In class, we considered how to describe the Lorentz force on a particle of charge \( q \) in an electric field \( E \) and magnetic field \( B \):

\[
F_{EM} = \frac{q}{c} \mathbf{v} \times \mathbf{B} + q \mathbf{E}
\]  

(1)

in the Lagrangian formulation.

Assuming we can describe these electric and magnetic fields in terms of the scalar and vector potentials according to

\[
E = -\nabla \phi - \frac{1}{c} \frac{\partial A}{\partial t},
\]

(2)

and

\[
B = \nabla \times \mathbf{A},
\]

(3)

the electromagnetic contribution to the Lagrangian is

\[
L_{EM} = \frac{q}{c} \mathbf{\dot{r}} \cdot \mathbf{A} - q\phi.
\]

(4)

1. Show that

\[
F_{EM}|_z = \frac{\partial L_{EM}}{\partial z} - \frac{d}{dt} \frac{\partial L_{EM}}{\partial \dot{z}}.
\]

(5)

2. Consider what happens to the Lagrangian and to the Lorentz force when the vector and scalar potentials are changed according to

\[
A' = A + \nabla \psi,
\]

(6)

and

\[
\phi' = \phi - \frac{1}{c} \frac{\partial \psi}{\partial t}.
\]

(7)

Here, the scalar function \( \psi(x, y, z, t) \) is arbitrary.

3. Write down a suitable Lagrangian function to describe the 3-dimensional motion of a particle of mass \( m \) and charge \( q \) moving in a uniform magnetic field

\[
B = B_0 \hat{z},
\]

(8)

where \( B_0 \) is a fixed constant.