In class, we considered how to describe the Lorentz force on a particle of charge $q$ in an electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$:

$$\mathbf{F}_{EM} = \frac{q}{c} \mathbf{v} \times \mathbf{B} + q \mathbf{E}$$

in the Lagrangian formulation.

Assuming we can describe these electric and magnetic fields in terms of the scalar and vector potentials according to

$$\mathbf{E} = -\nabla \phi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t},$$

and

$$\mathbf{B} = \nabla \times \mathbf{A},$$

the electromagnetic contribution to the Lagrangian is

$$L_{EM} = \frac{q}{c} \dot{\mathbf{r}} \cdot \mathbf{A} - q \phi.$$  \hspace{1cm} (4)

1. Show that

$$\mathbf{F}_{EM} \big|_z = \frac{d}{dt} \left( \frac{\partial L_{EM}}{\partial \dot{z}} \right) - \frac{\partial L_{EM}}{\partial z}.$$ \hspace{1cm} (5)

2. Consider what happens to the Lagrangian and to the Lorentz force when the vector and scalar potentials are changed according to

$$\mathbf{A}' = \mathbf{A} + \nabla \psi,$$

and

$$\phi' = \phi - \frac{\partial \psi}{\partial t}.$$ \hspace{1cm} (7)

Here, the scalar function $\psi(x, y, z, t)$ is arbitrary.