PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 14:

Chapter 9 -- Linear momentum

1. **Impulse and momentum in two dimensions**
2. **Center of mass**
3. **Collision analysis in two dimensions**

5	09/07/2012	Motion in 2d	4.1-4.3	4.3,4.50	09/10/2012
6	09/10/2012	Circular motion	4.4-4.6	4.29,4.30	09/12/2012
7	09/12/2012	Newton's laws	5.1-5.6	5.1,5.13	09/14/2012
8	09/14/2012	Newton's laws applied	5.7-5.8	5.20,5.30,5.48	09/17/2012
	09/17/2012	Review	1-5		
	09/19/2012	Exam	1-5		
9	09/21/2012	More applications of Newton's laws	6.1-6.4	6.3,6.14	09/24/2012
10	09/24/2012	Work	7.1-7.4	7.1,7.15	09/26/2012
11	09/26/2012	Kinetic energy	7.5-7.9	7.31,7.41,7.49	09/28/2012
12	09/28/2012	Conservation of energy	8.1-8.5	8.6,8.22,8.35	10/01/2012
13	10/01/2012	Momentum and collisions	9.1-9.4	9.15,9.18	10/03/2012
14	10/03/2012	Momentum and collisions	9.5-9.9	9.29,9.37	10/05/2012
	10/05/2012	Review	6-9		
	10/08/2012	Exam	6-9		
15	10/10/2012	Rotational motion	10.1-10.5		10/12/2012

Relationship between Newton's second law and linear momentum for a single particle:

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt} = \frac{d\mathbf{p}}{dt}$$

Using a little calculus:

$$\mathbf{F}dt = d\mathbf{p}$$

$$\int_i^f \mathbf{F}dt = \int_i^f d\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$$

Define "impulse": $\mathbf{I} = \int_i^f \mathbf{F}dt$

$$\mathbf{I} = \int_i^f \mathbf{F}dt = \mathbf{p}_f - \mathbf{p}_i$$

Physics of composite systems

Newton's second law:

$$\sum_i \mathbf{F}_i = \sum_i m_i \mathbf{a}_i = \sum_i m_i \frac{d\mathbf{v}_i}{dt} = \sum_i \frac{dm_i \mathbf{v}_i}{dt} = \frac{d}{dt}\left(\sum_i \mathbf{p}_i\right)$$

Note that if $\sum_i \mathbf{F}_i = 0$, then:

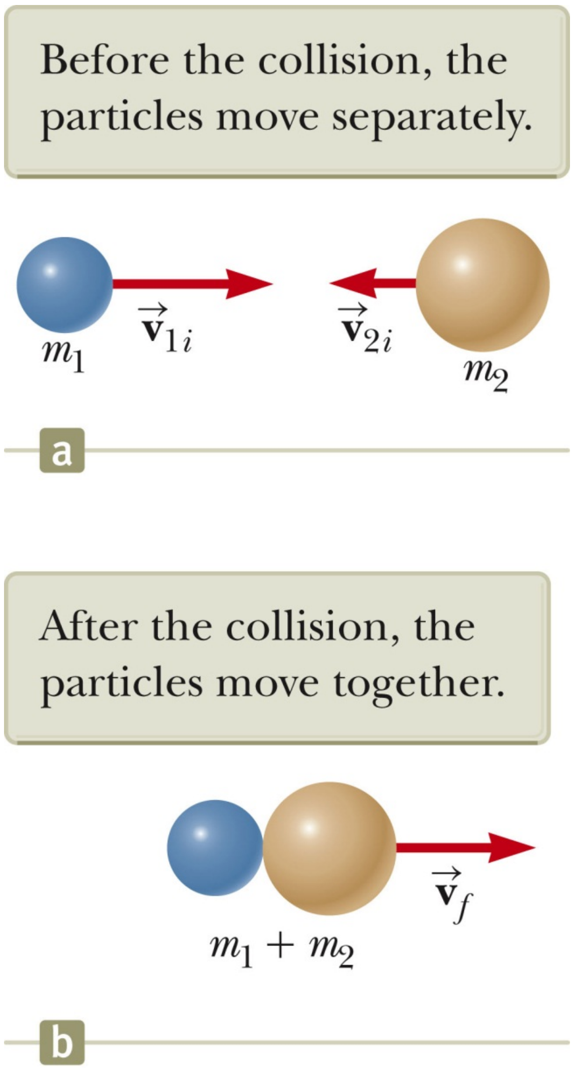
$$\frac{d}{dt}\left(\sum_i \mathbf{p}_i\right) = 0$$

$$\Rightarrow \sum_i \mathbf{p}_i = \text{(constant)}$$

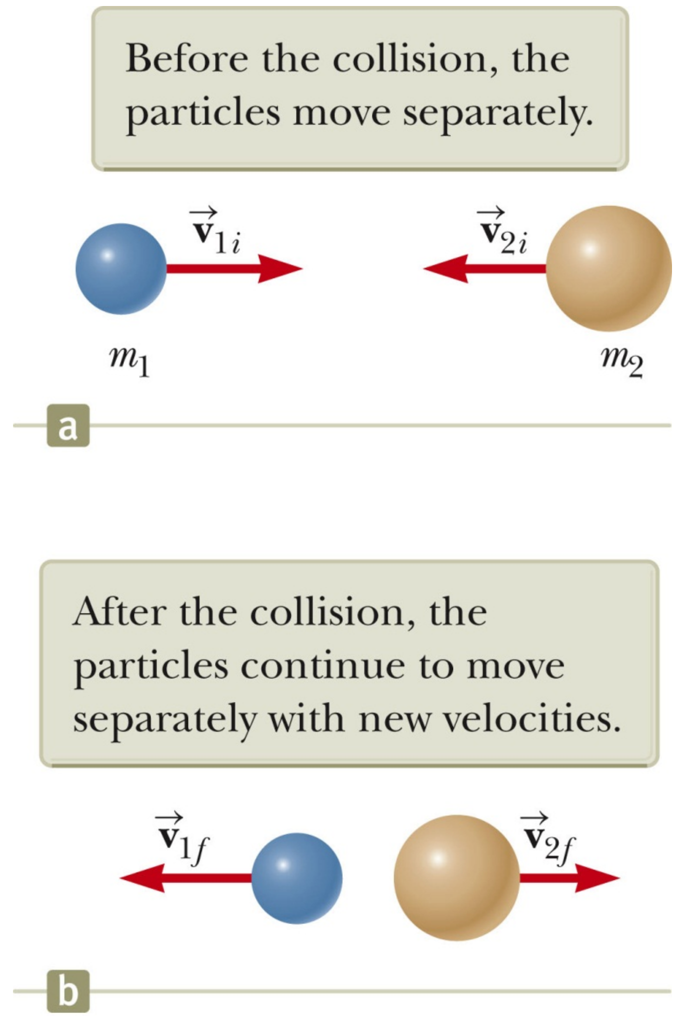
$$\Rightarrow \sum_i \mathbf{p}_{i\,\text{initial}} = \sum_i \mathbf{p}_{i\,\text{final}}$$

Examples of completely one-dimensional (head-on) collision; balls moving on a frictionless surface

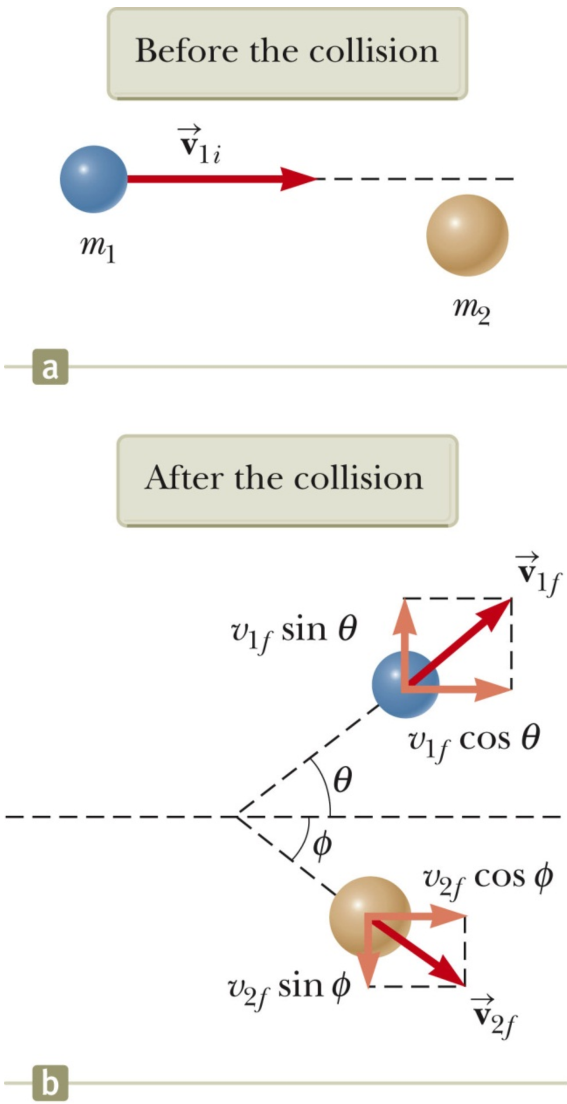
inelastic:



elastic:



Examples of two-dimensional collision; balls moving on a frictionless surface



$$\sum_i \mathbf{p}_{i\ \text{initial}} = \sum_i \mathbf{p}_{i\ \text{final}}$$

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$$

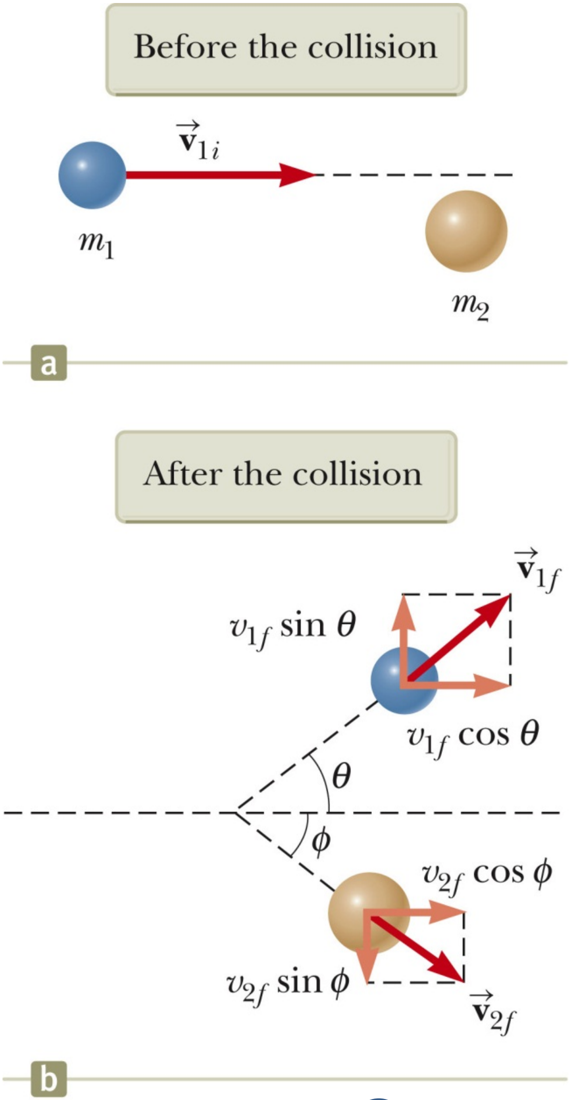
$$0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$$

$$\text{Knowns: } \ m_1, m_2, v_{1i}$$

$$\text{Unknowns: } \ v_{1f}, v_{2f}, \theta, \phi$$

$$\text{Need 2 more equations --}$$

Examples of two-dimensional collision; balls moving on a frictionless surface



$$\text{Suppose:} \quad m_1 = m_2 = 0.06kg, \quad v_{1i} = 2m/s,$$

$$v_{2f} = 1m/s, \quad \phi = 20^{\circ}$$

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$$

$$0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$$

$$v_{1f} \sin\theta = v_{2f} \sin\phi$$

$$= (1m/s)\sin 20^{o} = 0.342m/s$$

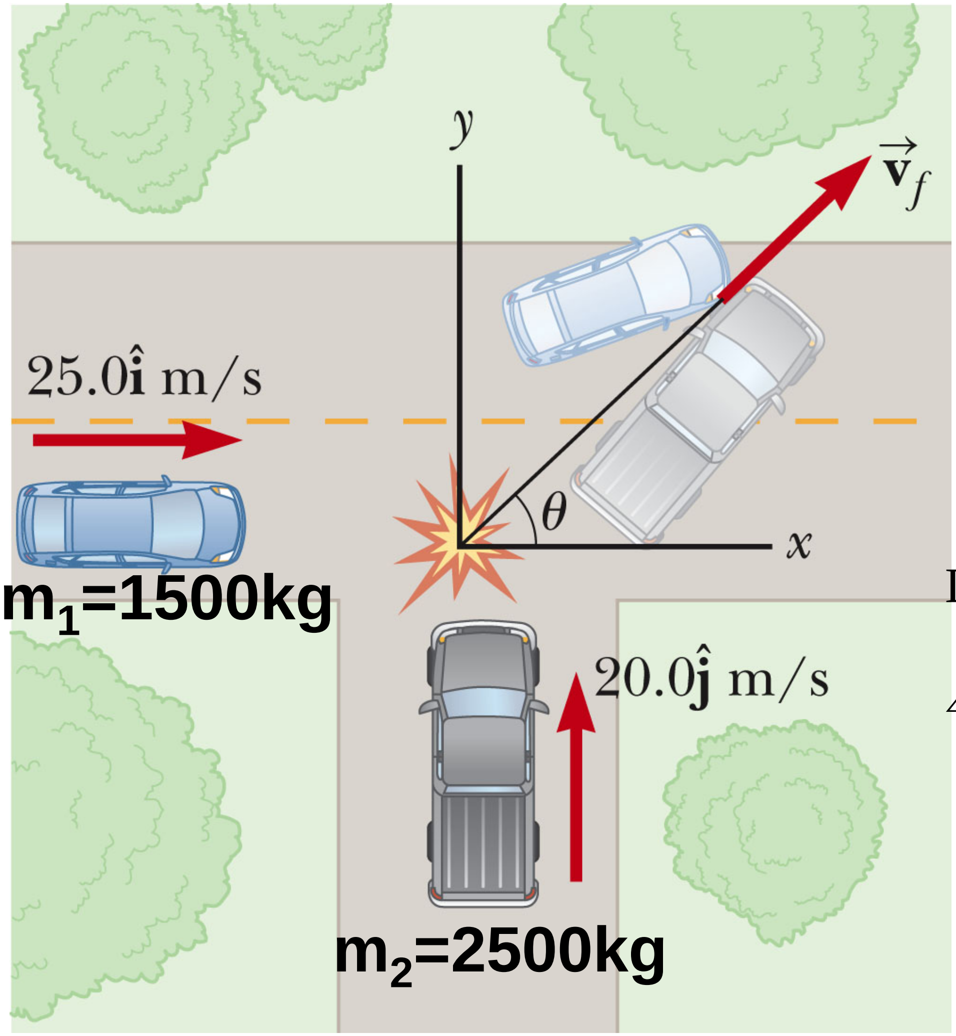
$$v_{1f} \cos\theta = v_{1i} - v_{2f} \cos\phi$$

$$= (2m/s) - (1m/s)(\cos 20^{o}) = 1.060m/s$$

$$\tan\theta = \frac{0.342}{1.060} \quad \Rightarrow \theta = 17.88^{\circ}$$

$$v_{1f} = \frac{0.342m/s}{\sin 17.88^{o}} = \frac{1.060m/s}{\cos 17.88^{o}} = 1.11m/s$$

Example: two-dimensional totally inelastic collision



$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$$

$$\mathbf{v}_f = \frac{m_1}{m_1 + m_2}\mathbf{v}_{1i} + \frac{m_2}{m_1 + m_2}\mathbf{v}_{2i}$$

$$\mathbf{v}_f = \frac{1500}{4000}25m/s\,\hat{\mathbf{i}} + \frac{2500}{4000}20m/s\,\hat{\mathbf{j}}$$

$$= 9.375m/s\,\hat{\mathbf{i}} + 12.5m/s\,\hat{\mathbf{j}}$$

Loss of kinetic energy in this case:

$$\Delta E = \frac{1}{2}(m_1 + m_2)|\mathbf{v}_f|^2 - \left(\frac{1}{2}m_1|\mathbf{v}_{1i}|^2 + \frac{1}{2}m_2|\mathbf{v}_{2i}|^2\right)$$

$$= \frac{1}{2}(4000)\left((9.375)^2 + (12.5)^2\right)$$

$$-\left(\frac{1}{2}1500(25)^2 + \frac{1}{2}2500(20)^2\right)$$

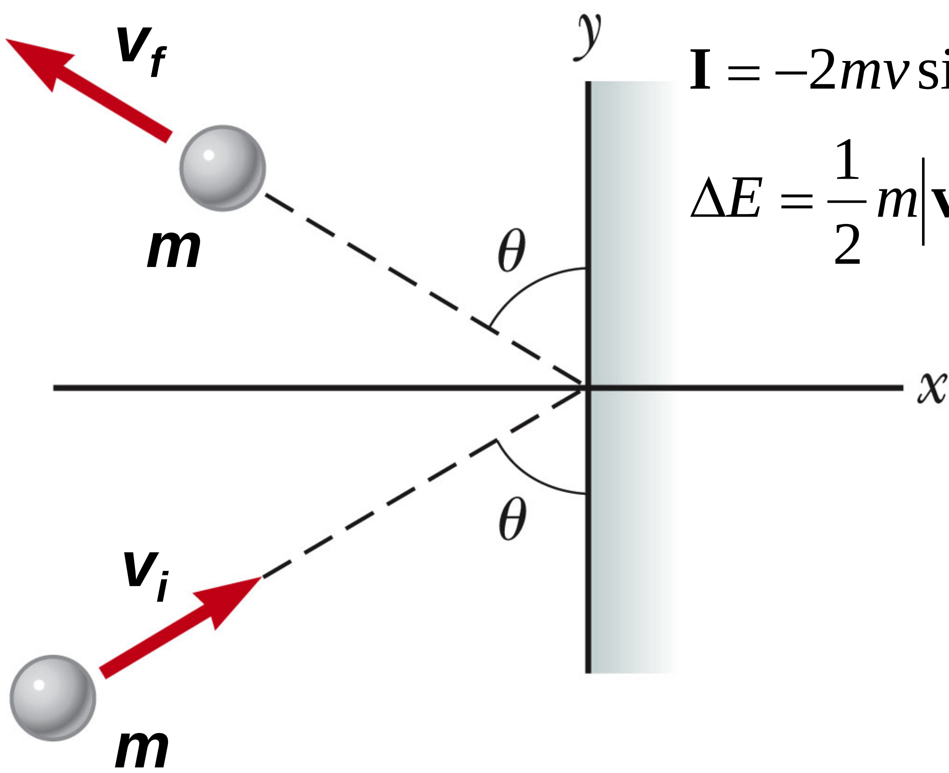
$$\Delta E = -4.8\times10^5\,J$$

iclicker exercise:

Can this analysis be used to analyze a real collision?

A. **Of course! The laws of physics must be obeyed.**
B. **Of course NOT! In physics class we only deal with idealized situations which never happen.**

Another example of 2-dimensional elastic collision:



In this case, the collision is elastic -- $|\mathbf{v}_i| = |\mathbf{v}_f| \equiv v$

$$\mathbf{I} = -2mv\sin\theta\hat{\mathbf{i}}$$

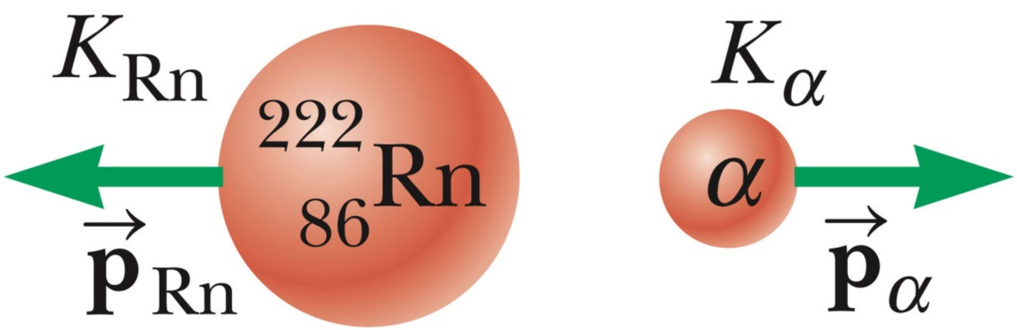
$$\Delta E = \frac{1}{2}m|\mathbf{v}_f|^2 - \frac{1}{2}m|\mathbf{v}_i|^2 = 0$$

Energy analysis of a simple nuclear reaction :

$$^{226}_{88}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + ^{4}_{2}\text{He}$$

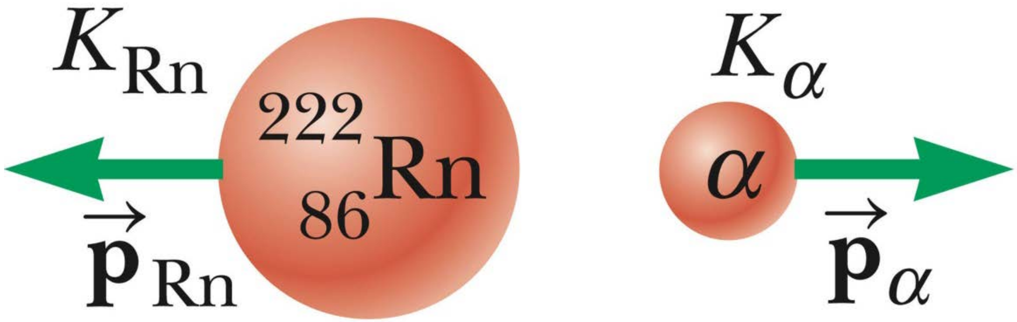
Before decay

Q=4.87 MeV



After decay

Energy analysis of a simple reaction : $^{226}_{88}\mathrm{Ra} \rightarrow ^{222}_{86}\mathrm{Rn} + ^{4}_{2}\mathrm{He}$



Q=4.87 MeV

$$\mathbf{p}_{Rn} = -\mathbf{p}_{He} \equiv \mathbf{p}$$

$$Q = \frac{p_{Rn}^{\;2}}{2m_{Rn}} + \frac{p_{He}^{\;2}}{2m_{He}} \approx \frac{p^2}{2m_u}\left(\frac{1}{222} + \frac{1}{4}\right)$$

$$E_{He} = \frac{p_{He}^{\;2}}{2m_{He}} = \frac{1/4}{1/222 + 1/4} Q = 0.98 \cdot Q = 4.8\mathrm{MeV}$$

The notion of the center of mass and the physics of composite systems

Newton's second law:

$$\sum_i \mathbf{F}_i = \sum_i m_i \mathbf{a}_i = \sum_i m_i \frac{d\mathbf{v}_i}{dt} = \sum_i \frac{d\left(m_i \mathbf{v}_i\right)}{dt} = \frac{d}{dt}\left(\sum_i \mathbf{p}_i\right)$$

or

$$\sum_i \mathbf{F}_i = \sum_i m_i \mathbf{a}_i = \sum_i m_i \frac{d^2\mathbf{r}_i}{dt^2} = \sum_i \frac{d^2\left(m_i \mathbf{r}_i\right)}{dt^2}$$

Define: $\mathbf{r}_{CM} \equiv \dfrac{\sum_i \left(m_i \mathbf{r}_i\right)}{M}$ $\qquad M \equiv \sum_i \left(m_i\right)$

$$\sum_i \mathbf{F}_i = \mathbf{F}_{total} = M \frac{d^2\mathbf{r}_{CM}}{dt^2}$$

General relation for center of mass:

$$\text{Define:} \quad \mathbf{r}_{CM} \equiv \frac{\sum_i (m_i \mathbf{r}_i)}{M} \qquad M \equiv \sum_i (m_i)$$

$$\sum_i \mathbf{F}_i = \mathbf{F}_{total} = M \frac{d^2 \mathbf{r}_{CM}}{dt^2}$$

$$\text{If } \mathbf{F}_{\text{total}} \equiv \sum_i \mathbf{F}_i = 0, \text{ then:}$$

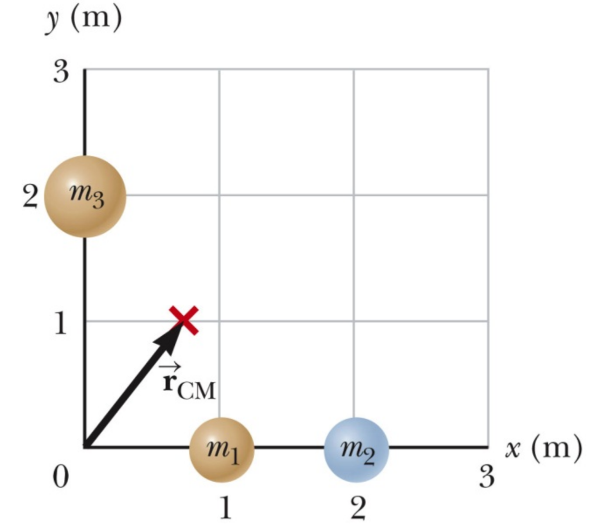
$$\frac{d}{dt}\left(\sum_i \mathbf{p}_i\right) = 0$$

$$\Rightarrow \sum_i \mathbf{p}_i = \text{(constant)}$$

$$\Rightarrow \sum_i \mathbf{p}_{i\,\text{initial}} = \sum_i \mathbf{p}_{i\,\text{final}} = M \frac{d\mathbf{r}_{CM}}{dt}$$

Finding the center of mass

$$\mathbf{r}_{CM} \equiv \frac{\sum_i (m_i \mathbf{r}_i)}{M} \qquad M \equiv \sum_i (m_i)$$



In this example: $m_1 = m_2 = 1kg;\ m_3 = 2kg$

$$\mathbf{r}_{CM} = \frac{m_1 x_1 \hat{\mathbf{i}} + m_2 x_2 \hat{\mathbf{i}} + m_3 y_3 \hat{\mathbf{j}}}{m_1 + m_2 + m_3}$$

$$\mathbf{r}_{CM} = \frac{(1)(1m)\hat{\mathbf{i}} + (1)(2m)\hat{\mathbf{i}} + (2)(2m)\hat{\mathbf{j}}}{4}$$

$$= 0.75m\hat{\mathbf{i}} + 1.00m\hat{\mathbf{j}}$$

Finding the center of mass

For a solid object composed of constant density material, the center of mass is located at the center of the object.