PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 14:

Chapter 9 -- Linear momentum

1. **Impulse and momentum**
2. **Conservation of linear momentum**
3. **Examples – collision analysis**

5	09/07/2012	Motion in 2d	4.1-4.3	4.3,4.50	09/10/2012
6	09/10/2012	Circular motion	4.4-4.6	4.29,4.30	09/12/2012
7	09/12/2012	Newton's laws	5.1-5.6	5.1,5.13	09/14/2012
8	09/14/2012	Newton's laws applied	5.7-5.8	5.20,5.30,5.48	09/17/2012
	09/17/2012	Review	1-5		
	09/19/2012	Exam	1-5		
9	09/21/2012	More applications of Newton's laws	6.1-6.4	6.3,6.14	09/24/2012
10	09/24/2012	Work	7.1-7.4	7.1,7.15	09/26/2012
11	09/26/2012	Kinetic energy	7.5-7.9	7.31,7.41,7.49	09/28/2012
12	09/28/2012	Conservation of energy	8.1-8.5	8.6,8.22,8.35	10/01/2012
13	10/01/2012	Momentum and collisions	9.1-9.4	9.15,9.18	10/03/2012
14	10/03/2012	Momentum and collisions	9.5-9.9	9.29,9.37	10/05/2012
	10/05/2012	Review	6-9		
	10/08/2012	Exam	6-9		
15	10/10/2012	Rotational motion	10.1-10.5		10/12/2012

PHY 113 General Physics I -- Section A

MWF 9 AM-9:50 PM	OPL 101	http://www.wfu.edu/~natalie/f12phy113/

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- General information
- Syllabus and homework assignments
- Lecture Notes
- For registered students

Tutorial sessions in Olin 101

- Sundays 5:00-7:00 PM -- Jiajie Xiao
- Mondays 5:00-7:00 PM -- Jiajie Xiao
- Tuesdays 5:00-7:00 PM -- Stephen Baker
- Wednesdays 5:00-7:00 PM -- Stephen Baker
- Thursdays 5:00-7:00 PM -- Loah Stevens
- Fridays 5:00-7:00 PM -- Loah Stevens

Note: Because of PHY 114 exams on Sunday and Monday, the tutorials those nights will be moved from Olin 101 – check for signs – this week only.

Summary of physics "laws"

Newton's second law:

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt}$$

Work-kinetic energy theorem:

$$W_{total}^{i\to f} = \int_i^f \mathbf{F}\cdot d\mathbf{r} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

For conservative forces: $W_{conservative}^{i\to f} = -\left(U\left(\mathbf{r}_f\right) - U\left(\mathbf{r}_i\right)\right)$

$$W_{total}^{i\to f} = W_{conservative}^{i\to f} + W_{non-conservative}^{i\to f}$$

$$\frac{1}{2}mv_i^2 + U\left(\mathbf{r}_i\right) + W_{non-conservative}^{i\to f} = \frac{1}{2}mv_f^2 + U\left(\mathbf{r}_f\right)$$

Another way to look at Newton's second law:

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt}$$

$$\text{Define "linear momentum":} \qquad m\mathbf{v} \equiv \mathbf{p}$$

$$\text{Units of linear momentum: } \mathrm{kg \cdot m/s}$$

iclicker question:

Why would you want to define linear momentum?

A. **To impress your friends.**
B. **To exercise your brain.**
C. **It might be helpful.**
D. **To distinguish it from angular momentum.**

Relationship between Newton's second law and linear momentum:

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt} = \frac{d\mathbf{p}}{dt}$$

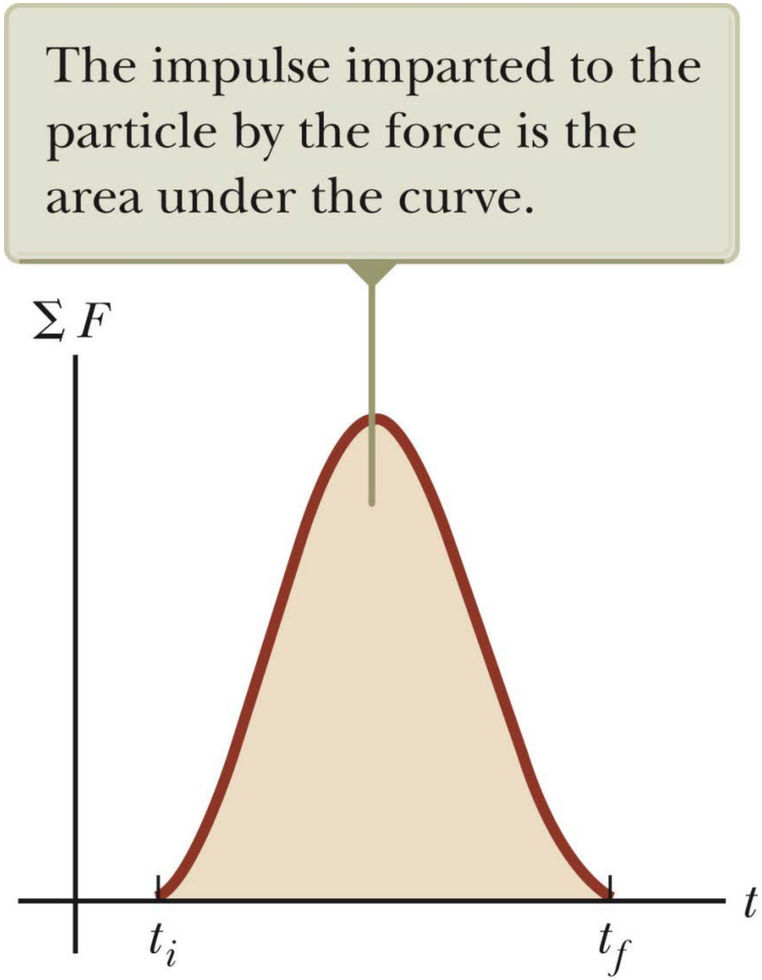
Using a little calculus:

$$\mathbf{F}dt = d\mathbf{p}$$

$$\int_i^f \mathbf{F}dt = \int_i^f d\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$$

Define "impulse": $I = \int_i^f \mathbf{F}dt$

$$I = \int_i^f \mathbf{F}dt = \mathbf{p}_f - \mathbf{p}_i$$



Example: Suppose the impulse $I = \int_i^f \mathbf{F}dt$ shown in this figure has the value of 1000 Ns which is imparted to an object of mass $m = 50$ kg, initially at rest. What is its final velocity?

$$I = \int_i^f \mathbf{F}dt = \mathbf{p}_f - \mathbf{p}_i$$

$$\mathbf{v}_f = \frac{I}{m} = \frac{1000}{50} m/s = 20 m/s$$

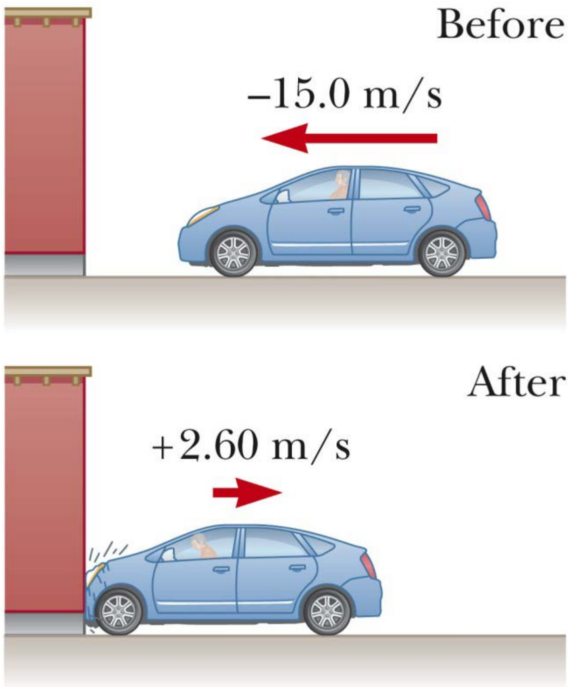
Suppose that a tennis ball with mass m=0.057 kg approaches a tennis racket at a speed of 45m/s. What is the impulse the racket must exert on the ball to return the ball in the opposite direction at the same speed. Assume that the motion is completely horizontal.

$$I = \left|\mathbf{p}_f - \mathbf{p}_i\right| = 2mv = 2(0.057)(45) = 5.13Ns$$

What is the average force exerted by the racket if the interaction lasted for 0.3 s?

$$\langle \mathrm{F} \rangle = \frac{I}{\Delta t} = \frac{5.13Ns}{0.3s} = 17.1N$$

Example: A 1500 kg car collides with a wall, with v_i= -15m/s and v_f=2.6m/s. What is the impulse exerted on the car?



$$I = \left|\mathbf{p}_f - \mathbf{p}_i\right| = m\left(v_f - v_i\right)$$
$$= (1500\text{kg})(2.6-(-15))m/s$$
$$= 26400 \text{ Ns}$$

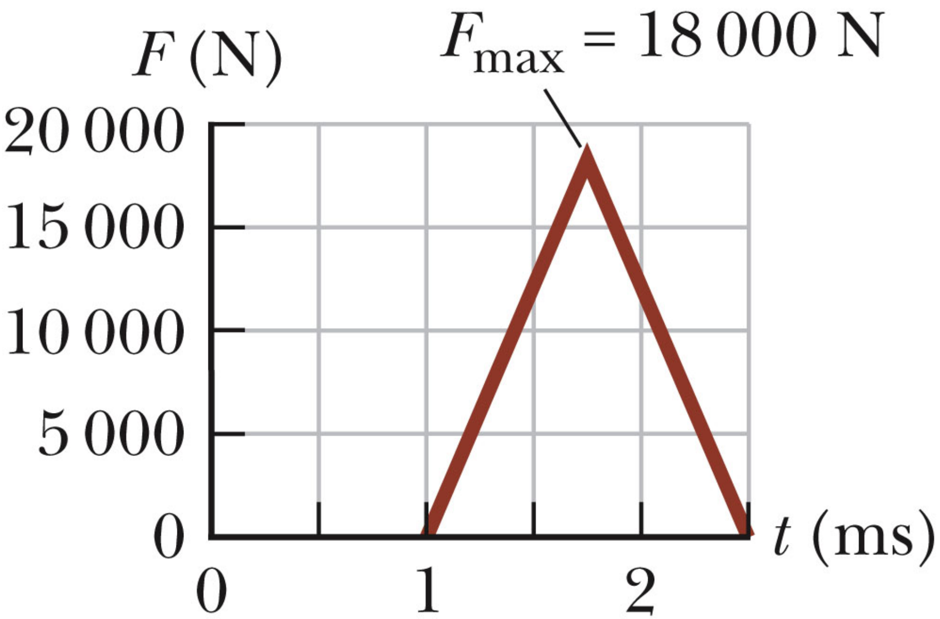
If the collision time is 0.15 s, what is the average force exerted on the car?

$$\langle \mathrm{F} \rangle = \frac{I}{\Delta t} = \frac{26400\,Ns}{0.15s} = 1.76\times 10^5\,N$$

Energy loss due to collision

$$\Delta E \equiv \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = -1.6368\times 10^5\,J$$

Example of graphical representation of F(t)



Estimate of impulse:

$$I = \int_{0.001}^{0.003} F(t)dt = \frac{1}{2}F_{max}(0.002s) = \frac{1}{2}(18000N)(0.002s) = 18Ns$$

Physics of composite systems

Newton's second law:

$$\sum_i \mathbf{F}_i = \sum_i m_i \mathbf{a}_i = \sum_i m_i \frac{d\mathbf{v}_i}{dt} = \sum_i \frac{dm_i \mathbf{v}_i}{dt} = \frac{d}{dt}\left(\sum_i \mathbf{p}_i\right)$$

Note that if $\sum_i \mathbf{F}_i = 0$, then:

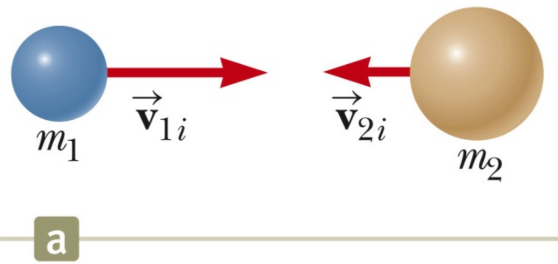
$$\frac{d}{dt}\left(\sum_i \mathbf{p}_i\right) = 0$$

$$\Rightarrow \sum_i \mathbf{p}_i = \text{(constant)}$$

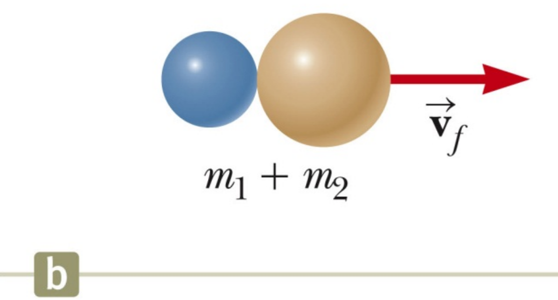
$$\Rightarrow \sum_i \mathbf{p}_{i\,\text{initial}} = \sum_i \mathbf{p}_{i\,\text{final}}$$

Example – completely inelastic collision; balls moving on a frictionless surface

Before the collision, the particles move separately.



After the collision, the particles move together.



$$\sum_i \mathbf{p}_{i\,\text{initial}} = \sum_i \mathbf{p}_{i\,\text{final}}$$

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$$

$$\mathbf{v}_f = \frac{m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i}}{m_1 + m_2}$$

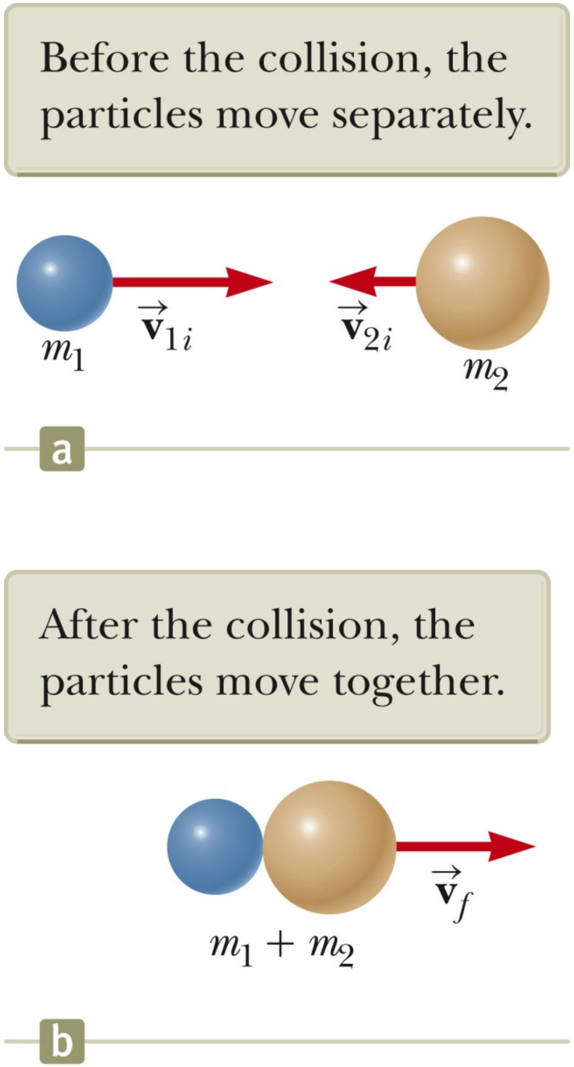
For $\quad m_1 = 0.3kg, \; m_2 = 0.5kg$

$$\mathbf{v}_{1i} = 2m/s\,\hat{\mathbf{i}}, \; \mathbf{v}_{2i} = -1m/s\,\hat{\mathbf{i}}$$

$$\mathbf{v}_f = \frac{(0.3)(2)\hat{\mathbf{i}} + (0.5)(-1)\hat{\mathbf{i}}}{(0.3)+(0.5)}m/s$$

$$= 0.125 \;\; m/s\;\hat{\mathbf{i}}$$

Energy loss in this example:



$$\Delta E = \frac{1}{2}(m_1 + m_2)\left|\mathbf{v}_f\right|^2 - \left(\frac{1}{2}m_1\left|\mathbf{v}_{1i}\right|^2 + \frac{1}{2}m_2\left|\mathbf{v}_{2i}\right|^2\right)$$

For $\quad m_1 = 0.3kg, \; m_2 = 0.5kg$

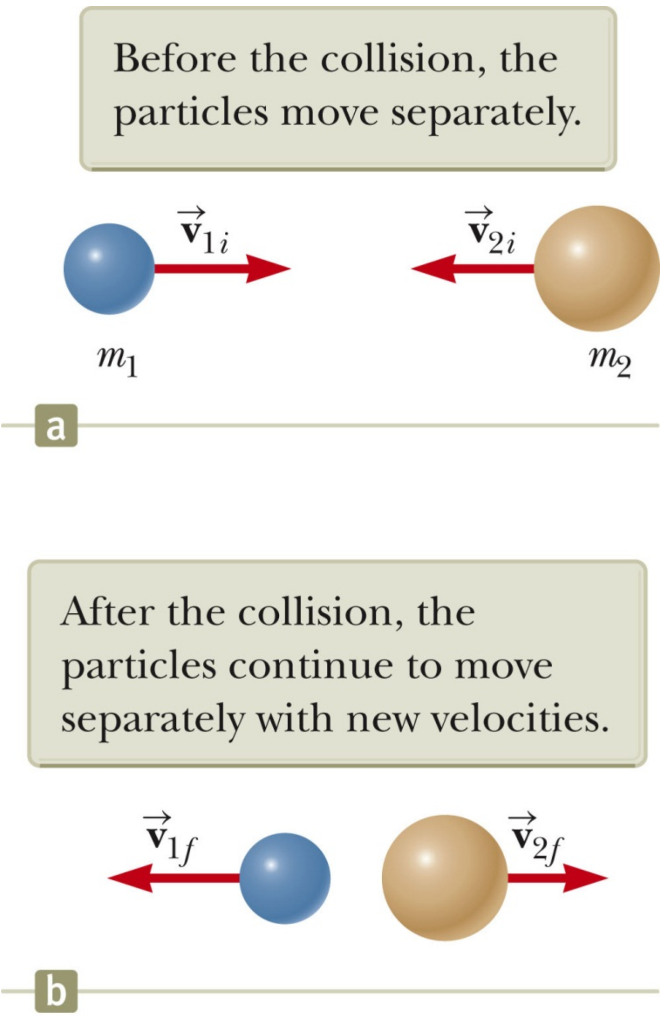
$$\mathbf{v}_{1i} = 2m/s\,\hat{\mathbf{i}}, \; \mathbf{v}_{2i} = -1m/s\,\hat{\mathbf{i}}$$

$$\mathbf{v}_f = 0.125 \; m/s\,\hat{\mathbf{i}}$$

$$\Delta E = \frac{1}{2}((0.3)+(0.5))\left|0.125\right|^2 - \left(\frac{1}{2}(0.3)\left|2\right|^2 + \frac{1}{2}(0.5)\left|1\right|^2\right)$$

$$= -0.84 \; J$$

Example – completely elastic collision; balls moving on a frictionless surface



$$\sum_i \mathbf{p}_{i\,\text{initial}} = \sum_i \mathbf{p}_{i\,\text{final}}$$

$$\sum_i \left(\frac{1}{2} m_i v_{i\,\text{initial}}^2\right) = \sum_i \left(\frac{1}{2} m_i v_{i\,\text{final}}^2\right)$$

$$m_1 \mathbf{v}_{1i} + m_2 \mathbf{v}_{2i} = m_1 \mathbf{v}_{1f} + m_2 \mathbf{v}_{2f}$$

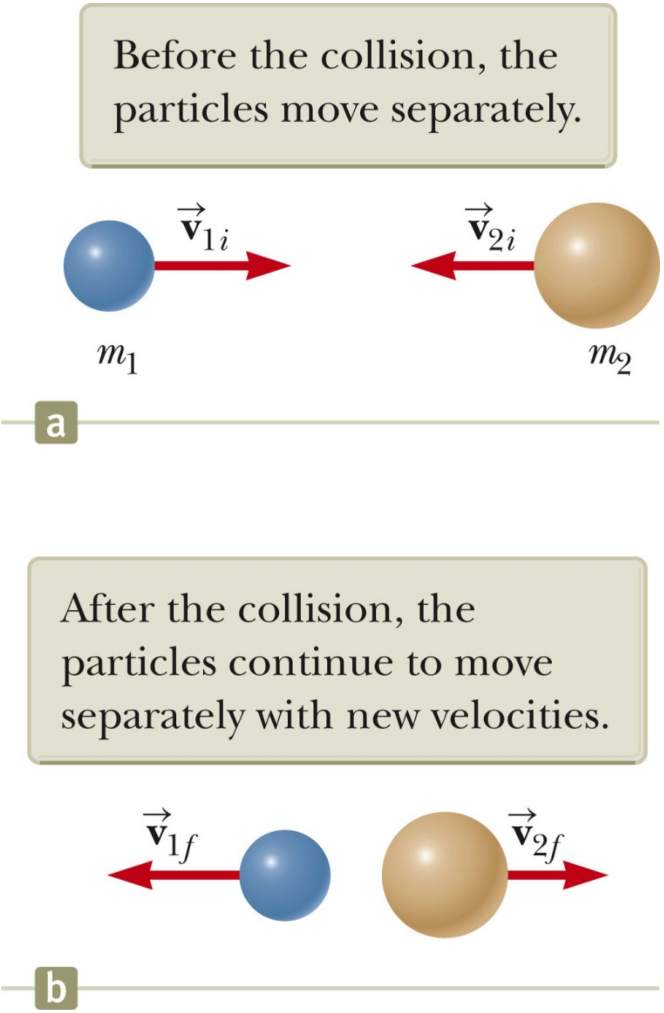
$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

For 1-d motion, after some algebra:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

Completely elastic collision; numerical example:



$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

For $\quad m_1 = 0.3kg, \; m_2 = 0.5kg$

$$\mathbf{v}_{1i} = 2m/s\,\hat{\mathbf{i}}, \; \mathbf{v}_{2i} = -1m/s\,\hat{\mathbf{i}}$$

$$v_{1f} = \left(\frac{(0.3)-(0.5)}{(0.3)+(0.5)}\right)(2m/s) + $$

$$\left(\frac{2(0.5)}{(0.3)+(0.5)}\right)(-1m/s) = -1.75m/s$$

$$v_{1f} = \left(\frac{2(0.3)}{(0.3)+(0.5)}\right)(2m/s) + $$

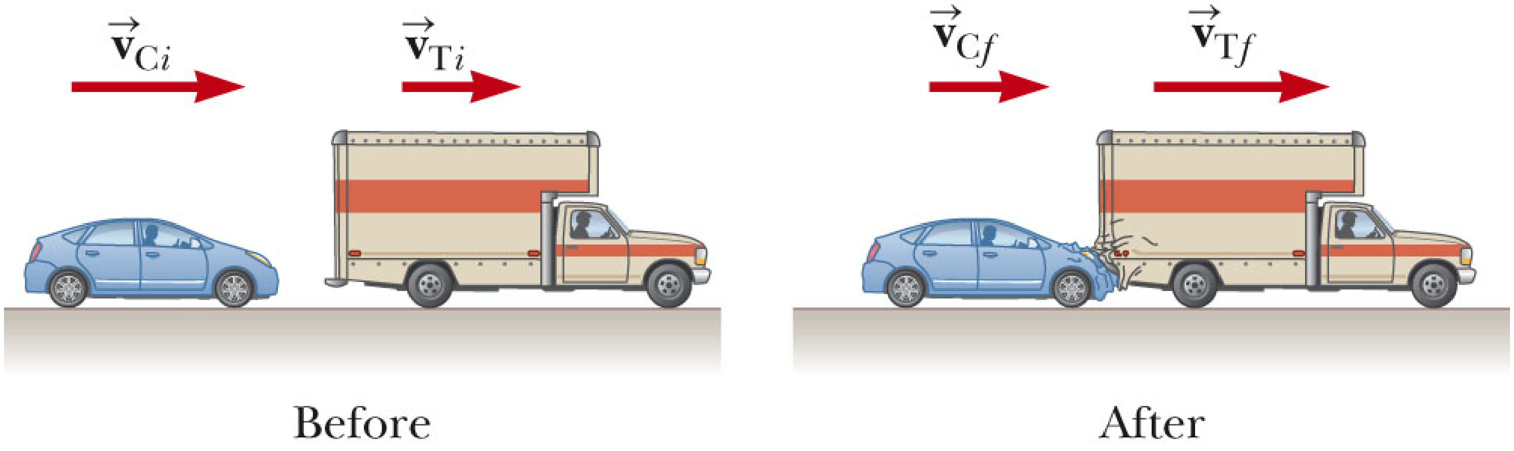
$$\left(\frac{(0.5)-(0.3)}{(0.3)+(0.5)}\right)(-1m/s) = 1.25m/s$$

iclicker exercise:

We have assumed that there is no net force acting on the system. What happens if there are interaction forces between the particles?

A. Analysis still applies
B. Analysis must be modified

Example from homework:

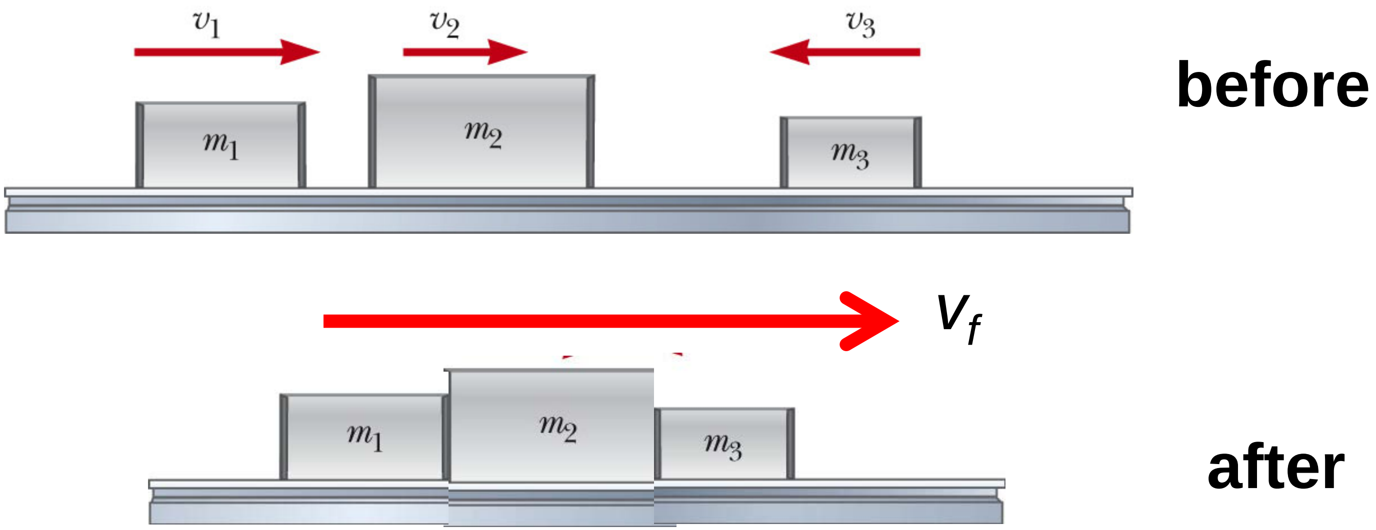


$$\sum_i \mathbf{p}_{i\ \text{initial}} = \sum_i \mathbf{p}_{i\ \text{final}}$$

$$m_C \mathbf{v}_{Ci} + m_T \mathbf{v}_{Ti} = m_C \mathbf{v}_{Cf} + m_T \mathbf{v}_{Tf}$$

$$\mathbf{v}_{Tf} = ?$$

Example from homework:



$$\sum_i \mathbf{p}_{i\,\text{initial}} = \sum_i \mathbf{p}_{i\,\text{final}}$$