September 1, 2010

PHY 741 – Problem Set #2

Finish reading Chapter 1 and start Chapter 2 in **Mahan**; homework is due Wednesday, September 1, 2010.

Find the current density vector $\mathbf{J}(\mathbf{r}, t)$ for the following probability amplitudes. Note that in cartesian coordinates, the gradient operator can be written:

$$\nabla f(\mathbf{r}, t) \equiv \hat{\mathbf{x}}\frac{\partial f(\mathbf{r}, t)}{\partial x} + \hat{\mathbf{y}}\frac{\partial f(\mathbf{r}, t)}{\partial y} + \hat{\mathbf{z}}\frac{\partial f(\mathbf{r}, t)}{\partial z}.$$

In spherical polar coordinates, the gradient operator can be written:

$$\nabla f(\mathbf{r}, t) \equiv \hat{\mathbf{r}}\frac{\partial f(\mathbf{r}, t)}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial f(\mathbf{r}, t)}{\partial \theta} + \hat{\phi}\frac{1}{r\,\sin\theta}\frac{\partial f(\mathbf{r}, t)}{\partial \phi}.$$

1.

$$\psi(\mathbf{r}, t) = \frac{1}{\sqrt{64\pi}}\left(\frac{Z}{a}\right)^{3/2}\frac{Zr}{a}\mathrm{e}^{-Zr/(2a)}\sin\theta\mathrm{e}^{-i\phi}\mathrm{e}^{+i\omega t},$$

where Z and a are constants and $\omega = Z^2e^2/(8a\hbar)$.

2.

$$\psi(\mathbf{r}, t) = \begin{cases} \left(A\mathrm{e}^{ikx} + B\mathrm{e}^{-ikx}\right)\mathrm{e}^{-i\hbar k^2 t/(2m)} & \text{for} \quad x \leq 0 \\ C\mathrm{e}^{iqx}\mathrm{e}^{-i\hbar k^2 t/(2m)} & \text{for} \quad x > 0, \end{cases}$$

where k and q are constants as are A, B, and C.