Announcements

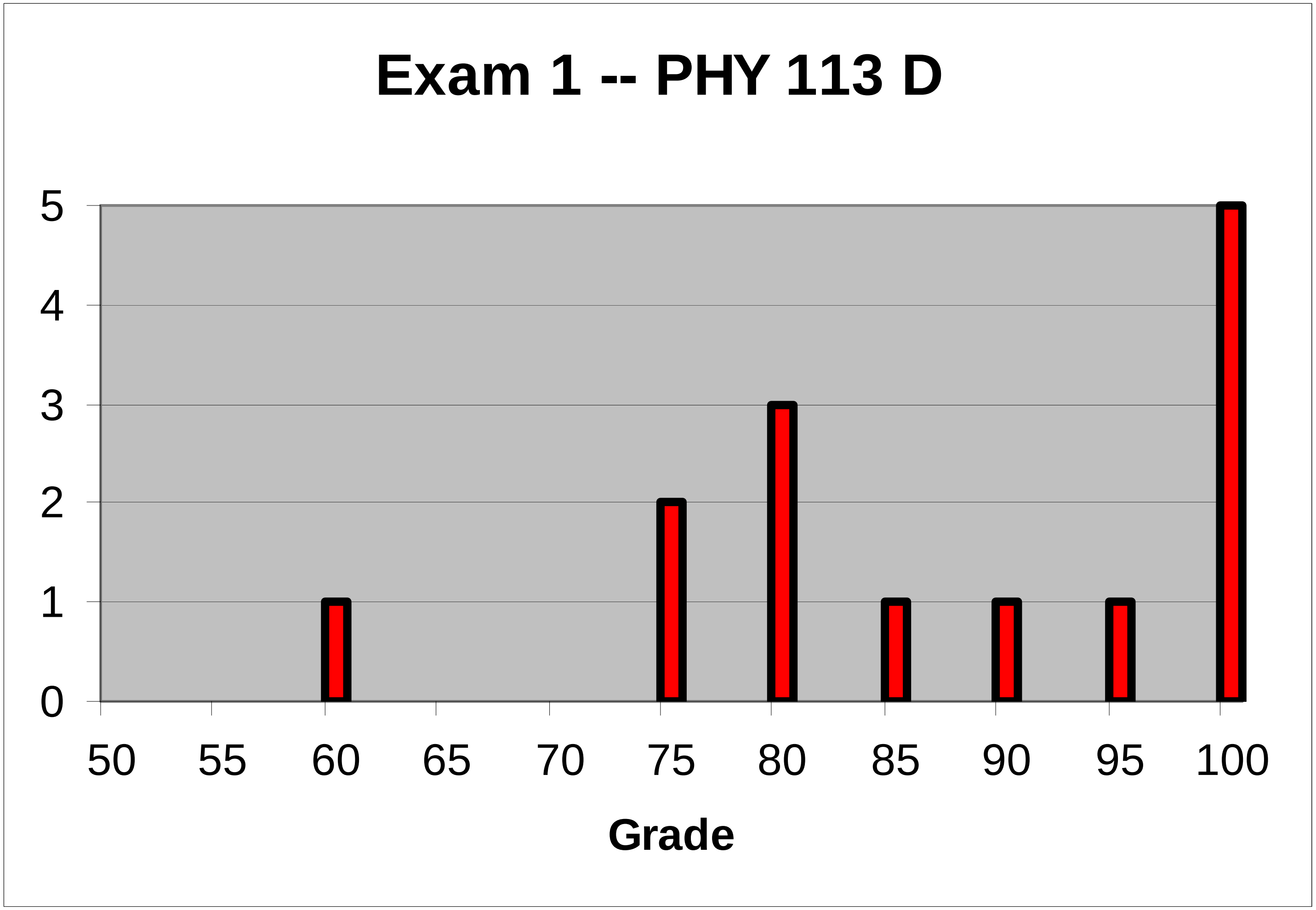
1. **Exam 1 will be returned at the end of class. Please rework the exam, to help "solidify" your knowledge of this material. (Up to 10 extra credit points granted for reworked exam – turn in old exam, corrections on a separate paper – due Tuesday, Oct. 7.)**
2. **Physics colloquium today – 4 PM Olin 101**

   SPS meeting at 11:30 AM (free pizza for physics majors and potential physics majors)
3. **Today's topic – Chapter 9 HRW**

   Center of mass

   Definition of momentum

   Conservation of momentum


Exam 1 -- PHY 113 D
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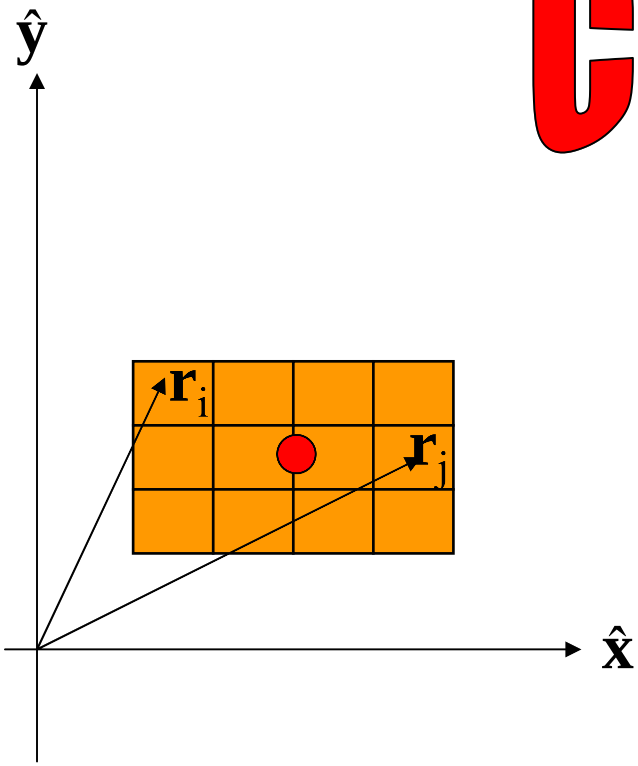
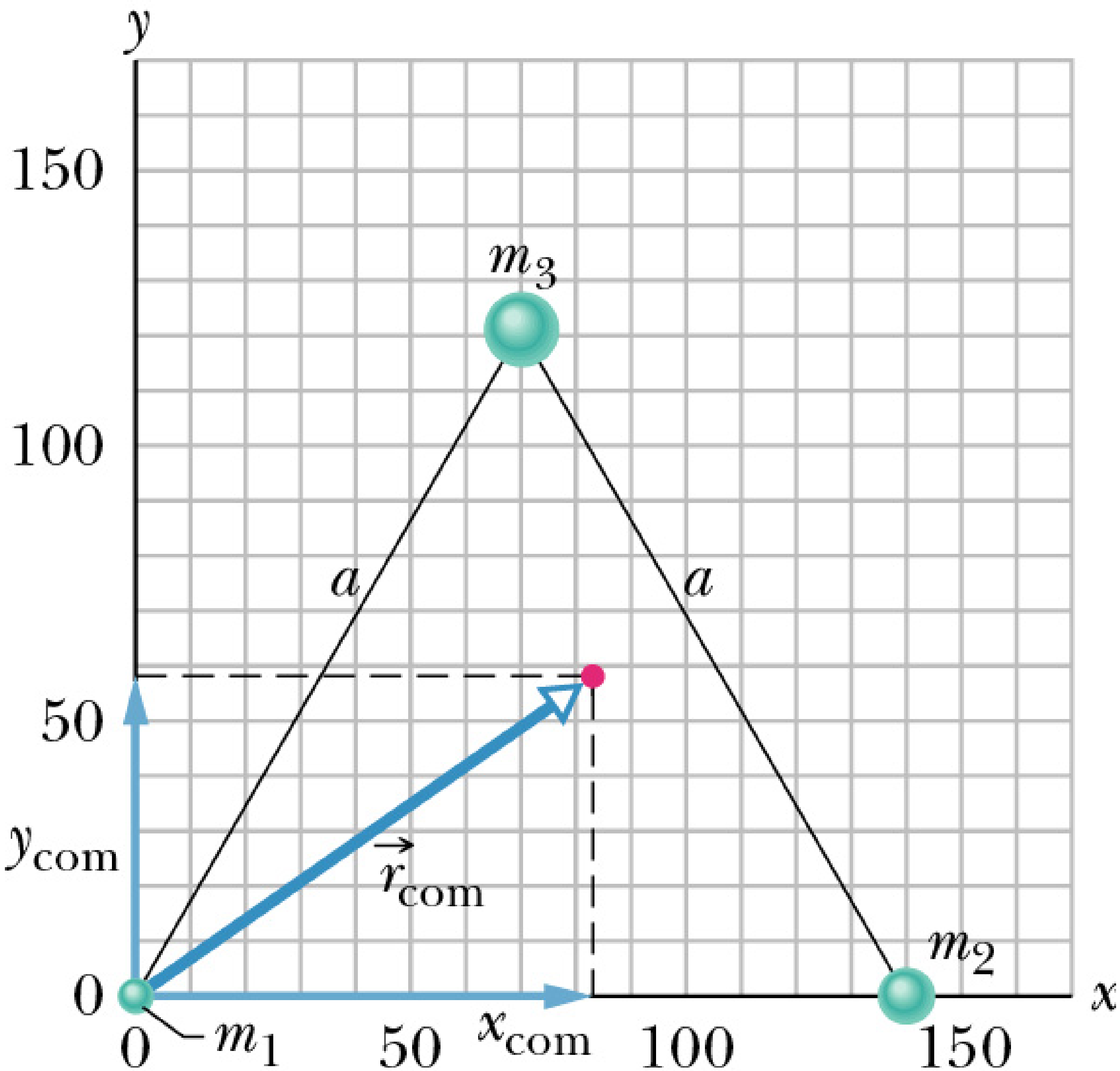
Center
of mass



$$\mathbf{r}_{\mathrm{COM}} \equiv \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}$$

Example:



m_1=1.2kg

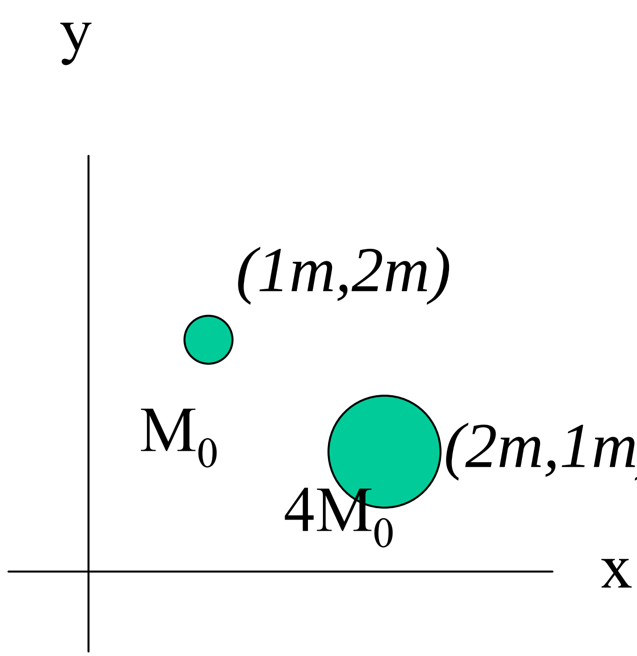
m_2=2.5kg

m_3=3.4kg

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

Another example of center of mass:



$$\mathbf{r}_{com} = \left(\frac{M_0 \cdot 1 + 4M_0 \cdot 2}{5M_0}\mathbf{i} + \frac{M_0 \cdot 2 + 4M_0 \cdot 1}{5M_0}\mathbf{j} \right) m$$

$$= (1.8\mathbf{i} + 1.2\mathbf{j}) m$$

Position of the center of mass: $$\mathbf{r}_{com} \equiv \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}$$

Velocity of the center of mass: $$\mathbf{v}_{com} \equiv \frac{\sum_i m_i \mathbf{v}_i}{\sum_i m_i}$$

Acceleration of the center of mass: $$\mathbf{a}_{com} \equiv \frac{\sum_i m_i \mathbf{a}_i}{\sum_i m_i}$$

Physics of composite systems:

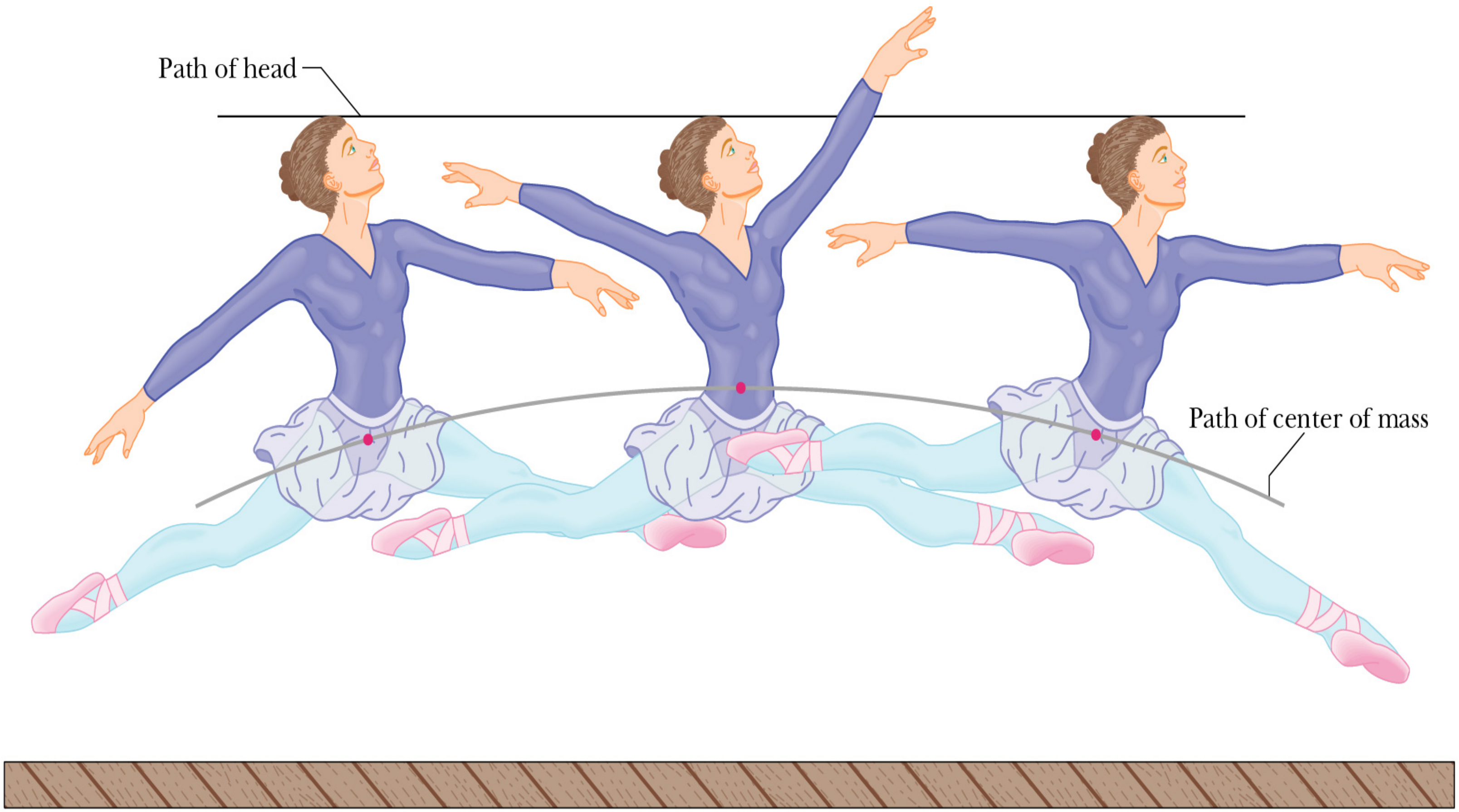
$$\sum_i \mathbf{F}_i = \sum_i m_i \mathbf{a}_i = \sum_i \frac{d m_i \mathbf{v}_i}{dt} = \sum_i \frac{d\mathbf{p}_i}{dt}$$

Center-of-mass velocity:

$$\mathbf{v}_{com} \equiv \frac{\sum_i m_i \mathbf{v}_i}{\sum_i m_i} \equiv \frac{\sum_i m_i \mathbf{v}_i}{M}$$

Note that:

$$\sum_i \mathbf{F}_i \equiv \mathbf{F}_{total} = M \frac{d\mathbf{v}_{\mathrm{com}}}{dt}$$


Path of head
Path of center of mass

A new way to look at Newton's second law:

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt} \equiv \frac{d\mathbf{p}}{dt}$$

Define linear momentum $\mathbf{p} = \mathrm{m}\mathbf{v}$

Consequences:

1. If $\mathbf{F} = 0$ ➔ $\frac{d\mathbf{p}}{dt} = 0$ ➔ $\mathbf{p} = \text{constant}$

2. For system of particles: $\sum_i \mathbf{F}_i = \sum_i \frac{d\mathbf{p}_i}{dt}$

   If $\sum_i \mathbf{F}_i = 0 \quad \Rightarrow \sum_i \frac{d\mathbf{p}_i}{dt} = 0 \quad \Rightarrow \sum_i \mathbf{p}_i = \text{constant}$

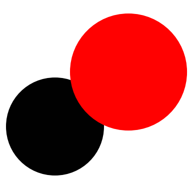
Conservation of (linear) momentum:

If $\frac{d\mathbf{p}}{dt} = \mathbf{F} = 0$, $\mathbf{p}$ = constant

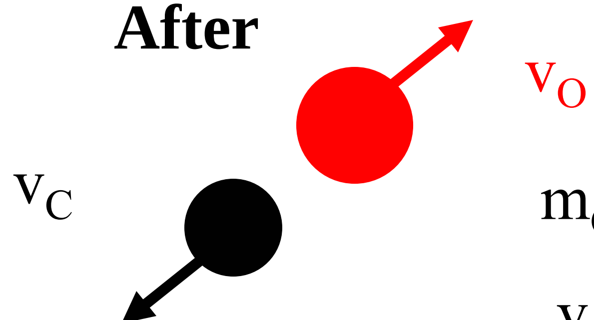
Example:

Suppose a molecule of CO is initially at rest, and it suddenly decomposes into separate C and O atoms. In this process the chemical binding energy E_0 is transformed into mechanical energy of the C and O atoms. What can you say about the motion of these atoms after the decomposition?

Before



After



$m_C v_C - m_O v_O = 0$

$v_C = v_O \; m_O / m_C$

Further analysis:

$$E_0 = \tfrac{1}{2} m_C v_C^2 + \tfrac{1}{2} m_O v_O^2 = \tfrac{1}{2} m_C v_C^2 (1 + m_C/m_O)$$

$$\Rightarrow \tfrac{1}{2} m_C v_C^2 = E_0/(1 + m_C/m_O) \sim E_0/(1 + 12/16) = 4/7\ E_0$$

$$\Rightarrow \tfrac{1}{2} m_O v_O^2 = 3/7\ E_0$$

Extra credit opportunity:

Work through the details of the above analysis and verify the results for yourself (perhaps with a different diatomic molecule).

Peer instruction question:

Suppose a nucleus which has an initial mass of $M_i = 238\ m_0$ (where m_0 denotes a standard mass unit),suddenly decomposes into two smaller nuclei with $M_1 = 234\ m_0$ and $M_2 = 4\ m_0$.

If the velocity of nucleus #1 is V_1 what is the velocity of nucleus #2?

(a) $-0.017V_1$ (b) $-V_1$ (c) $-59.5V_1$

After the decomposition which nucleus has more energy?

(a) M_1 (b) M_2 (c) The nuclei have the same energy.

(d) Not enough information is given.

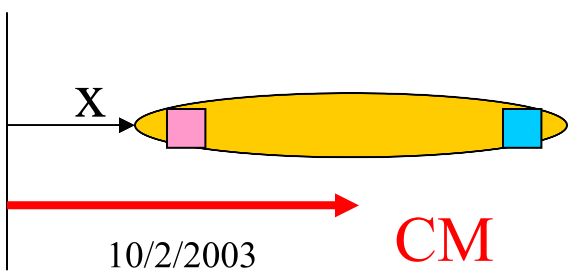
Peer instruction question:

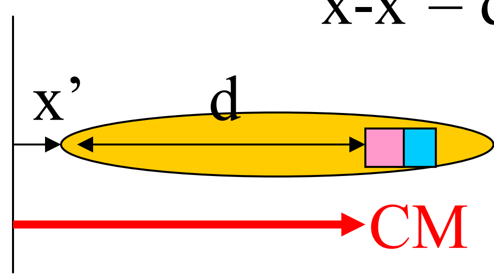
Romeo (60 kg) entertains Juliet (40 kg) by playing his guitar from the rear of their boat (100 kg) which is at rest in still water. Romeo is 2m away from Juliet who is in the front of the boat. After the serenade, Juliet carefully moves to the rear of the boat (away from shore) to plant a kiss on Romeo’s cheek. How does the boat move relative to the shore in this process? (Initially Juliet is closest to the shore.)

(a) 0.2 m away from shore (b) 0.4 m away from shore

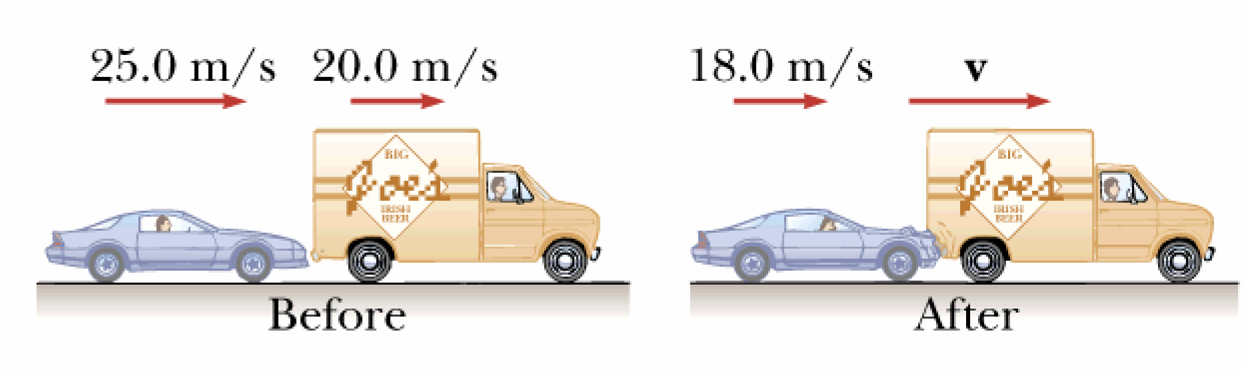
(c) 0.2 m toward shore (d) 0.4 m toward shore

$x - x' = d\, m_J / M_{total} = 0.4$ m





Another example:

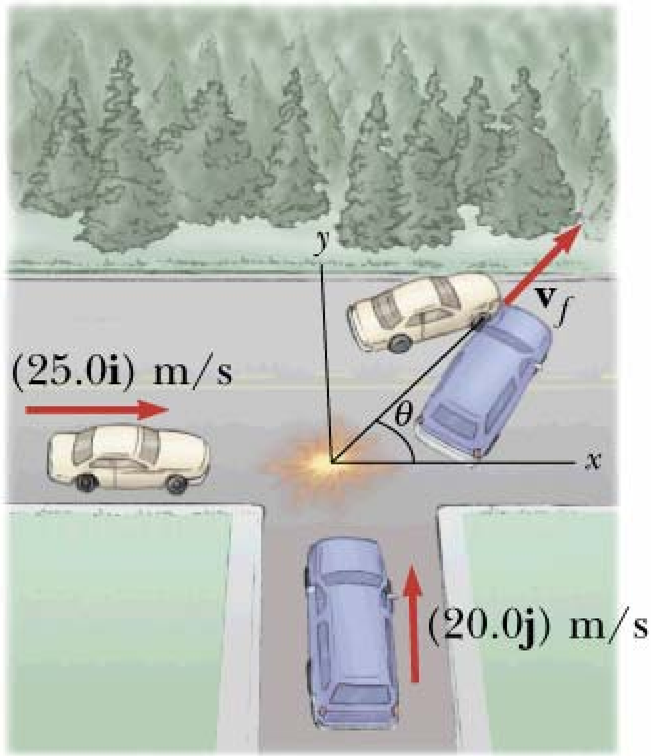


In this case, mechanical energy is not conserved. (Where does it go?) However, to our level of approximation, we will assume that momentum is conserved.

Before: After:

$$m_c v_{0c} + m_T v_{0T} = m_c v_c + m_T v$$

Note: In general, momentum is a **vector** quantity.



$$m_c \mathbf{v}_{0c} + m_T \mathbf{v}_{0T} \quad = \quad (m_c + m_T)\, \mathbf{v}_f$$

$$\mathbf{v}_f = \frac{m_c}{m_c + m_T} v_{0c}\mathbf{i} + \frac{m_T}{m_c + m_T} v_{0T}\,\mathbf{j}$$

5. HRW6 9.P.024. [53272] A 0.165 kg cue ball with an initial speed of 1.60 m/s bounces off the rail in a game of pool, as shown from overhead in Fig. 9-33. For *x* and *y* axes located as shown, the bounce reverses the *y* component of the ball's velocity but does not alter the *x* component.

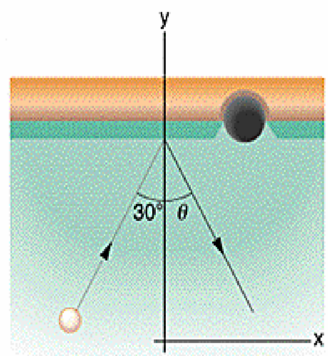


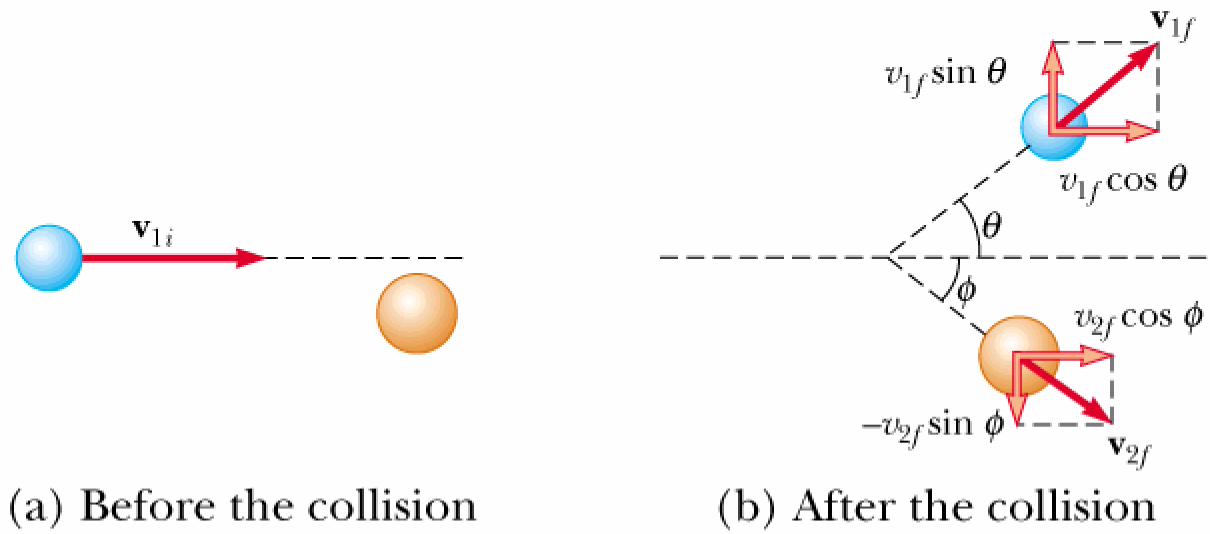
Figure 9-33.

(a) What is θ in Fig. 9-33?

[0.133333333] ____ °

(b) What is the change in the ball's linear momentum in unit-vector notation? (The fact that the ball rolls is not relevant to either question.)

[0.133333333] ____ **i** kg · m/s + [0.133333333] ____ **j** kg · m/s



Statement of conservation of momentum:

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\varphi$$

$$0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\varphi$$

If mechanical (kinetic) energy is conserved, then:

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

Peer instruction question:

Given the previous example, summarized with these equations:

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\varphi$$

$$0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\varphi$$

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

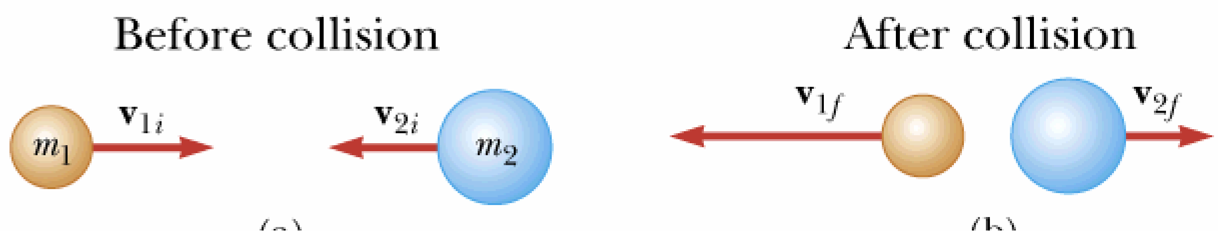
which of the following statements are true?

(a) It is in principle possible to solve the above equations uniquely.

(b) It is not possible to solve the above equations uniquely because the mathematics is too difficult.

(c) It is not possible to solve the above equations uniquely because there is missing physical information.

Figure 9.10



One dimensional case:

Conservation of momentum: $m_1v_{1i} + m_2v_{2i} = m_1v_{1f}+m_2v_{2f}$

Conservation of energy:

$$\tfrac{1}{2} m_1v_{1i}^2 + \tfrac{1}{2} m_2v_{2i}^2 = \tfrac{1}{2} m_1v_{1f}^2+ \tfrac{1}{2} m_2v_{2f}^2$$

Extra credit: Show that

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}$$

Summary

Linear momentum: **p** = m**v**

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

Generalization for a composite system:

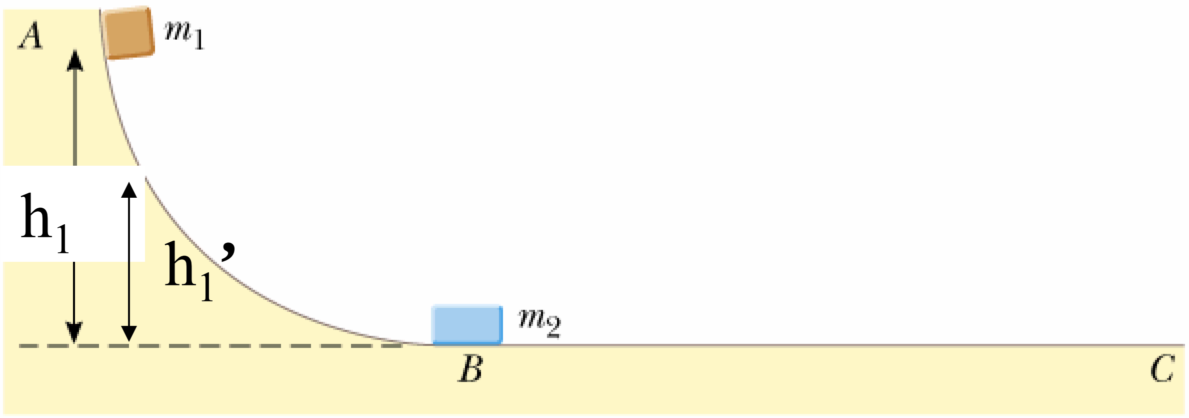
$$\sum_i \mathbf{F}_i = \sum_i \frac{d\mathbf{p}_i}{dt}$$

Conservation of momentum:

If $\sum_i \mathbf{F}_i = 0; \Rightarrow \sum_i \frac{d\mathbf{p}_i}{dt} = 0; \Rightarrow \sum_i \mathbf{p}_i \quad = \text{(constant)}$

Energy may also be conserved (for example, in an "elastic" collision)

Another example



1. m_1 falls a distance h_1 – energy conserved
2. m_1 collides with m_2 – momentum and energy conserved
3. m_1 moves back up the incline to a height h_1'

Extra credit:

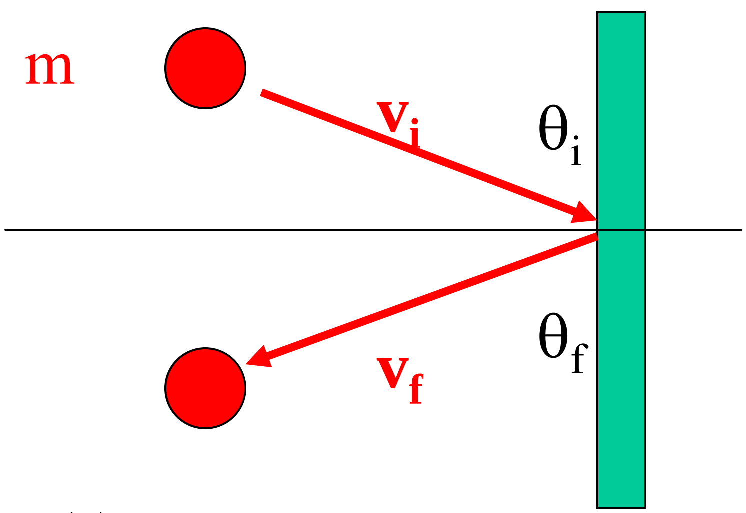
Show that

$$h_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 h_1$$

Notion of impulse:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \qquad \rightarrow \mathbf{F}\Delta t = \Delta\mathbf{p}$$

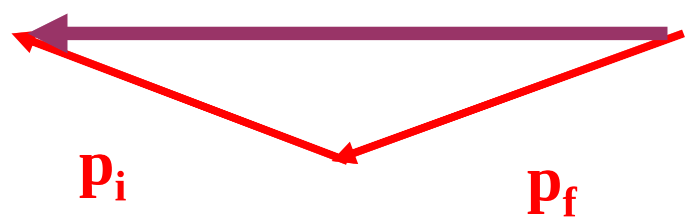
Example:



$\Delta\mathbf{p} = m\mathbf{v_f} - m\mathbf{v_i}$

$\Delta\mathbf{p} = (mv_f \sin\theta_f + mv_f \sin\theta_f)\ \mathbf{i}$

$\quad + (-mv_f \cos\theta_f + mv_f \cos\theta_f)\ \mathbf{j}$



Notion of impulse:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

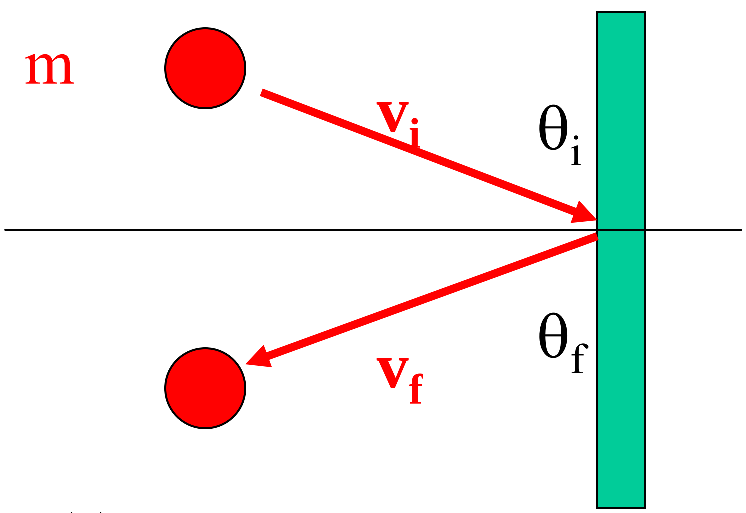
cause ⟷ **effect**

$$\Delta\mathbf{p} = m\mathbf{v_f} - m\mathbf{v_i}$$

Example:

$$\Delta\mathbf{p} = (mv_f \sin\theta_f + mv_f \sin\theta_f)\,\mathbf{i}$$

$$+(-mv_f \cos\theta_f + mv_f \cos\theta_f)\,\mathbf{j}$$



$$\Delta\mathbf{p} = \mathbf{p_f} - \mathbf{p_i} = \mathbf{F}\,\Delta t$$