PHY 711 – Assignment #8

September 11, 2008

This exercise is designed to illustrate the differences between partial and total derivatives.

1. Consider an arbitrary function of the form $f = f(q, \dot{q}, t)$, where it is assumed that $q = q(t)$ and $\dot{q} \equiv dq/dt$.

   (a) Evaluate
   $$\frac{\partial}{\partial q}\frac{df}{dt} - \frac{d}{dt}\frac{\partial f}{\partial q}.$$

   (b) Evaluate
   $$\frac{\partial}{\partial \dot{q}}\frac{df}{dt} - \frac{d}{dt}\frac{\partial f}{\partial \dot{q}}.$$

   (c) Evaluate
   $$\frac{df}{dt}$$
   .

   (d) Now suppose that
   $$f(q, \dot{q}, t) = q\dot{q}t, \quad \text{where} \quad q(t) = \mathrm{e}^{-t/\tau}.$$
   Here τ is a constant. Evaluate df/dt using the expression you just derived. Now find $f(t)$ and take its time derivative directly to check your previous results.