PHY 711 – Problem Set # 5

Continue reading Chapter 3 of Fetter & Walecka.

1. Suppose you want to extremize an integral of the form:

$$\mathcal{I} \equiv \int_{x_1}^{x_2} \phi(y, y', y''; x) \, dx,$$

where $y = y(x)$, $y' \equiv \frac{dy}{dx}$, and $y'' \equiv \frac{d^2y}{dx^2}$, and where $\phi$ is a given function. Find the generalization of the Euler-Lagrange equation (Eqs. 17.19 & 34 in your text) for this function $\phi$. In order to solve this problem, you should assume that the end point values and slopes are all fixed (not varying). That is, $y(x_1) = A$, $y(x_2) = B$, $y'(x_1) = C$, and $y'(x_2) = D$, where $A$, $B$, and $C$, and $D$ are all fixed values.

2. Extra Credit

Consider an integral of the form:

$$E = \int d^3r \, F[\rho(r), g(r)],$$

where $\rho(r)$ is a function of position in three dimensions, and $g(r) \equiv |\nabla \rho(r)|$. Show that the functional variation of $E$ with respect to $\rho(r)$ is given by:

$$\delta E = \int d^3r \left\{ \frac{\partial F}{\partial \rho} - \frac{\partial F}{\partial g} \left( \frac{\nabla^2 \rho}{g} - \frac{\nabla \rho \cdot \nabla g}{g^2} \right) - \frac{\partial^2 F}{\partial \rho \partial g} g - \frac{\partial^2 F}{\partial^2 g} \frac{\nabla \rho \cdot \nabla g}{g^3} \right\} \delta \rho(r).$$