This exercise is designed to illustrate the differences between partial and total derivatives.

1. Consider an arbitrary function of the form \( f = f(q, \dot{q}, t) \), where it is assumed that \( q = q(t) \) and \( \dot{q} \equiv dq/dt \).

   (a) Evaluate
   \[
   \frac{\partial}{\partial q} \frac{df}{dt} - \frac{df}{dt} \frac{\partial}{\partial q}.
   \]

   (b) Evaluate
   \[
   \frac{\partial}{\partial \dot{q}} \frac{df}{dt} - \frac{df}{dt} \frac{\partial}{\partial \dot{q}}.
   \]

   (c) Evaluate
   \[
   \frac{df}{dt}.
   \]

   (d) Now suppose that
   \[ f(q, \dot{q}, t) = q\dot{q}t, \quad \text{where} \quad q(t) = e^{-t/\tau}. \]
   
   Here \( \tau \) is a constant. Evaluate \( df/dt \) using the expression you just derived. Now find \( f(t) \) and take its time derivative directly to check your previous results.