PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 17:

Chapter 10 – rotational motion

1. Angular variables
2. Rotational energy
3. Moment of inertia

13	10/01/2012	Momentum and collisions	9.1-9.4	9.15,9.18	10/03/2012
14	10/03/2012	Momentum and collisions	9.5-9.9	9.29,9.37	10/05/2012
	10/05/2012	Review	6-9		
	10/08/2012	Exam	6-9		
15	10/10/2012	Rotational motion	10.1-10.5	10.6, 10.13, 10.25	10/12/2012
16	10/12/2012	Torque	10.6-10.9	10.37, 10.55	10/15/2012
17	10/15/2012	Angular momentum	11.1-11.5	11.11, 11.34	10/17/2012
18	10/17/2012	Equilibrium	12.1-12.4		10/22/2012
	10/19/2012	*Fall Break*			
19	10/22/2012	Simple harmonic motion	15.1-15.3		10/24/2012
20	10/24/2012	Resonance	15.4-15.7		10/26/2012
21	10/26/2012	Gravitational force	13.1-13.3		10/29/2012
22	10/29/2012	Kepler's laws and satellite motion	13.4-13.6		10/31/2012
	10/31/2012	Review	10-13,15		
	11/02/2012	Exam	10-13,15		
23	11/05/2012	Fluid mechanics	14.1-14.4		11/07/2012

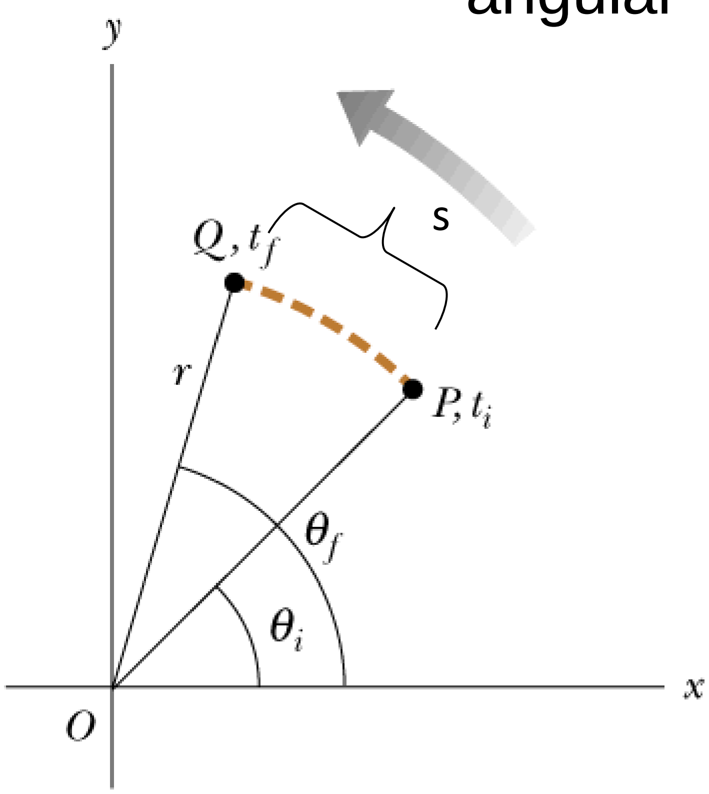
Angular motion

angular "displacement" → $\theta(t)$

angular "velocity" → $\omega(t) = \frac{d\theta}{dt}$

angular "acceleration" → $\alpha(t) = \frac{d\omega}{dt}$

Serway, Physics for Scientists and Engineers, 5/e
Figure 10.2

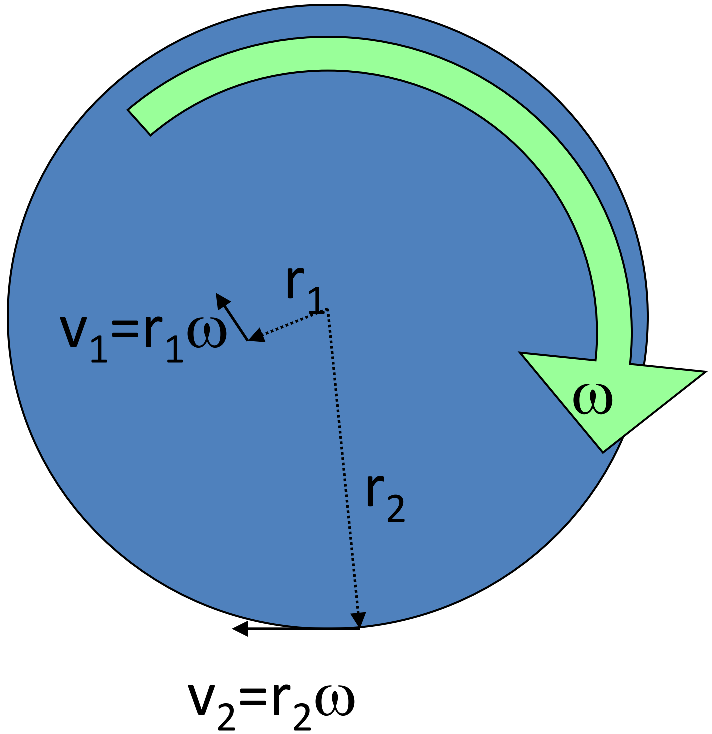


"natural" unit == 1 radian

Relation to linear variables: $s_\theta = r\,(\theta_f - \theta_i)$

$v_\theta = r\,\omega$

$a_\theta = r\,\alpha$

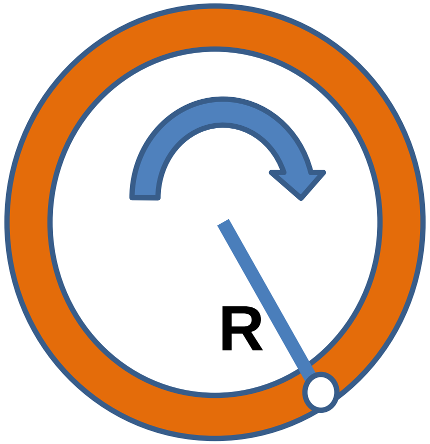


Special case of constant angular acceleration: $\alpha = \alpha_0$:

$$\omega(t) = \omega_i + \alpha_0\, t$$

$$\theta(t) = \theta_i + \omega_i\, t + \tfrac{1}{2}\, \alpha_0\, t^2$$

$$(\omega(t))^2 = \omega_i^2 + 2\, \alpha_0\, (\theta(t) - \theta_i)$$



A wheel is initially rotating at a rate of *f*=30 rev/sec.

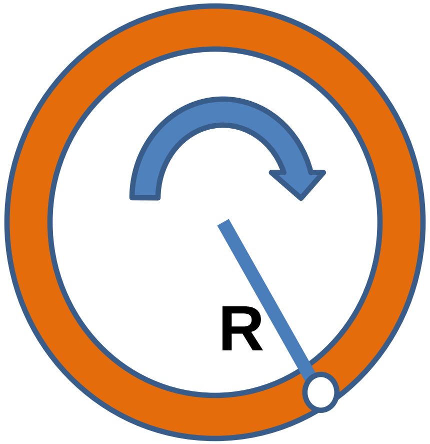
What is the angular velocity?

$$\omega = 2\pi f = 2\pi(30) \quad \text{rad/s}$$

$$= 188.495 \ \text{rad/s}$$

What is the speed of a dot on the rim of the wheel at a radius $R = 0.5\text{m}$?

$$v = \omega R = (188.495 \ \text{rad/s})(0.5\text{m}) = 94.247 m/s$$



A wheel is initially rotating at a rate of *f*=30 rev/sec. Because of a constant angular deceleration, the wheel comes to rest in 3 seconds.

What is the angular deceleration?

$$\alpha = \frac{0 - 2\pi f}{3s} = \frac{-2\pi(30) \quad \text{rad/s}}{3s}$$

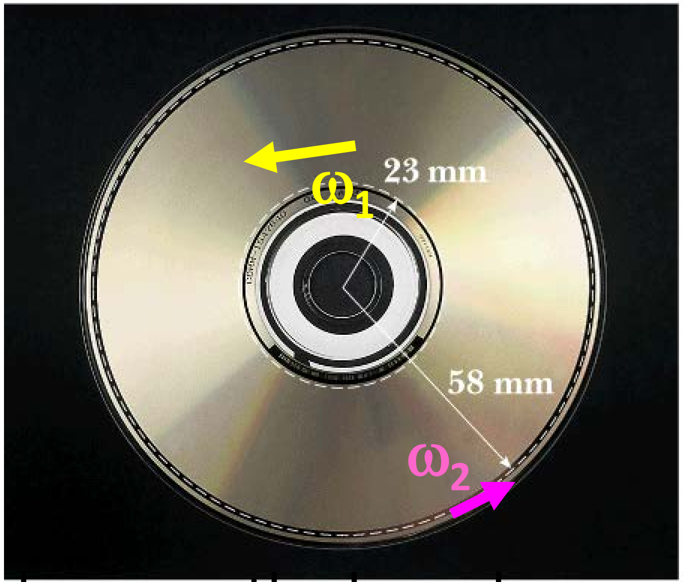
$$= -62.83 \ \text{rad/s}^2$$

What is the deceleration of a dot on the rim of the wheel at a radius $R = 0.5\text{m}$?

$$a = \alpha R = (-62.83 \, \text{rad/s}^2)(0.5\text{m}) = -31.42 m/s^2$$

Serway, Physics for Scientists and Engineers, 5/e
Figure 10.6

Example: Compact disc motion



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In a compact disk, each spot on the disk passes the laser-lens system at a constant linear speed of $v_\theta = 1.3$ m/s.

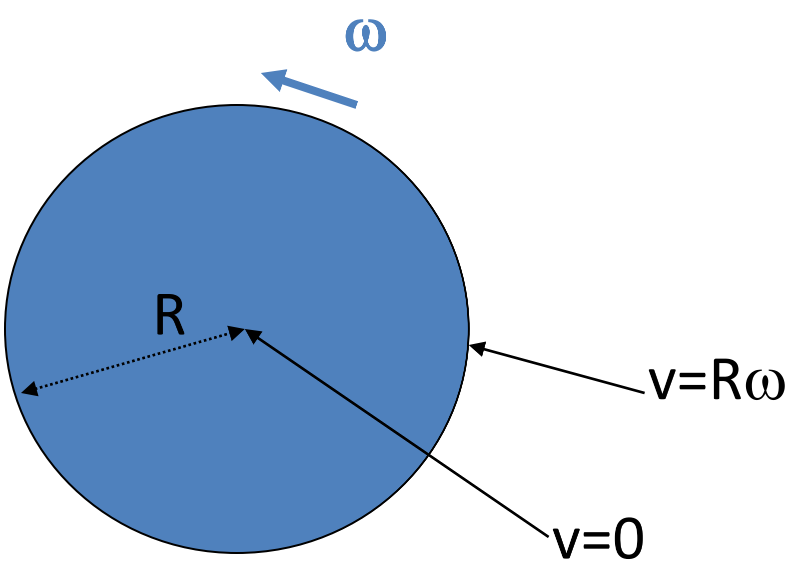
$\omega_1=v_\theta/r_1=56.5$ rad/s

$\omega_2=v_\theta/r_2=22.4$ rad/s

What is the average angular acceleration of the CD over the time interval $\Delta t=4473$ s as the spot moves from the inner to outer radii?

$\alpha = (\omega_2-\omega_1)/\Delta t =-0.0076 \text{ rad/s}^2$

Object rotating with constant angular velocity (α = 0)



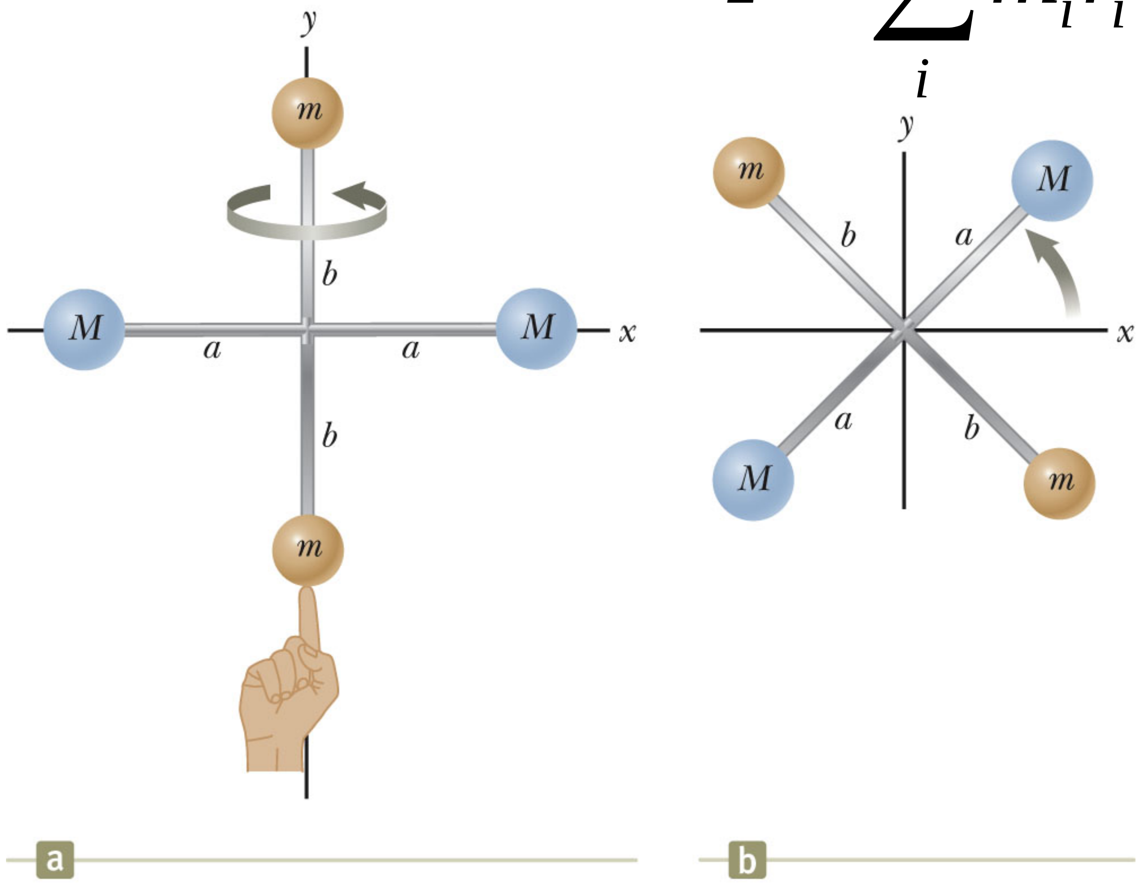
Kinetic energy associated with rotation:

$$K = \sum_i \tfrac{1}{2} m_i v_i^2 = \sum_i \tfrac{1}{2} m_i r_i^2 \omega^2 \equiv \tfrac{1}{2} I \omega^2;$$

$$\text{where}: I \equiv \sum_i m_i r_i^2$$ "moment of inertia"

Moment of inertia:

$$I \equiv \sum_i m_i r_i^2$$

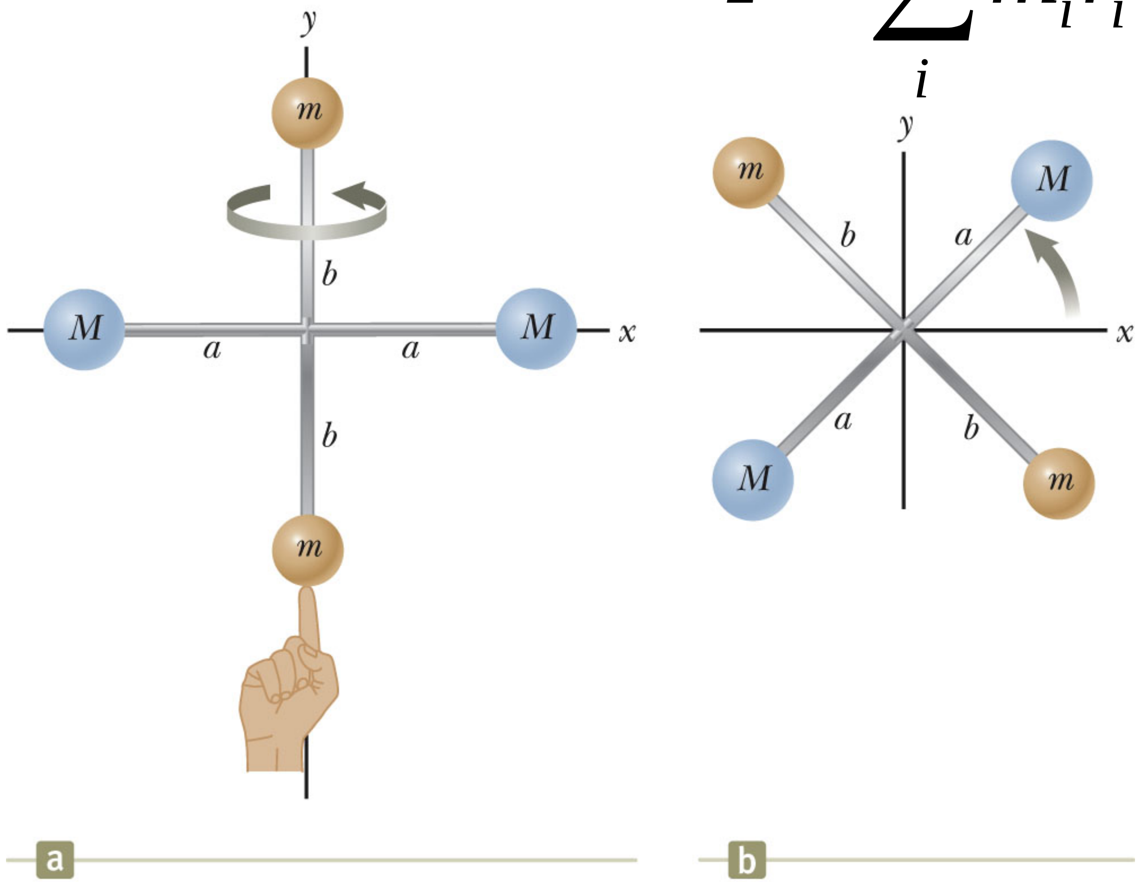


iclicker exercise: **Which case has the larger I?**

A. a **B. b**

Moment of inertia:

$$I \equiv \sum_i m_i r_i^2$$

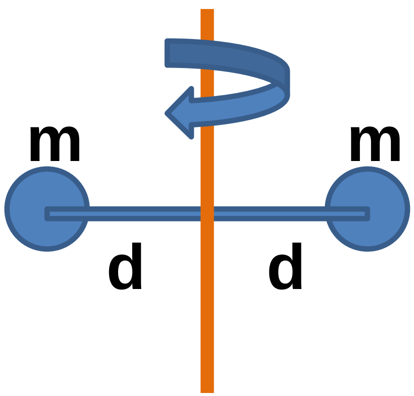


$$I = 2Ma^2$$

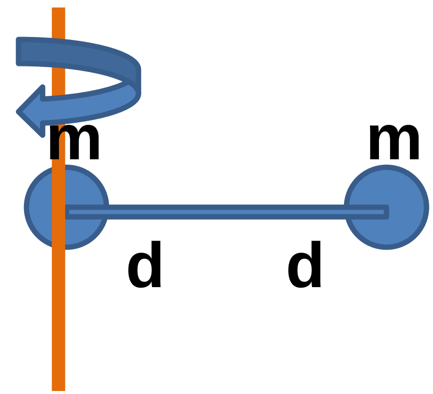
$$I = 2Ma^2 + 2mb^2$$

Note that the moment of inertia depends on both

a) **The position of the rotational axis**
b) **The direction of rotation**



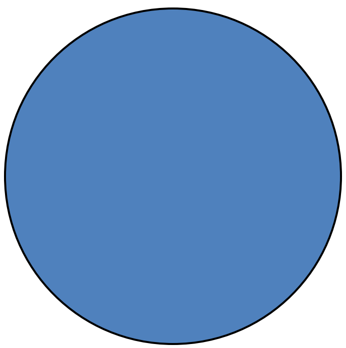
$I=2md^2$



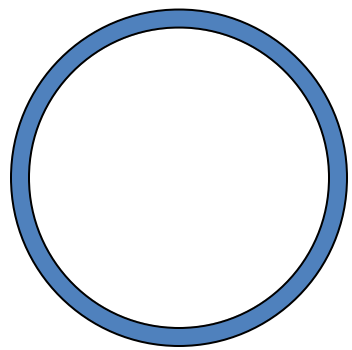
$I=m(2d)^2=4md^2$

iclicker question:

Suppose each of the following objects each has the same total mass M and outer radius R and each is rotating counter-clockwise at an constant angular velocity of ω=3 rad/s. Which object has the greater kinetic energy?

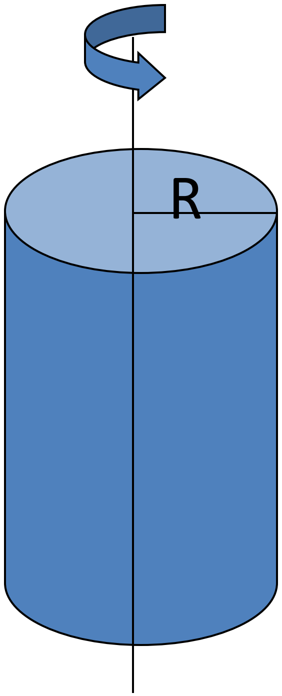


(a) (Solid disk)



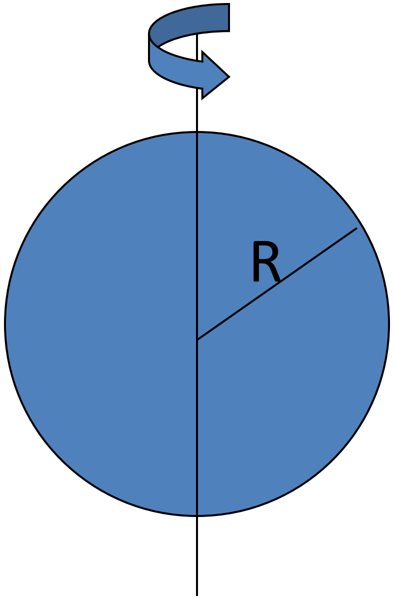
(b) (circular ring)

Various moments of inertia:



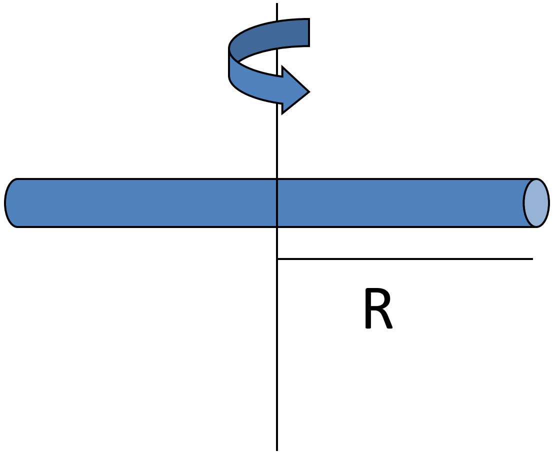
solid cylinder:

$I=1/2\ MR^2$



solid sphere:

$I=2/5\ MR^2$

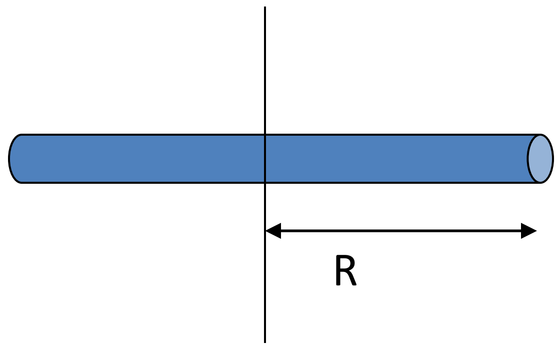


solid rod:

$I=1/3\ MR^2$

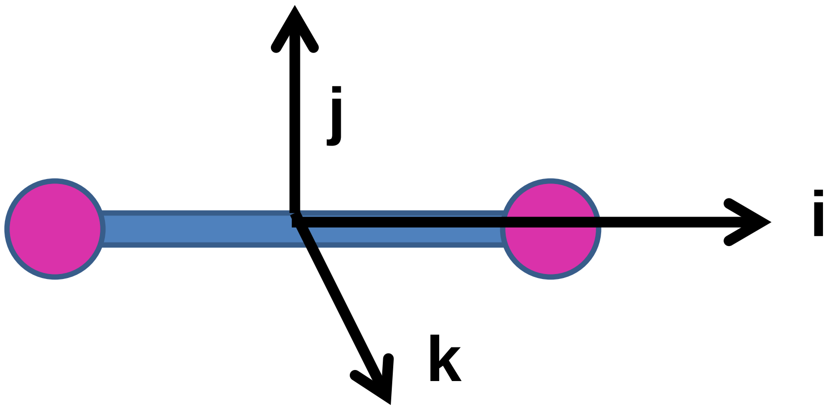
Calculation of moment of inertia:

Example -- moment of inertia of solid rod through an axis perpendicular rod and passing through center:



$$I = \sum_i m_i r_i^2 = \int_{-R}^{R} \left(\frac{M}{2R}\right) dr\, r^2 = \left(\frac{M}{2R}\right) \int_{-R}^{R} r^2 dr = \frac{1}{3} MR^2$$

Note that any solid object has 3 moments of inertia; some times two or more can be equal



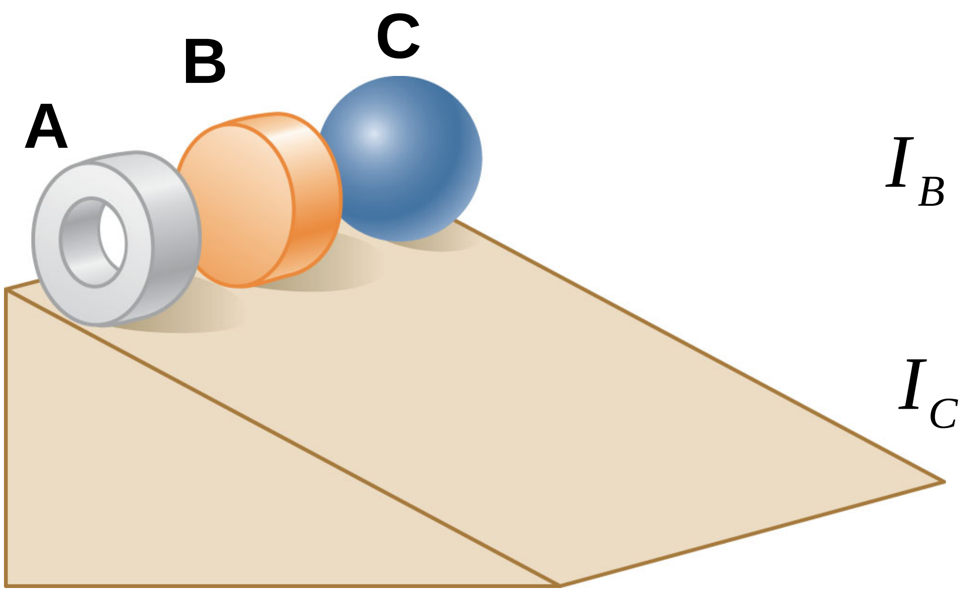
iclicker exercise:

Which moment of inertia is the smallest?

(A) i (B) j (C) k

iclicker exercise:

Three round balls, each having a mass M and radius R, start from rest at the top of the incline. After they are released, they roll without slipping down the incline. Which ball will reach the bottom first?



$$I_A = MR^2$$

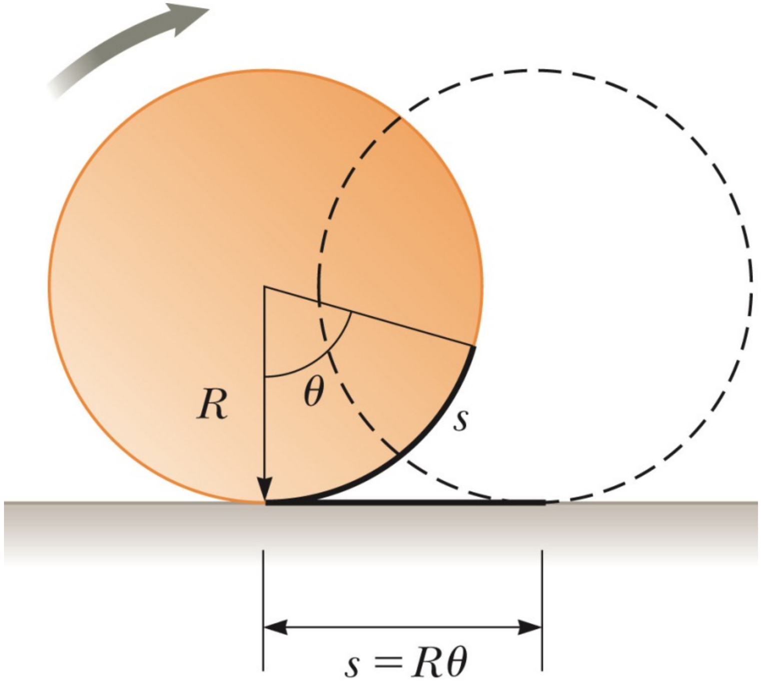
$$I_B = \frac{1}{2}MR^2 = 0.5MR^2$$

$$I_C = \frac{2}{5}MR^2 = 0.4MR^2$$

Total kinetic energy of rolling object :

$$K_{total} = K_{rolling} + K_{CM}$$

$$= \frac{1}{2} I\omega^2 + \frac{1}{2} M v_{CM}{}^2$$



Note that :

$$\omega = \frac{d\theta}{dt}$$

$$\frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega = v_{CM}$$

$$K_{total} = K_{rolling} + K_{CM}$$

$$= \frac{1}{2}\frac{I}{R^2}(R\omega)^2 + \frac{1}{2} M v_{CM}{}^2$$

$$= \frac{1}{2}\left(\frac{I}{R^2} + M\right) v_{CM}{}^2$$

iclicker exercise:

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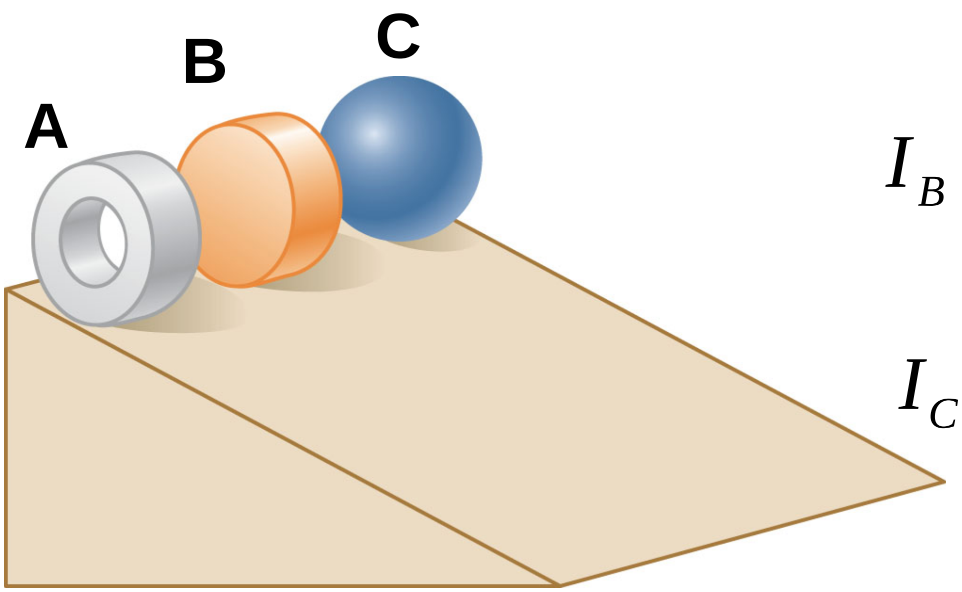

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