PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 16:

Review of Chapters 5-9

1. **Circular motion**
2. **Work & kinetic energy**
3. **Impulse and momentum**

5	09/07/2012	Motion in 2d	4.1-4.3	4.3,4.50	09/10/2012
6	09/10/2012	Circular motion	4.4-4.6	4.29,4.30	09/12/2012
7	09/12/2012	Newton's laws	5.1-5.6	5.1,5.13	09/14/2012
8	09/14/2012	Newton's laws applied	5.7-5.8	5.20,5.30,5.48	09/17/2012
	09/17/2012	Review	1-5		
	09/19/2012	Exam	1-5		
9	09/21/2012	More applications of Newton's laws	6.1-6.4	6.3,6.14	09/24/2012
10	09/24/2012	Work	7.1-7.4	7.1,7.15	09/26/2012
11	09/26/2012	Kinetic energy	7.5-7.9	7.31,7.41,7.49	09/28/2012
12	09/28/2012	Conservation of energy	8.1-8.5	8.6,8.22,8.35	10/01/2012
13	10/01/2012	Momentum and collisions	9.1-9.4	9.15,9.18	10/03/2012
14	10/03/2012	Momentum and collisions	9.5-9.9	9.29,9.37	10/05/2012
	10/05/2012	Review	6-9		
	10/08/2012	Exam	6-9		
15	10/10/2012	Rotational motion	10.1-10.5		10/12/2012

Format of Wednesday's exam

What to bring:

1. **Clear, calm head**
2. **Equation sheet (turn in with exam)**
3. **Scientific calculator**
4. **Pencil or pen**

(Note: labtops, cellphones, and other electronic equipment must be off or in sleep mode.)

Timing:

May begin as early as 8 AM; must end ≤ 9:50 AM

Probable exam format

- **4-5 problems similar to homework and class examples; focus on Chapters 6-9 of your text.**
- **Full credit awarded on basis of analysis steps as well as final answer**

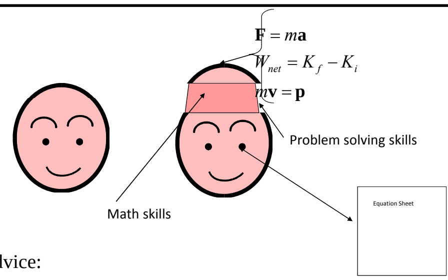
Examples of what to include on equation sheet

Given information on exam	Suitable for equation sheet
Universal constants (such as g=9.8m/s^2)	Trigonometric relations and definition of dot product
Particular constants (such as μ_s, μ_k)	Simple derivative and integral relationships
Unit conversion factors if needed	Definition of work, potential energy, kinetic energy
	Work-kinetic energy theorem
	Relationship between force, potential energy, and work for conservative systems
	Relationship of impulse and momentum; conservation of momentum
	Elastic and inelastic collisions
	Center of mass

iclicker exercise:
Which of the following quantities are vectors:
A. Work
B. Kinetic energy
C. Impulse
D. Time
E. None of these

iclicker exercise:
Which of the following quantities are scalars:
A. Momentum
B. Center of mass
C. Force
D. Potential energy
E. None of these

Some concepts introduced in Chapters 6-9 that were not emphasized in class:
1. Equations of motion in the presence of air or fluid friction
2. So called “fictitious” forces due to accelerating reference frames
3. Rocket propulsion



Advice:

1. Keep basic concepts and equations at the top of your head.
2. Practice problem solving and math skills
3. Develop an equation sheet that you can consult.

Problem solving steps

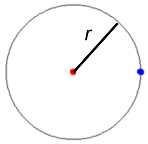
1. Visualize problem – labeling variables
2. Determine which basic physical principle(s) apply
3. Write down the appropriate equations using the variables defined in step 1.
4. Check whether you have the correct amount of information to solve the problem (same number of knowns and unknowns).
5. Solve the equations.
6. Check whether your answer makes sense (units, order of magnitude, etc.).

Review of some concepts:

Newton's second law :

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt}$$

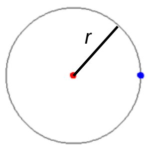
Newton's second law for the case of uniform circular motion



$$\mathbf{F} = m\mathbf{a}$$

$$\mathbf{a}_c = -\frac{v^2}{r}\hat{\mathbf{r}}$$

Example: Suppose a race car driver maintains of speed of v=40m/s around a horizontal level circular track of radius r=100m. Assuming that static friction keeps the car on the circular path, what must be the minimum coefficient of static friction for the car-road surface?



$$\mathbf{F} = m\mathbf{a}$$

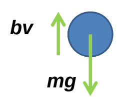
$$\mathbf{a}_c = -\frac{v^2}{r}\hat{\mathbf{r}}$$

$$f = \frac{mv^2}{r}; \qquad f \le \mu_s mg$$

$$\mu_s mg \ge \frac{mv^2}{r}$$

$$\mu_s \ge \frac{v^2}{gr} = \frac{(40)^2}{(9.8)(100)} = 1.63$$

Models of air friction forces



For small velocities : $F_{\text{air}} = -bv$

For larger velocities : $F_{air} = -Dv^2$

Denoting up direction as + and assuming $v < 0$:

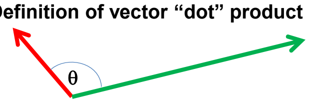
$$-bv - mg = ma$$

Solution to differential equation :

$$-bv - mg = m\frac{dv}{dt}$$

$$v(t) = -\frac{mg}{b}\left(1 - e^{-bt/m}\right)$$

Definition of vector "dot" product



$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$$

Example: $A = 5, \quad B = 15, \quad \theta = 120^\circ$

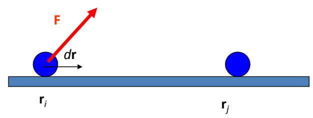
$$\mathbf{A}\cdot\mathbf{B} = (5)(15)\cos 120^\circ = -37.5 \quad \text{(scalar)}$$

Component form :

$$\left(5\hat{\mathbf{i}} + 6\hat{\mathbf{j}}\right)\cdot\left(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\right) = (5)(3) + (6)(2) = 27$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = 1 \qquad \hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = 0$$

Definition of work:



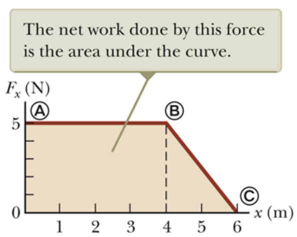
$$W_{i\to f} = \int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F}\cdot d\mathbf{r}$$

Units of work:

Work = (Newtons)(meters) ≡ (Joules)

1 J = 0.239 cal

Example:



$$W_{i\to f} = \int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F}\cdot d\mathbf{r} = \int_{x_i}^{x_f} F_x dx = (5N)(4m) + \tfrac{1}{2}(5N)(2m) = 25J$$

Introduction of the notion of Kinetic energy

Some more details:

Consider Newton's second law:

$\mathbf{F}_{total} = m\,\mathbf{a}$ ➔ $\mathbf{F}_{total}\cdot d\mathbf{r} = m\,\mathbf{a}\cdot d\mathbf{r}$

$$\int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F}_{total}\cdot d\mathbf{r} = \int_{\mathbf{r}_i}^{\mathbf{r}_f} m\mathbf{a}\cdot d\mathbf{r} = \int_{\mathbf{r}_i}^{\mathbf{r}_f} m\frac{d\mathbf{v}}{dt}\cdot d\mathbf{r} = \int_{t_i}^{t_f} m\frac{d\mathbf{v}}{dt}\cdot\frac{d\mathbf{r}}{dt}dt = \int_{t_i}^{t_f} m\frac{d\mathbf{v}}{dt}\cdot\mathbf{v}dt$$

$$\int_{t_i}^{t_f} m\frac{d\mathbf{v}}{dt}\cdot\mathbf{v}dt = \int_{\mathbf{v}_i}^{\mathbf{v}_f} m d\mathbf{v}\cdot\mathbf{v} = \int_i^f d\left(\frac{1}{2}m\mathbf{v}\cdot\mathbf{v}\right) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\Rightarrow W_{total} = \underbrace{\tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2}_{\text{Kinetic energy (joules)}} = K_f - K_i$$

Special case of "conservative" forces
➔conservative ⟵➔ non-dissipative

For non-dissipative forces $\mathbf{F}$, it is possible to write

$$W_{i\to f} \equiv \int_i^f \mathbf{F}\cdot d\mathbf{r} = -\left(U\left(\mathbf{r}_f\right) - U\left(\mathbf{r}_i\right)\right)$$

Example of gravity near surface of Earth:

$$W_{i\to f} \equiv \int_i^f \mathbf{F}\cdot d\mathbf{r} = -mg\left(y_f - y_i\right) = -\left(mgy_f - mgy_i\right)$$

$$\Rightarrow U(\mathbf{r}_f) = mgy_f \qquad \text{and} \qquad U(\mathbf{r}_i) = mgy_i$$

Special case of "conservative" forces
➔conservative ⟵➔ non-dissipative

For non-dissipative forces $\mathbf{F}$, it is possible to write

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Example of motion of mass on elastic spring:

$$W_{i\to f} \equiv \int_i^f \mathbf{F}\cdot d\mathbf{r} = -\int_i^f kx\,dx = = -\left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right)$$

$$\Rightarrow U(\mathbf{r}_f) = \tfrac{1}{2}kx_f^2 \qquad \text{and} \qquad U(\mathbf{r}_i) = \tfrac{1}{2}kx_i^2$$

General work-kinetic energy theorem:

$$W_{i\to f} = \int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F}_{total}\cdot d\mathbf{r} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

For conservative forces:

$$W_{i\to f} = \int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F}_{total}\cdot d\mathbf{r} = -\left(U\left(\mathbf{r}_f\right) - U\left(\mathbf{r}_i\right)\right)$$

$$\Rightarrow \frac{1}{2}mv_i^2 + U\left(\mathbf{r}_i\right) = \frac{1}{2}mv_f^2 + U\left(\mathbf{r}_f\right) = E$$

Example:

Suppose a mass m is attached to a spring with spring constant k and compressed by a distance x_i. Assuming there are no other forces acting, what is the velocity of the mass when the spring is released $(x_f = 0)$?

$$m = 0.5kg; k = 70N/m; x_i = 0.1m$$

$$\frac{1}{2}mv_i^2 + U(\mathbf{r}_i) = \frac{1}{2}mv_f^2 + U(\mathbf{r}_f) = E$$

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 \quad \Rightarrow v_f = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{70}{0.5}}(0.1) = 1.2m/s$$

Summary of physics "laws"

Newton's second law:

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt}$$

Work - kinetic energy theorem:

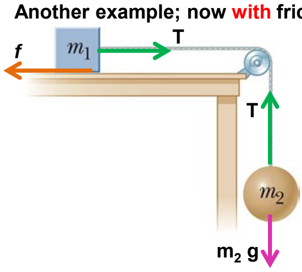
$$W_{total}^{i \to f} = \int_i^f \mathbf{F} \cdot d\mathbf{r} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

For conservative forces: $W_{conservative}^{i \to f} = -\left(U(\mathbf{r}_f) - U(\mathbf{r}_i)\right)$

$$W_{total}^{i \to f} = W_{conservative}^{i \to f} + W_{non-conservative}^{i \to f}$$

$$\frac{1}{2}mv_i^2 + U(\mathbf{r}_i) + W_{non-conservative}^{i \to f} = \frac{1}{2}mv_f^2 + U(\mathbf{r}_f)$$

Another example; now with friction



Mass m_1 (=0.2kg) slides horizontally on a table with kinetic friction μ_k=0.5 and is initially at rest. What is its velocity when it moves a distance Δx=0.1m (and m_2 (=0.3kg) falls Δy=0.1m)?

$$W = (T - f)\Delta x = \frac{1}{2}m_1 v_f^2 - \underbrace{\cancel{\frac{1}{2}m_1 v_i^2}}_{0}$$

$$T - f = m_1 a$$
$$T - m_2 g = -m_2 a$$
$$\Rightarrow T = \frac{m_1 m_2}{m_1 + m_2} g + \frac{m_2}{m_1 + m_2} f$$
$$W = (T - f)\Delta x = \frac{m_1 m_2}{m_1 + m_2} g\Delta x - \frac{m_1}{m_1 + m_2} f\Delta x$$

$$W = (T - f)\Delta x = \frac{1}{2} m_1 v_f^2$$
$$v_f = \sqrt{\frac{2m_2 g\Delta x - 2f\Delta x}{m_1 + m_2}} \qquad f = \mu_k m_1 g$$
$$v_f = \sqrt{2g\Delta x}\sqrt{\frac{m_2 - \mu_k m}{m_1 + m_2}} = \sqrt{2(9.8)(0.1)}\sqrt{\frac{0.3 - (0.5)(0.2)}{0.3 + 0.2}} = 0.885 m/s$$

Relationship between Newton's second law and linear momentum for a single particle:

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt} = \frac{d\mathbf{p}}{dt}$$

Using a little calculus:

$$\mathbf{F}dt = d\mathbf{p}$$
$$\int_i^f \mathbf{F}dt = \int_i^f d\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i$$

Define "impulse": $\mathbf{I} = \int_i^f \mathbf{F}dt$

$$\mathbf{I} = \int_i^f \mathbf{F}dt = \mathbf{p}_f - \mathbf{p}_i$$

Physics of composite systems

Newton's second law:

$$\sum_i \mathbf{F}_i = \sum_i m_i \mathbf{a}_i = \sum_i m_i \frac{d\mathbf{v}_i}{dt} = \sum_i \frac{dm_i \mathbf{v}_i}{dt} = \frac{d}{dt}\left(\sum_i \mathbf{p}_i\right)$$

Note that if $\sum_i \mathbf{F}_i = 0$, then:

$$\frac{d}{dt}\left(\sum_i \mathbf{p}_i\right) = 0$$
$$\Rightarrow \sum_i \mathbf{p}_i = \text{ (constant)}$$
$$\Rightarrow \sum_i \mathbf{p}_{i\,\text{initial}} = \sum_i \mathbf{p}_{i\,\text{final}}$$

General relation for center of mass:

$$\text{Define: } \mathbf{r}_{CM} \equiv \frac{\sum_i (m_i \mathbf{r}_i)}{M} \qquad M \equiv \sum_i (m_i)$$

$$\sum_i \mathbf{F}_i = \mathbf{F}_{total} = M \frac{d^2 \mathbf{r}_{CM}}{dt^2}$$

$$\text{If } \mathbf{F}_{\text{total}} \equiv \sum_i \mathbf{F}_i = 0, \text{ then:}$$

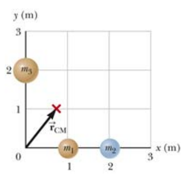
$$\frac{d}{dt}\left(\sum_i \mathbf{p}_i\right) = 0$$

$$\Rightarrow \sum_i \mathbf{p}_i = \text{(constant)}$$

$$\Rightarrow \sum_i \mathbf{p}_{i\,\text{initial}} = \sum_i \mathbf{p}_{i\,\text{final}} = M \frac{d\mathbf{r}_{CM}}{dt}$$

Finding the center of mass

$$\mathbf{r}_{CM} \equiv \frac{\sum_i (m_i \mathbf{r}_i)}{M} \qquad M \equiv \sum_i (m_i)$$



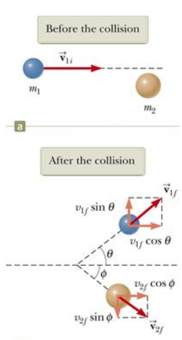
In this example: $m_1 = m_2 = 1kg;\ m_3 = 2kg$

$$\mathbf{r}_{CM} = \frac{m_1 x_1 \hat{\mathbf{i}} + m_2 x_2 \hat{\mathbf{i}} + m_3 y_3 \hat{\mathbf{j}}}{m_1 + m_2 + m_3}$$

$$\mathbf{r}_{CM} = \frac{(1)(1m)\hat{\mathbf{i}} + (1)(2m)\hat{\mathbf{i}} + (2)(2m)\hat{\mathbf{j}}}{4}$$

$$= 0.75m\hat{\mathbf{i}} + 1.00m\hat{\mathbf{j}}$$

Examples of two-dimensional collision; balls moving on a frictionless surface



$$\sum_i \mathbf{p}_{i\,\text{initial}} = \sum_i \mathbf{p}_{i\,\text{final}}$$

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$$

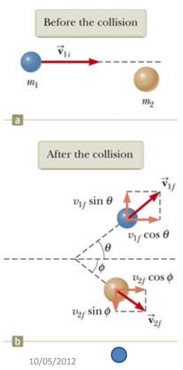
$$0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$$

Knowns: m_1, m_2, v_{1i}

Unknowns: $v_{1f}, v_{2f}, \theta, \phi$

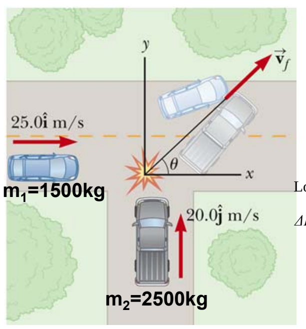
Need 2 more equations --

Examples of two-dimensional collision; balls moving on a frictionless surface



$$\text{Suppose: } m_1 = m_2 = 0.06kg, \quad v_{1i} = 2m/s,$$
$$v_{2f} = 1m/s, \quad \phi = 20^o$$
$$m_1 v_{1i} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\phi$$
$$0 = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\phi$$
$$v_{1f}\sin\theta = v_{2f}\sin\phi$$
$$= (1m/s)\sin 20^o = 0.342m/s$$
$$v_{1f}\cos\theta = v_{1i} - v_{2f}\cos\phi$$
$$= (2m/s) - (1m/s)(\cos 20^o) = 1.060m/s$$
$$\tan\theta = \frac{0.342}{1.060} \quad \Rightarrow \theta = 17.88^o$$
$$v_{1f} = \frac{0.342m/s}{\sin 17.88^o} = \frac{1.060m/s}{\cos 17.88^o} = 1.11m/s$$

Example: two-dimensional totally inelastic collision



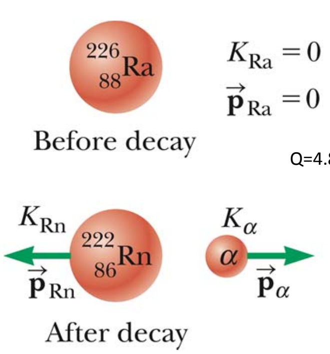
$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$$
$$\mathbf{v}_f = \frac{m_1}{m_1+m_2}\mathbf{v}_{1i} + \frac{m_2}{m_1+m_2}\mathbf{v}_{2i}$$
$$\mathbf{v}_f = \frac{1500}{4000}25m/s\,\hat{\mathbf{i}} + \frac{2500}{4000}20m/s\,\hat{\mathbf{j}}$$
$$= 9.375m/s\,\hat{\mathbf{i}} + 12.5m/s\,\hat{\mathbf{j}}$$

Loss of kinetic energy in this case:

$$\Delta E = \frac{1}{2}(m_1+m_2)|\mathbf{v}_f|^2 - \left(\frac{1}{2}m_1|\mathbf{v}_{1i}|^2 + \frac{1}{2}m_2|\mathbf{v}_{2i}|^2\right)$$
$$= \frac{1}{2}(4000)\left((9.375)^2 + (12.5)^2\right)$$
$$-\left(\frac{1}{2}1500(25)^2 + \frac{1}{2}2500(20)^2\right)$$
$$\Delta E = -4.8\times 10^5 J$$

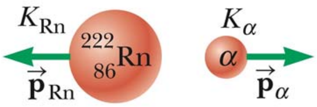
Energy analysis of a simple nuclear reaction :

$${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$



Q=4.87 MeV

Energy analysis of a simple reaction : $^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}$



Q=4.87 MeV

$$\mathbf{p}_{Rn} = -\mathbf{p}_{He} \equiv \mathbf{p}$$

$$Q = \frac{p_{Rn}{}^2}{2m_{Rn}} + \frac{p_{He}{}^2}{2m_{He}} \approx \frac{p^2}{2m_u}\left(\frac{1}{222} + \frac{1}{4}\right)$$

$$E_{He} = \frac{p_{He}{}^2}{2m_{He}} = \frac{1/4}{1/222 + 1/4} Q = 0.98 \cdot Q = 4.8\mathrm{MeV}$$