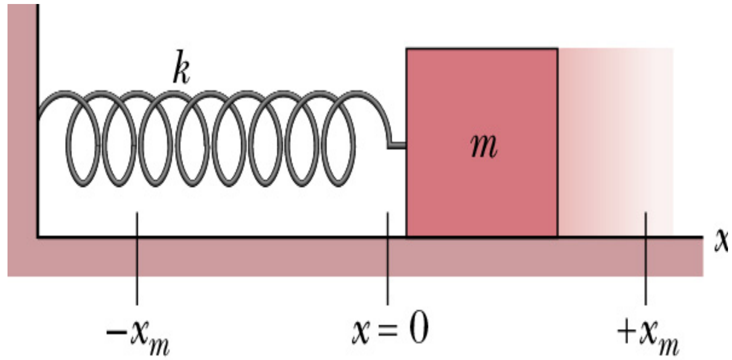


If $F_{track}=0$, then $\frac{1}{2}mv_t^2=\frac{1}{2}mgR$

$W_{net} = K_f - K_i \Rightarrow mg(h-2R)= \frac{1}{2}mv_t^2$

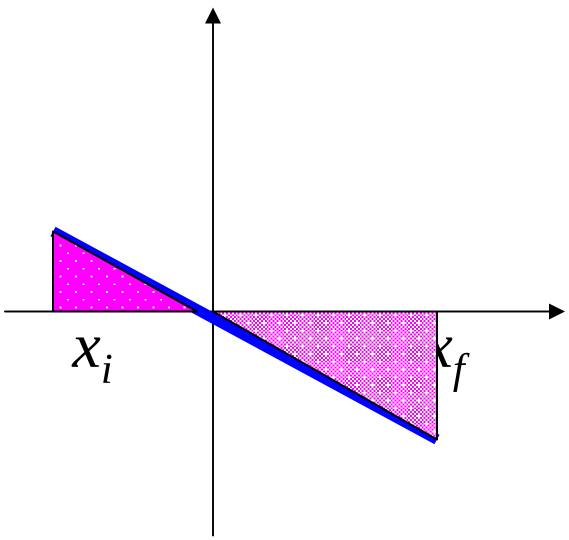
$\Rightarrow h=5/2R$

Hooke's "law" (model force)



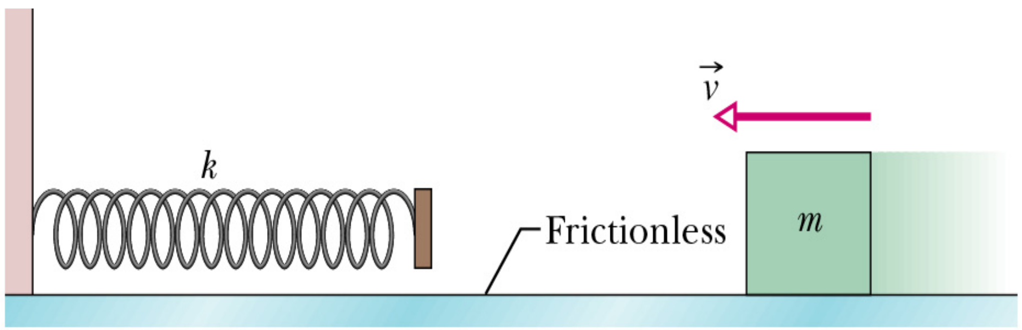
F = –kx

$$W_{i\to f} = \int_{x_i}^{x_f} -kx\,dx = -\tfrac{1}{2}k\left(x_f^2 - x_i^2\right)$$



Example problem:

A mass m with initial velocity v compresses a spring with Hooke's law constant k by a distance x_f. What is x_f when the mass momentarily comes to rest?



$$K_i = \tfrac{1}{2} m v^2$$

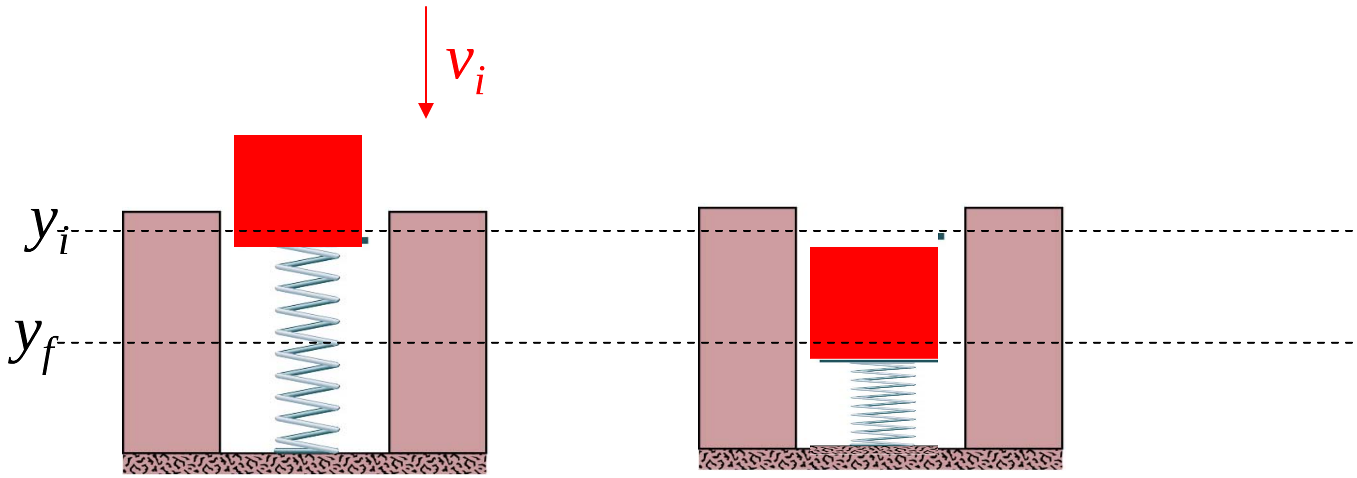
$$K_f = 0$$

$$W_{i \to f} = -\tfrac{1}{2} k x_f^2$$

$$W_{i \to f} = K_f - K_i \qquad \Rightarrow -\tfrac{1}{2} k x_f^2 = -\tfrac{1}{2} m v^2$$

$$x_f = \sqrt{\frac{m}{k}}\, v$$

Example: gravity and Hooke's law work --



$$K_i = \tfrac{1}{2} m v_i^2$$

$$K_f = 0$$

$$W_{i\to f} = mg(y_i - y_f) - \tfrac{1}{2} k\left(y_i - y_f\right)^2$$

$$\tfrac{1}{2} k\left(y_i - y_f\right)^2 - mg\left(y_i - y_f\right) - \tfrac{1}{2} m v_i^2 = 0$$

Power: $P = \frac{dW}{dt}$

Units: 1 J/s ≡ 1 Watt

Special case:

Constant force with no acceleration

$$dW = \mathbf{F} \cdot d\mathbf{r}$$

$$\frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} = \mathbf{F} \cdot \mathbf{v}$$

Energy forms

1. Work: $W = \int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F} \cdot d\mathbf{r}$

2. Kinetic energy: K= ½ $m\, v^2$

3. Work-kinetic energy theorem: $W_{total}^{i\to f} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

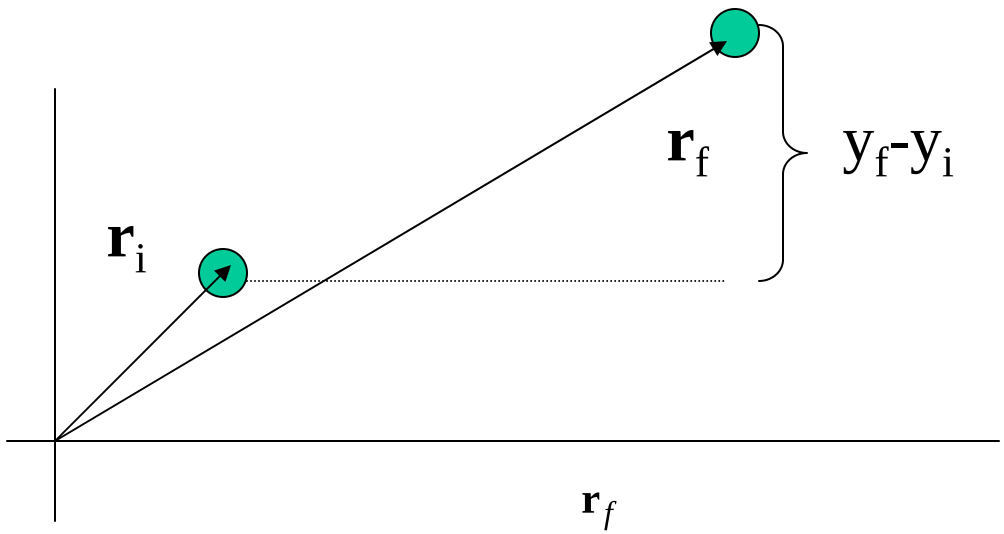
4. Different kinds of contribution to total work:

   a. Conservative (reversible) -- gravity, Hooke's law

   b. Dissipative (irreversible) -- friction

Conservative forces – reversible work

Consider the work done by gravity $F_g = -mg\,\mathbf{j}$



$$W_{i\to f} = \int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F} \cdot d\mathbf{r} = -mg(y_f - y_i)$$

Note that the work of gravity is: $\begin{cases} > 0 \text{ if } & y_f < y_i \\ < 0 \text{ if } & y_f > y_i \end{cases}$

Conservative forces – reversible work

Define potential energy: $U(\mathbf{r}) = -\int_{\mathbf{r}_{ref}}^{\mathbf{r}} \mathbf{F} \cdot d\mathbf{r}' = -W(\mathbf{r}_{ref} \rightarrow \mathbf{r})$

Consider the work done by gravity: F_g = -mg **j**

$$W_{i\rightarrow f} = \int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F} \cdot d\mathbf{r}' = -mg(y_f - y_i) = U(y_i) - U(y_f)$$

➔ $U_{gravity}(y)$ = mgy (assuming y_{ref} = 0)

Note: Depends only on position (y) (not on path)

$$\mathbf{F} = -\nabla U(y) = -mg\mathbf{j}$$

Conservative forces – reversible work – more examples

Define potential energy: $U(\mathbf{r}) = -\int_{\mathbf{r}_{ref}}^{\mathbf{r}} \mathbf{F} \cdot d\mathbf{r}' = -W(\mathbf{r}_{ref} \rightarrow \mathbf{r})$

Consider the work done by Hooke's law force: $F_s = -kx\,\mathbf{i}$

$$W_{i\rightarrow f} = \int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F} \cdot d\mathbf{r}' = -k\int_{x_i}^{x_f} x\,dx = -\tfrac{1}{2}k\left(x_f^2 - x_i^2\right) = U(x_i) - U(x_f)$$

➔ $U_s(x) = \frac{1}{2} kx^2$ (assuming $x_{ref} = 0$)

Note: Depends only on position (x) (not on path)

$$\mathbf{F} = -\nabla U(x) = -kx\,\mathbf{i}$$

Using gravitational potential energy in Work-Kinetic energy theorem

$$W_{total}^{i\rightarrow f} = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2$$

Suppose that the total work is done by conservative forces:

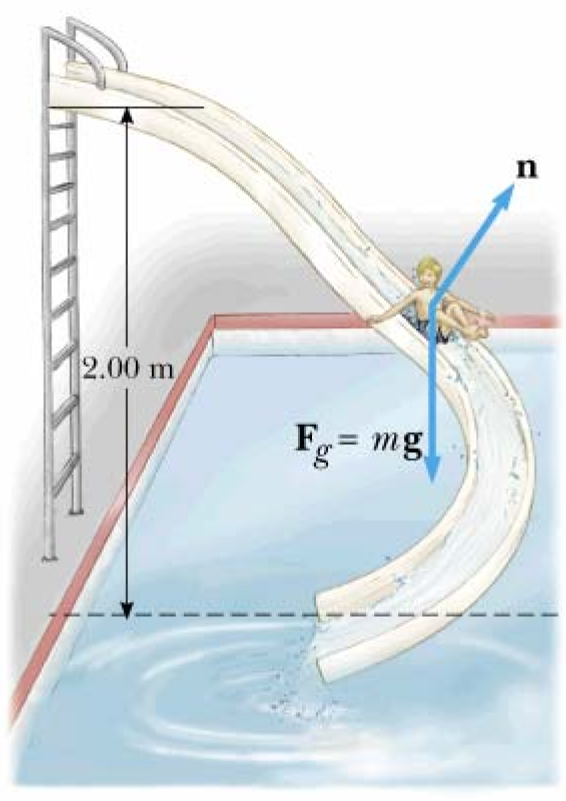
$$W_{total}^{i\rightarrow f} = U(\mathbf{r}_i) - U(\mathbf{r}_f) = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2$$

$$\tfrac{1}{2} m v_f^2 + U(\mathbf{r}_f) = \tfrac{1}{2} m v_i^2 + U(\mathbf{r}_i) \equiv \text{total mechanical energy}$$

More generally:

$$W_{total}^{i\rightarrow f} = W_{conservative}^{i\rightarrow f} + W_{dissipative}^{i\rightarrow f} = U(\mathbf{r}_i) - U(\mathbf{r}_f) + W_{dissipative}^{i\rightarrow f}$$

$$\tfrac{1}{2} m v_f^2 + U(\mathbf{r}_f) = \tfrac{1}{2} m v_i^2 + U(\mathbf{r}_i) + W_{dissipative}^{i\rightarrow f}$$



A child of mass m=20kg starts from rest at the top of a 2m slide and has a speed of v_f=3m/s at the end of the ride. How much friction energy does the child generate?

$$\tfrac{1}{2} m v_f^2 + U(\mathbf{r}_f) = \tfrac{1}{2} m v_i^2 + U(\mathbf{r}_i) + W_{dissipative}^{i \rightarrow f}$$

$$\tfrac{1}{2} m v_f^2 + 0 = 0 + mgh + W_{friction}$$

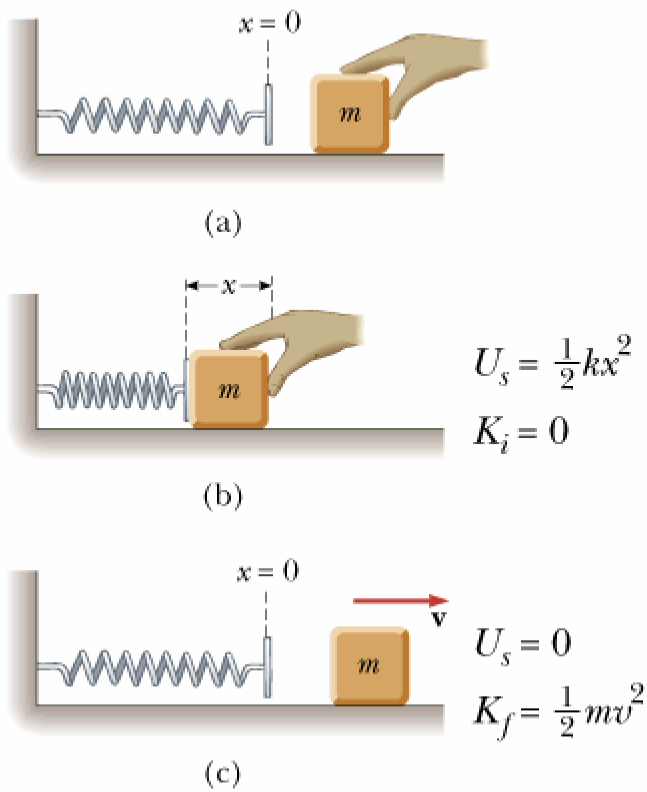
$$\tfrac{1}{2}(20\text{kg})(3\text{m/s})^2 = (20\text{kg})(9.8\text{m/s}^2)(2\text{m}) + W_{friction}$$

$$W_{friction} = -302 \text{ J}$$

Spring force: $F_{spring} = -k(x-x_0)$

$$U_{spring}(x) = -\int_{x_0}^{x} (-k(x'-x_0)dx' = \tfrac{1}{2}k(x-x_0)^2$$

Serway, Physics for Scientists and Engineers, 5/e
Figure 8.2



$$\tfrac{1}{2}mv_f^2 + U(\mathbf{r}_f) = \tfrac{1}{2}mv_i^2 + U(\mathbf{r}_i) + W_{dissipative}^{i\to f}$$

$$\tfrac{1}{2}mv_f^2+0=0+\tfrac{1}{2}k(x_i-x_0)^2+0$$

$$v_f = \sqrt{\frac{k}{m}}\,|x_i - x_0|$$

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Review -- Work-Kinetic energy theorem:

$$W_{total}^{i \to f} = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2$$

Analyzing work in terms of "conservative" and "non-conservative":

$$W_{total}^{i \to f} = W_{conservative}^{i \to f} + W_{non-conservative}^{i \to f} = U(\mathbf{r}_i) - U(\mathbf{r}_f) + W_{non-conservative}^{i \to f}$$

Rearranging terms to form total mechanical energy:

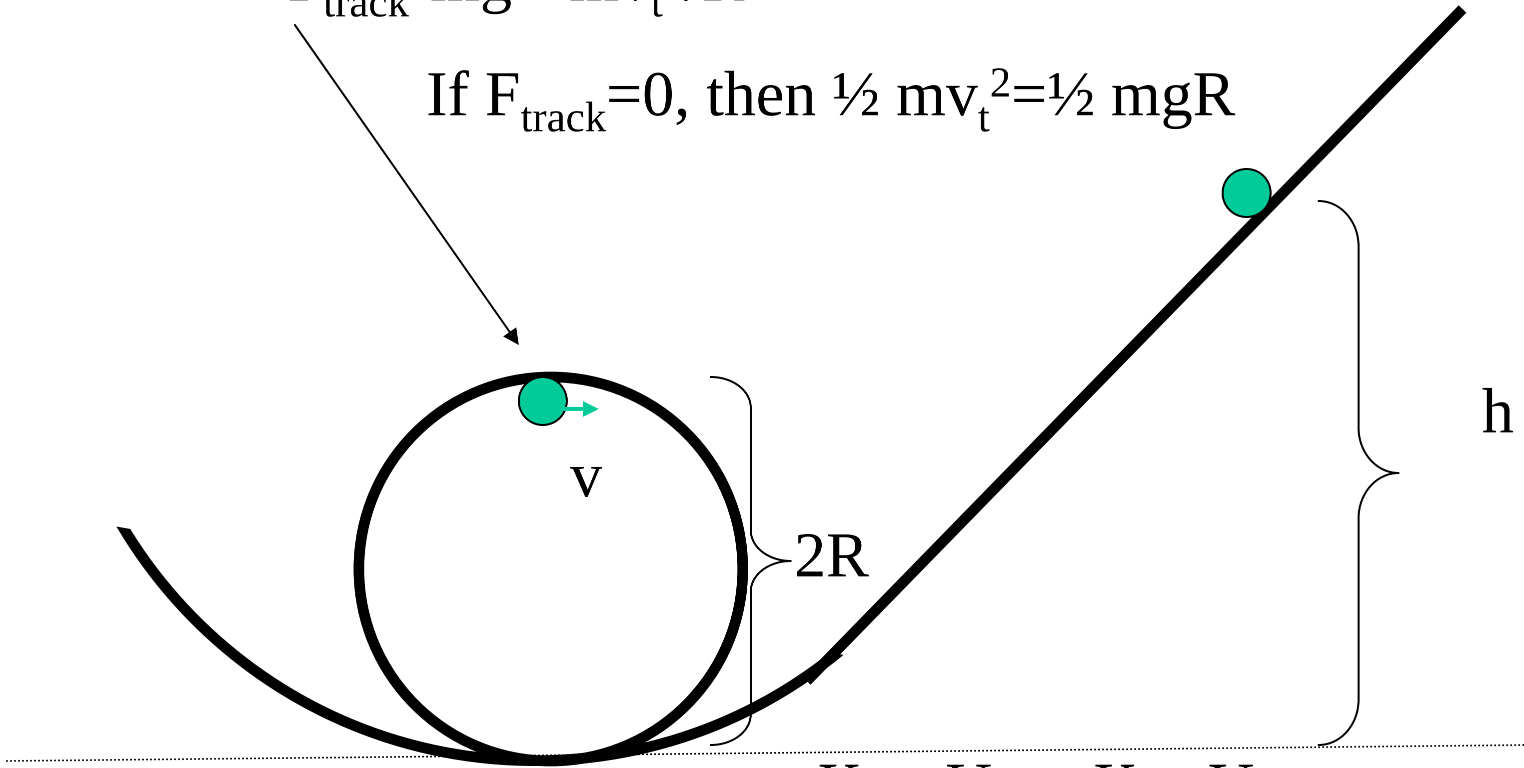
$$\tfrac{1}{2} m v_f^2 + U(\mathbf{r}_f) = \tfrac{1}{2} m v_i^2 + U(\mathbf{r}_i) + W_{non-conservative}^{i \to f}$$

Statement of conservation of energy: If $W_{non-conservative}^{i \to f} = 0$

$$E_{mechanical} = \tfrac{1}{2} m v^2 + U(\mathbf{r}) = (\text{constant}); \Rightarrow \frac{dE_{mech}}{dt} = 0$$

$-F_{track} - mg = -mv_t^2/R$

If $F_{track}=0$, then $\frac{1}{2} mv_t^2 = \frac{1}{2} mgR$



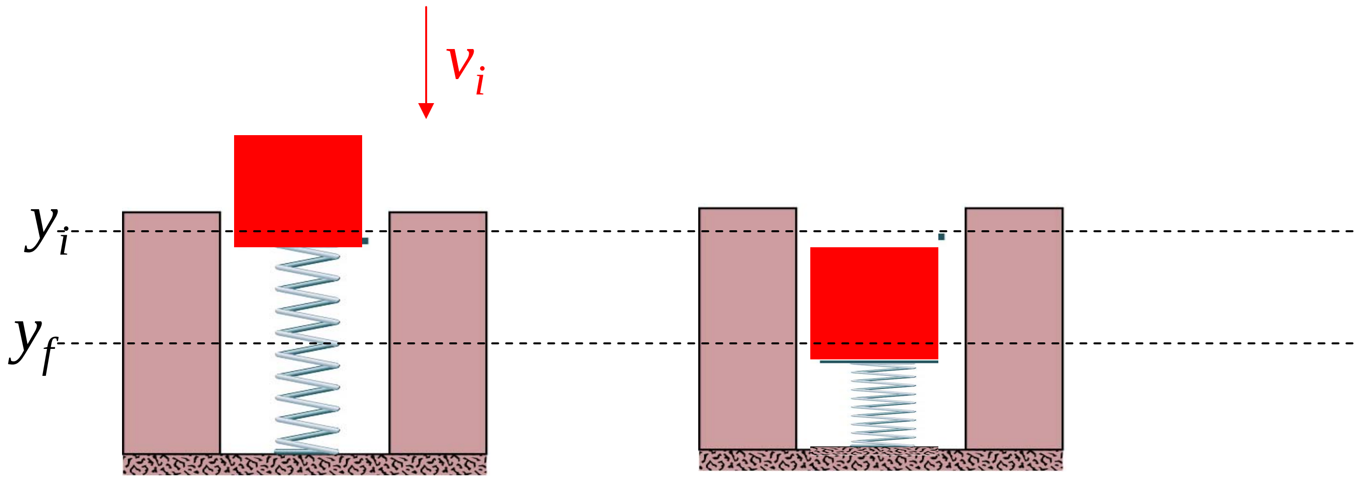
$$K_f + U_f = K_i + U_i$$

$$\tfrac{1}{2} mv_t^2 + mg(2R) = 0 + mgh$$

$$\tfrac{1}{2} mgR + mg(2R) = mgh$$

$$\Rightarrow h = \tfrac{5}{2} R$$

Example: gravity and Hooke's law work --



$$K_f + U_f = K_i + U_i$$

$$0 + mgy_f + \tfrac{1}{2}k\left(y_f - y_i\right)^2 = \tfrac{1}{2}mv_i^2 + mgy_i + 0$$

$$\tfrac{1}{2}k\left(y_i - y_f\right)^2 - mg\left(y_i - y_f\right) - \tfrac{1}{2}mv_i^2 = 0$$

More on Hooke's law -- $U = ½ k x^2$

$E = ½ mv^2 + ½ k x^2$