PHY 711 – Assignment #8

September 15, 2007

In class, we considered how to describe the Lorentz force on a particle of charge q in an electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$:

$$\mathbf{F_{EM}} = \frac{q}{c}\mathbf{v} \times \mathbf{B} + q\mathbf{E} \tag{1}$$

in the Lagrangian formulation.

Assuming we can describe these electric and magnetic fields in terms of the scalar and vector potentials according to

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}, \tag{2}$$

and

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{3}$$

the electromagnetic contribution to the Lagrangian is

$$L_{EM} = \frac{q}{c}\dot{\mathbf{r}} \cdot \mathbf{A} - q\phi. \tag{4}$$

1. Show that

$$\mathbf{F_{EM}}\rfloor_z = \frac{\partial L_{EM}}{\partial z} - \frac{d}{dt}\frac{\partial L_{EM}}{\partial \dot{z}}. \tag{5}$$

2. Consider what happens to the Lagrangian and to the Lorentz force when the vector and scalar potentials are changed according to

$$\mathbf{A}' = \mathbf{A} + \nabla\psi, \tag{6}$$

   and

$$\phi' = \phi - \frac{1}{c}\frac{\partial \psi}{\partial t}. \tag{7}$$

   Here, the scalar function $\psi(x, y, z, t)$ is arbitrary.

3. Write down a suitable Lagrangian function to describe the 3-dimensional motion of a particle of mass m and charge q moving in a uniform magnetic field

$$\mathbf{B} = B_0\hat{\mathbf{z}}, \tag{8}$$

   where B_0 is a fixed constant.