**PHY 113 A General Physics I**
**9-9:50 AM MWF Olin 101**

**Plan for Lecture 17:**

**Chapter 10 – rotational motion**

1. **Angular variables**
2. **Rotational energy**
3. **Moment of inertia**



| | | | | | |
|---|---|---|---|---|---|
| 13 | 10/01/2012 | Momentum and collisions | 9.1-9.4 | 9.15,9.18 | 10/03/2012 |
| 14 | 10/03/2012 | Momentum and collisions | 9.5-9.9 | 9.29,9.37 | 10/05/2012 |
| | 10/05/2012 | Review | 6-9 | | |
| | 10/08/2012 | Exam | 6-9 | | |
| 15 | 10/10/2012 | Rotational motion | 10.1-10.5 | 10.6, 10.13, 10.25 | 10/12/2012 |
| 16 | 10/12/2012 | Torque | 10.6-10.9 | 10.37, 10.55 | 10/15/2012 |
| 17 | 10/15/2012 | Angular momentum | 11.1-11.5 | 11.11, 11.34 | 10/17/2012 |
| 18 | 10/17/2012 | Equilibrium | 12.1-12.4 | | 10/22/2012 |
| | 10/19/2012 | *Fall Break* | | | |
| 19 | 10/22/2012 | Simple harmonic motion | 15.1-15.3 | | 10/24/2012 |
| 20 | 10/24/2012 | Resonance | 15.4-15.7 | | 10/26/2012 |
| 21 | 10/26/2012 | Gravitational force | 13.1-13.3 | | 10/29/2012 |
| 22 | 10/29/2012 | Kepler's laws and satellite motion | 13.4-13.6 | | 10/31/2012 |
| | 10/31/2012 | Review | 10-13,15 | | |
| | 11/02/2012 | Exam | 10-13,15 | | |
| 23 | 11/05/2012 | Fluid mechanics | 14.1-14.4 | | 11/07/2012 |



**Angular motion**

angular "displacement" → $\theta(t)$

angular "velocity" → $\omega(t) = \frac{d\theta}{dt}$

angular "acceleration" → $\alpha(t) = \frac{d\omega}{dt}$

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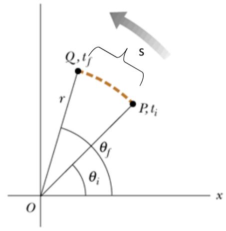
Figure 10.2

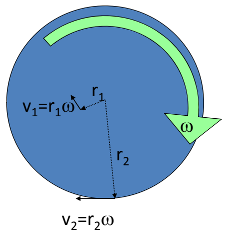



"natural" unit == 1 radian

Relation to linear variables: $s_\theta = r\,(\theta_f - \theta_i)$

$v_\theta = r\,\omega$

$a_\theta = r\,\alpha$



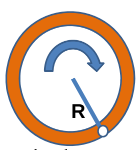

Special case of constant angular acceleration: $\alpha = \alpha_0$:

$\omega(t) = \omega_i + \alpha_0 t$

$\theta(t) = \theta_i + \omega_i t + \frac{1}{2} \alpha_0 t^2$

$(\omega(t))^2 = \omega_i^2 + 2 \alpha_0 (\theta(t) - \theta_i)$

10/10/2012 PHY 113 A Fall 2012 -- Lecture 17 4




**A wheel is initially rotating at a rate of *f*=30 rev/sec.**

What is the angular velocity?

$$\omega = 2\pi f = 2\pi(30) \text{ rad/s}$$
$$= 188.495 \text{ rad/s}$$

What is the speed of a dot on the rim of the wheel at a radius $R = 0.5$m?

$$v = \omega R = (188.495 \text{ rad/s})(0.5\text{m}) = 94.247 m/s$$

10/10/2012 PHY 113 A Fall 2012 -- Lecture 17 5


**A wheel is initially rotating at a rate of *f*=30 rev/sec. Because of a constant angular deceleration, the wheel comes to rest in 3 seconds.**

What is the angular deceleration?

$$\alpha = \frac{0 - 2\pi f}{3s} = \frac{-2\pi(30) \text{ rad/s}}{3s}$$
$$= -62.83 \text{ rad/s}^2$$

What is the deceleration of a dot on the rim of the wheel at a radius $R = 0.5$m?

$$a = \alpha R = (-62.83 \text{ rad/s}^2)(0.5\text{m}) = -31.42 m/s^2$$

10/10/2012 PHY 113 A Fall 2012 -- Lecture 17 6

Example: Compact disc motion

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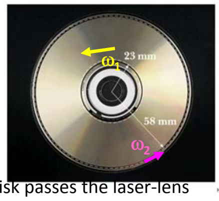



Harcourt, Inc.

In a compact disk, each spot on the disk passes the laser-lens system at a constant linear speed of $v_\theta$ = 1.3 m/s.

$\omega_1=v_\theta/r_1=56.5$ rad/s

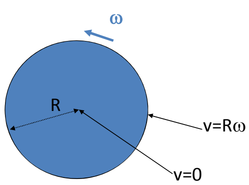
$\omega_2=v_\theta/r_2=22.4$ rad/s

What is the average angular acceleration of the CD over the time interval $\Delta t$=4473 s as the spot moves from the inner to outer radii?

$\alpha = (\omega_2-\omega_1)/\Delta t = -0.0076$ rad/s$^2$



Object rotating with constant angular velocity ($\alpha$ = 0)



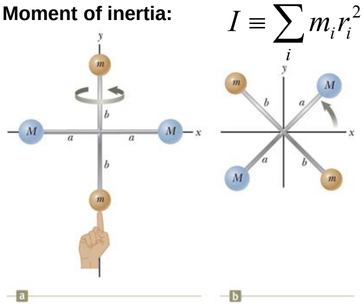

Kinetic energy associated with rotation:

$$K = \sum_i \tfrac{1}{2} m_i v_i^2 = \sum_i \tfrac{1}{2} m_i r_i^2 \omega^2 \equiv \tfrac{1}{2} I \omega^2;$$

$$\text{where}: I \equiv \sum_i m_i r_i^2$$ "moment of inertia"



**Moment of inertia:** $I \equiv \sum_i m_i r_i^2$

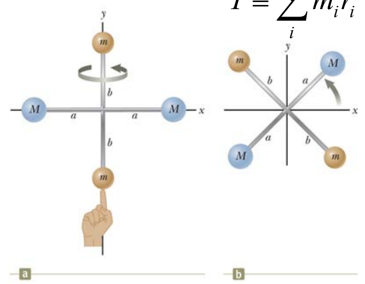



*iclicker exercise:* **Which case has the larger I?**

**A. a**

**B. b**

**Moment of inertia:** $I \equiv \sum_i m_i r_i^2$



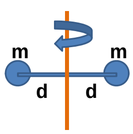

$I = 2Ma^2$ $\qquad$ $I = 2Ma^2 + 2mb^2$



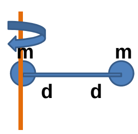
**Note that the moment of inertia depends on both**

**a) The position of the rotational axis**

**b) The direction of rotation**




**$I=2md^2$**

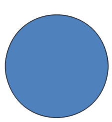



**$I=m(2d)^2=4md^2$**

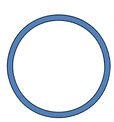


***iclicker question***:

Suppose each of the following objects each has the same total mass M and outer radius R and each is rotating counter-clockwise at an constant angular velocity of $\omega$=3 rad/s. Which object has the greater kinetic energy?

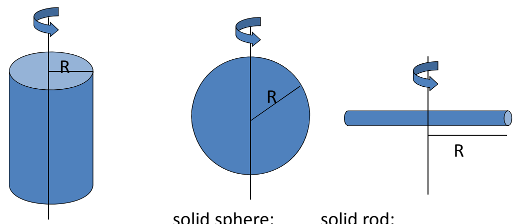


(a) (Solid disk)



(b) (circular ring)

Various moments of inertia:




solid cylinder: $I=1/2\ MR^2$

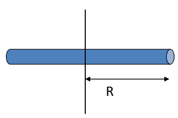
solid sphere: $I=2/5\ MR^2$

solid rod: $I=1/3\ MR^2$



Calculation of moment of inertia:

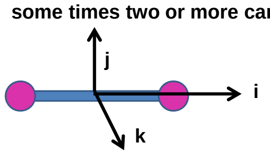
Example -- moment of inertia of solid rod through an axis perpendicular rod and passing through center:




$$I = \sum_i m_i r_i^2 = \int_{-R}^{R} \left(\frac{M}{2R}\right) dr\, r^2 = \left(\frac{M}{2R}\right) \int_{-R}^{R} r^2 dr = \frac{1}{3} MR^2$$



**Note that any solid object has 3 moments of inertia; some times two or more can be equal**



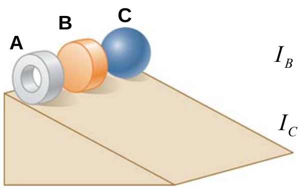

***iclicker exercise:***

**Which moment of inertia is the smallest?**

**(A) i (B) j (C) k**

**iclicker exercise:**

**Three round balls, each having a mass M and radius R, start from rest at the top of the incline. After they are released, they roll without slipping down the incline. Which ball will reach the bottom first?**




$$I_A = MR^2$$

$$I_B = \frac{1}{2}MR^2 = 0.5MR^2$$

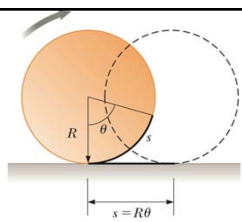
$$I_C = \frac{2}{5}MR^2 = 0.4MR^2$$



Total kinetic energy of rolling object:

$$K_{total} = K_{rolling} + K_{CM}$$

$$= \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{CM}{}^2$$




Note that:

$$\omega = \frac{d\theta}{dt}$$

$$\frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega = v_{CM}$$

$$K_{total} = K_{rolling} + K_{CM}$$

$$= \frac{1}{2}\frac{I}{R^2}(R\omega)^2 + \frac{1}{2}Mv_{CM}{}^2$$

$$= \frac{1}{2}\left(\frac{I}{R^2} + M\right)v_{CM}{}^2$$



**iclicker exercise:**

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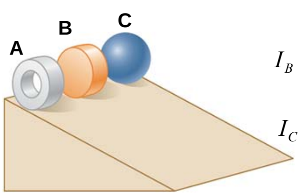


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