

PHY 113 A General Physics I

9-9:50 AM MWF Olin 101

Plan for Lecture 14:

Chapter 9 -- Linear momentum

1. Impulse and momentum

2. Conservation of linear momentum

3. Examples – collision analysis

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5	09/07/2012	Motion in 2d	<a href="#">4.1-4.3</a>	<a href="#">4.3-4.50</a>	09/10/2012
6	09/10/2012	Circular motion	<a href="#">4.4-4.6</a>	<a href="#">4.29-4.30</a>	09/12/2012
7	09/12/2012	Newton's laws	<a href="#">5.1-5.6</a>	<a href="#">5.1-5.13</a>	09/14/2012
8	09/14/2012	Newton's laws applied	<a href="#">5.7-5.8</a>	<a href="#">5.20-5.30-5.48</a>	09/17/2012
	09/17/2012	Review	<a href="#">1-5</a>		
	09/19/2012	Exam	1-5		
9	09/21/2012	More applications of Newton's laws	<a href="#">6.1-6.4</a>	<a href="#">6.3-6.14</a>	09/24/2012
10	09/24/2012	Work	<a href="#">7.1-7.4</a>	<a href="#">7.1-7.15</a>	09/26/2012
11	09/26/2012	Kinetic energy	<a href="#">7.5-7.9</a>	<a href="#">7.31-7.41-7.49</a>	09/28/2012
12	09/28/2012	Conservation of energy	<a href="#">8.1-8.5</a>	<a href="#">8.6-8.22-8.35</a>	10/01/2012
13	10/01/2012	Momentum and collisions	<a href="#">9.1-9.4</a>	<a href="#">9.15-9.19</a>	10/03/2012
14	10/03/2012	Momentum and collisions	<a href="#">9.5-9.9</a>	<a href="#">9.29-9.37</a>	10/05/2012
	10/05/2012	Review	<a href="#">6-9</a>		
	10/08/2012	Exam	6-9		
15	10/10/2012	Rotational motion	<a href="#">10.1-10.5</a>		10/12/2012

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PHY 113 General Physics I -- Section A

MWF 9 AM-9:50 PM OPL 101 <http://www.wfu.edu/~natalie/f12phy113/>

Instructor: [Natalie Holzwarth](#) Phone:758-5510 Office:300 OPL e-mail:[natalie@wfu.edu](mailto:natalie@wfu.edu)

Tutorial sessions in Olin 101

- General information
- Syllabus and homework assignments
- Lecture Notes
- For registered students

- Sundays 5:00-7:00 PM -- Jiajie Xiao
- Mondays 5:00-7:00 PM -- Jiajie Xiao
- Tuesdays 5:00-7:00 PM -- Stephen Baker
- Wednesdays 5:00-7:00 PM -- Stephen Baker
- Thursdays 5:00-7:00 PM -- Leah Stevens
- Fridays 5:00-7:00 PM -- Leah Stevens

**Note: Because of PHY 114 exams on Sunday and Monday, the tutorials those nights will be moved from Olin 101 – check for signs – this week only.**

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**Summary of physics "laws"**

Newton's second law :

$$\mathbf{F} = m\mathbf{a} = m \frac{d\mathbf{v}}{dt}$$

Work - kinetic energy theorem :

$$W_{total}^{i \rightarrow f} = \int_i^f \mathbf{F} \cdot d\mathbf{r} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

For conservative forces :  $W_{conservative}^{i \rightarrow f} = -(U(\mathbf{r}_f) - U(\mathbf{r}_i))$

$$W_{total}^{i \rightarrow f} = W_{conservative}^{i \rightarrow f} + W_{non-conservative}^{i \rightarrow f}$$

$$\frac{1}{2}mv_i^2 + U(\mathbf{r}_i) + W_{non-conservative}^{i \rightarrow f} = \frac{1}{2}mv_f^2 + U(\mathbf{r}_f)$$

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**Another way to look at Newton's second law:**

$$\mathbf{F} = m\mathbf{a} = m \frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt}$$

Define "linear momentum":  $m\mathbf{v} \equiv \mathbf{p}$

Units of linear momentum: kg · m/s

**iclicker question:**

Why would you want to define linear momentum?

- A. To impress your friends.
- B. To exercise your brain.
- C. It might be helpful.
- D. To distinguish it from angular momentum.

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**Relationship between Newton's second law and linear momentum:**

$$\mathbf{F} = m\mathbf{a} = m \frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt} = \frac{d\mathbf{p}}{dt}$$

Using a little calculus :

$$\mathbf{F}dt = d\mathbf{p}$$

$$\int_i^f \mathbf{F}dt = \int_i^f d\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$$

Define "impulse":  $I = \int_i^f \mathbf{F}dt$

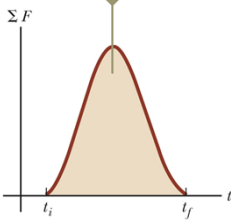
$$I = \int_i^f \mathbf{F}dt = \mathbf{p}_f - \mathbf{p}_i$$

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The impulse imparted to the particle by the force is the area under the curve.



Example: Suppose the impulse  $I = \int_i^f \mathbf{F} dt$  shown in this figure has the value of 1000 Ns which is imparted to an object of mass  $m = 50$  kg, initially at rest. What is its final velocity?

$$I = \int_i^f \mathbf{F} dt = \mathbf{p}_f - \mathbf{p}_i$$

$$\mathbf{v}_f = \frac{I}{m} = \frac{1000}{50} \text{ m/s} = 20 \text{ m/s}$$

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Suppose that a tennis ball with mass  $m=0.057$  kg approaches a tennis racket at a speed of 45m/s. What is the impulse the racket must exert on the ball to return the ball in the opposite direction at the same speed. Assume that the motion is completely horizontal.

$$I = |\mathbf{p}_f - \mathbf{p}_i| = 2mv = 2(0.057)(45) = 5.13 \text{ Ns}$$

What is the average force exerted by the racket if the interaction lasted for 0.3 s?

$$\langle F \rangle = \frac{I}{\Delta t} = \frac{5.13 \text{ Ns}}{0.3 \text{ s}} = 17.1 \text{ N}$$

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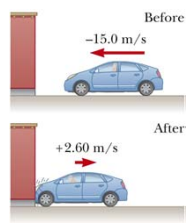
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**Example: A 1500 kg car collides with a wall, with  $v_i = -15 \text{ m/s}$  and  $v_f = 2.6 \text{ m/s}$ . What is the impulse exerted on the car?**



$$I = |\mathbf{p}_f - \mathbf{p}_i| = m(v_f - v_i) = (1500 \text{ kg})(2.6 - (-15)) \text{ m/s} = 26400 \text{ Ns}$$

If the collision time is 0.15 s, what is the average force exerted on the car?

$$\langle F \rangle = \frac{I}{\Delta t} = \frac{26400 \text{ Ns}}{0.15 \text{ s}} = 1.76 \times 10^5 \text{ N}$$

Energy loss due to collision

$$\Delta E = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2) = -1.6368 \times 10^5 \text{ J}$$

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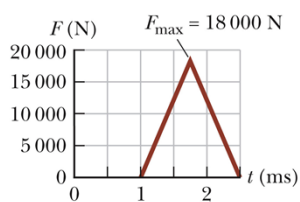
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### Example of graphical representation of $F(t)$



Estimate of impulse:

$$I = \int_{0.001}^{0.003} F(t) dt = \frac{1}{2} F_{\max} (0.002\text{ s}) = \frac{1}{2} (18\,000\text{ N}) (0.002\text{ s}) = 18\text{ N s}$$

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### Physics of composite systems

Newton's second law :

$$\sum_i \mathbf{F}_i = \sum_i m_i \mathbf{a}_i = \sum_i m_i \frac{d\mathbf{v}_i}{dt} = \sum_i \frac{dm_i \mathbf{v}_i}{dt} = \frac{d}{dt} \left( \sum_i \mathbf{p}_i \right)$$

Note that if  $\sum_i \mathbf{F}_i = 0$ , then :

$$\frac{d}{dt} \left( \sum_i \mathbf{p}_i \right) = 0$$

$$\Rightarrow \sum_i \mathbf{p}_i = (\text{constant})$$

$$\Rightarrow \sum_i \mathbf{p}_{i \text{ initial}} = \sum_i \mathbf{p}_{i \text{ final}}$$

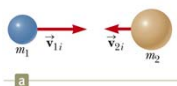
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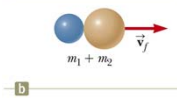
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### Example – completely **inelastic** collision; balls moving on a frictionless surface

Before the collision, the particles move separately.



After the collision, the particles move together.



$$\sum_i \mathbf{p}_{i \text{ initial}} = \sum_i \mathbf{p}_{i \text{ final}}$$

$$m_1 \mathbf{v}_{1i} + m_2 \mathbf{v}_{2i} = (m_1 + m_2) \mathbf{v}_f$$

$$\mathbf{v}_f = \frac{m_1 \mathbf{v}_{1i} + m_2 \mathbf{v}_{2i}}{m_1 + m_2}$$

For  $m_1 = 0.3\text{ kg}$ ,  $m_2 = 0.5\text{ kg}$

$$\mathbf{v}_{1i} = 2\text{ m/s} \hat{\mathbf{i}}, \quad \mathbf{v}_{2i} = -1\text{ m/s} \hat{\mathbf{i}}$$

$$\mathbf{v}_f = \frac{(0.3)(2)\hat{\mathbf{i}} + (0.5)(-1)\hat{\mathbf{i}}}{(0.3) + (0.5)} \text{ m/s}$$

$$= 0.125 \text{ m/s} \hat{\mathbf{i}}$$

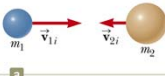
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**Energy loss in this example:**

Before the collision, the particles move separately.



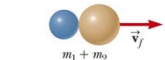
$$\Delta E = \frac{1}{2}(m_1 + m_2)|\mathbf{v}_f|^2 - \left( \frac{1}{2}m_1|\mathbf{v}_{1i}|^2 + \frac{1}{2}m_2|\mathbf{v}_{2i}|^2 \right)$$

For  $m_1 = 0.3\text{kg}$ ,  $m_2 = 0.5\text{kg}$

$$\mathbf{v}_{1i} = 2\text{m/s}\hat{\mathbf{i}}, \quad \mathbf{v}_{2i} = -1\text{m/s}\hat{\mathbf{i}}$$

$$\mathbf{v}_f = 0.125\text{ m/s}\hat{\mathbf{i}}$$

After the collision, the particles move together.



$$\Delta E = \frac{1}{2}((0.3) + (0.5))(0.125)^2 - \left( \frac{1}{2}(0.3)(2)^2 + \frac{1}{2}(0.5)(1)^2 \right)$$

$$= -0.84\text{ J}$$

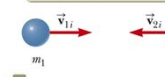
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**Example – completely elastic collision; balls moving on a frictionless surface**

Before the collision, the particles move separately.



$$\sum_i \mathbf{p}_{i\text{ initial}} = \sum_i \mathbf{p}_{i\text{ final}}$$

$$\sum_i \left( \frac{1}{2} m_i v_{i\text{ initial}}^2 \right) = \sum_i \left( \frac{1}{2} m_i v_{i\text{ final}}^2 \right)$$

$$m_1 \mathbf{v}_{1i} + m_2 \mathbf{v}_{2i} = m_1 \mathbf{v}_{1f} + m_2 \mathbf{v}_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

For 1-d motion, after some algebra :

$$v_{1f} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left( \frac{2m_2}{m_1 + m_2} \right) v_{2i}$$

$$v_{2f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{1i} + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) v_{2i}$$

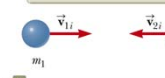
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**Completely elastic collision; numerical example:**

Before the collision, the particles move separately.



$$v_{1f} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left( \frac{2m_2}{m_1 + m_2} \right) v_{2i}$$

$$v_{2f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{1i} + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) v_{2i}$$

For  $m_1 = 0.3\text{kg}$ ,  $m_2 = 0.5\text{kg}$

$$\mathbf{v}_{1i} = 2\text{m/s}\hat{\mathbf{i}}, \quad \mathbf{v}_{2i} = -1\text{m/s}\hat{\mathbf{i}}$$

$$v_{1f} = \left( \frac{(0.3) - (0.5)}{(0.3) + (0.5)} \right) (2\text{m/s}) +$$

$$\left( \frac{2(0.5)}{(0.3) + (0.5)} \right) (-1\text{m/s}) = -1.75\text{m/s}$$

$$v_{1f} = \left( \frac{2(0.3)}{(0.3) + (0.5)} \right) (2\text{m/s}) +$$

$$\left( \frac{(0.5) - (0.3)}{(0.3) + (0.5)} \right) (-1\text{m/s}) = 1.25\text{m/s}$$

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**iclicker exercise:**

We have assumed that there is no net force acting on the system. What happens if there are interaction forces between the particles?

- A. Analysis still applies
- B. Analysis must be modified

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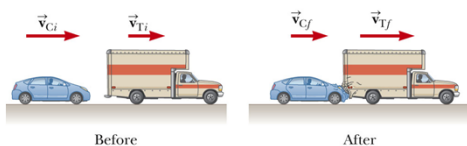
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**Example from homework:**

$$\sum_i \mathbf{p}_{i \text{ initial}} = \sum_i \mathbf{p}_{i \text{ final}}$$

$$m_C \mathbf{v}_{Ci} + m_T \mathbf{v}_{Ti} = m_C \mathbf{v}_{Cf} + m_T \mathbf{v}_{Tf}$$

$$\mathbf{v}_{Tf} = ?$$

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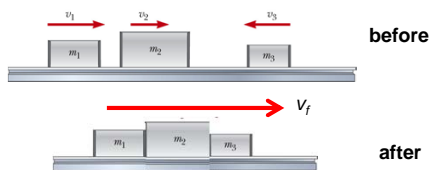
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**Example from homework:**

$$\sum_i \mathbf{p}_{i \text{ initial}} = \sum_i \mathbf{p}_{i \text{ final}}$$

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