PHY 711 – Assignment #6

1. In class, we considered the famous Brachistochrone problem in which a mass slides down a frictionless track with shape $y(x)$, starting at $y(0) = 2a$ and ending at $y(a\pi) = 0$. The integral that calculates the travel time T between these two points is:

$$\sqrt{2g}T = \int_0^{a\pi} \sqrt{\frac{1 + (dy/dx)^2}{2a - y}}\, dx. \tag{1}$$

Here g denotes the gravitational acceleration. Evaluate this integral for the follow two shapes $y(x)$. Numerically compare these two results (as factors of $\sqrt{a}$) to determine which is larger. Explain.

   (a) In parametric form with $0 \le \theta \le \pi$:
   $x = a(\theta - \sin\theta)$
   $y = a(1 + \cos\theta)$

   (b) In conventional form with $0 \le x \le a\pi$:
   $y(x) = 2a - \frac{2}{\pi}x$