# PHY 113 A General Physics I
# 9-9:50 AM MWF Olin 101

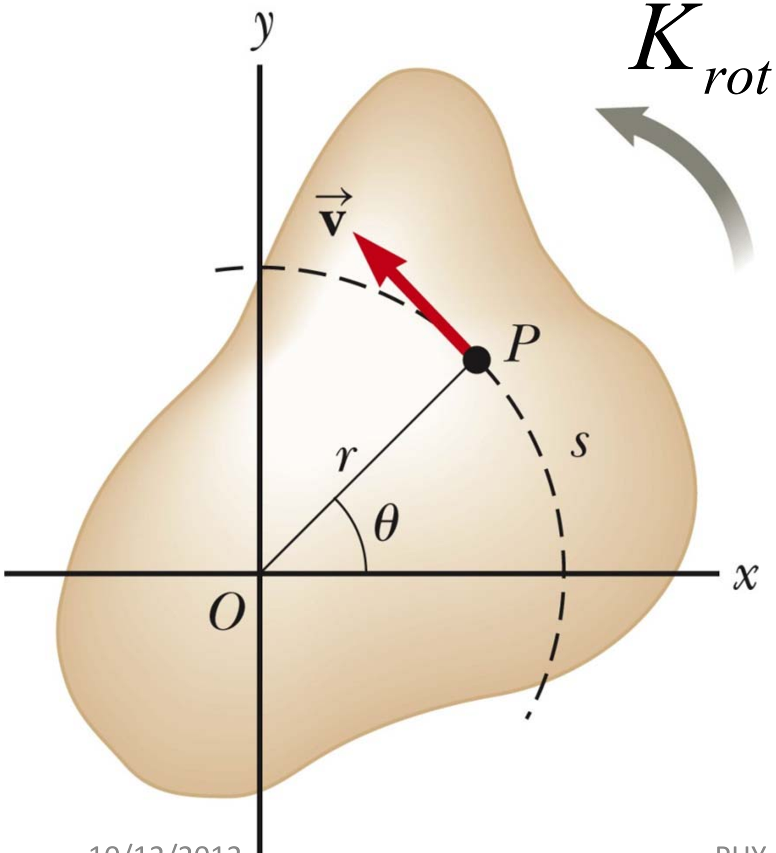
## Plan for Lecture 18:

## Chapter 10 – rotational motion

1. Torque
2. Conservation of energy including both translational and rotational motion

| | | | | | |
|---|---|---|---|---|---|
| **13** | 10/01/2012 | Momentum and collisions | 9.1-9.4 | 9.15,9.18 | 10/03/2012 |
| **14** | 10/03/2012 | Momentum and collisions | 9.5-9.9 | 9.29,9.37 | 10/05/2012 |
| | 10/05/2012 | Review | 6-9 | | |
| | 10/08/2012 | Exam | 6-9 | | |
| **15** | 10/10/2012 | Rotational motion | 10.1-10.5 | 10.6, 10.13, 10.25 | 10/12/2012 |
| **16** | 10/12/2012 | Torque | 10.6-10.9 | 10.37, 10.55 | 10/15/2012 |
| **17** | 10/15/2012 | Angular momentum | 11.1-11.5 | 11.11, 11.34 | 10/17/2012 |
| **18** | 10/17/2012 | Equilibrium | 12.1-12.4 | | 10/22/2012 |
| | 10/19/2012 | *Fall Break* | | | |
| **19** | 10/22/2012 | Simple harmonic motion | 15.1-15.3 | | 10/24/2012 |
| **20** | 10/24/2012 | Resonance | 15.4-15.7 | | 10/26/2012 |
| **21** | 10/26/2012 | Gravitational force | 13.1-13.3 | | 10/29/2012 |
| **22** | 10/29/2012 | Kepler's laws and satellite motion | 13.4-13.6 | | 10/31/2012 |
| | 10/31/2012 | Review | 10-13,15 | | |
| | 11/02/2012 | Exam | 10-13,15 | | |
| **23** | 11/05/2012 | Fluid mechanics | 14.1-14.4 | | 11/07/2012 |

# Review of rotational energy associated with a rigid body



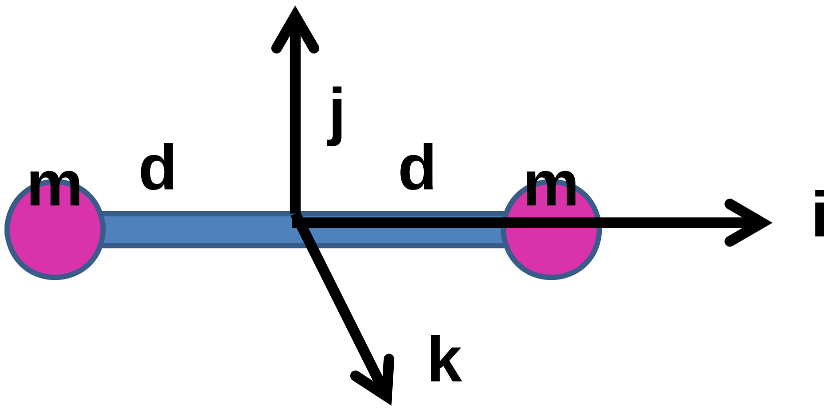

Rotational energy:

$$K_{rot} \equiv \frac{1}{2}\sum_i m_i v_i^2 = \frac{1}{2}\sum_i m_i (\omega r_i)^2$$

$$= \frac{1}{2}\sum_i m_i r_i^2 \omega^2 \equiv \frac{1}{2} I \omega^2$$

$$\text{where } I \equiv \sum_i m_i r_i^2$$

**Note that for a given center of rotation, any solid object has 3 moments of inertia; some times two or more can be equal**



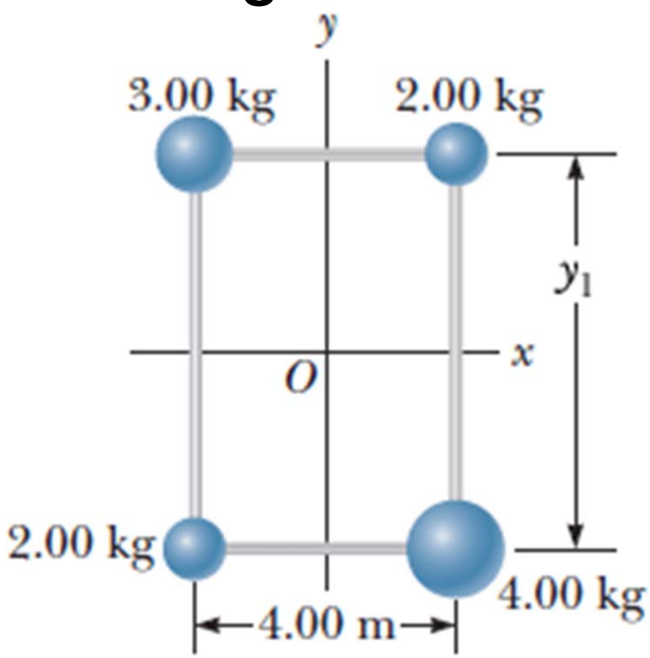

*iclicker exercise:*

**Which moment of inertia is the smallest?**

(A) i (B) j (C) k

$I_A=0$ $I_B=2md^2$ $I_C=2md^2$

## From Webassign:

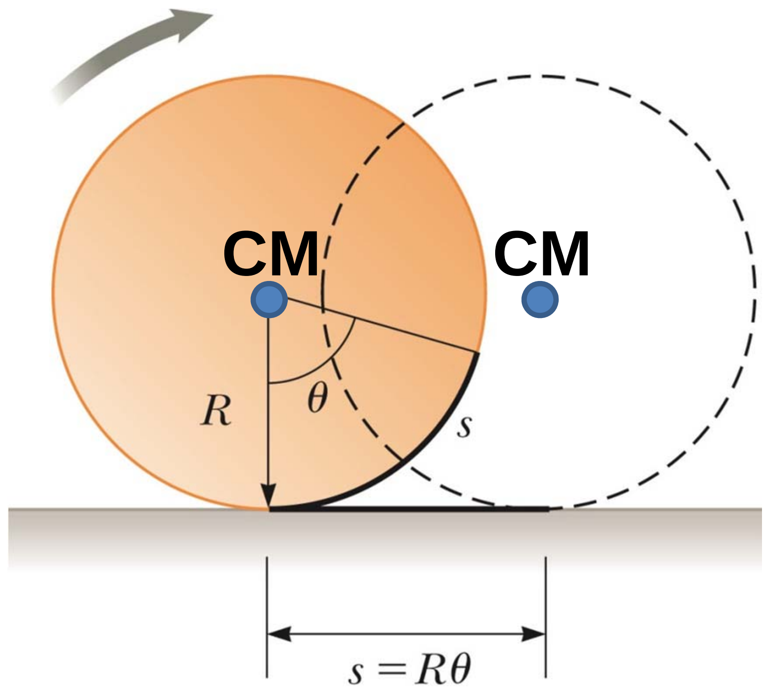



$$I_{yy} = \sum_i m_i x_i^2 = \left((3)(2)^2 + (2)(2)^2 + (2)(2)^2 + (4)(2)^2\right) kg \cdot m^2$$

Total kinetic energy of rolling object :

$$K_{total} = K_{rot} + K_{CM}$$

$$= \frac{1}{2} I\omega^2 + \frac{1}{2} M v_{CM}{}^2$$




Note that :

$$\omega = \frac{d\theta}{dt}$$

$$\frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega = v_{CM}$$

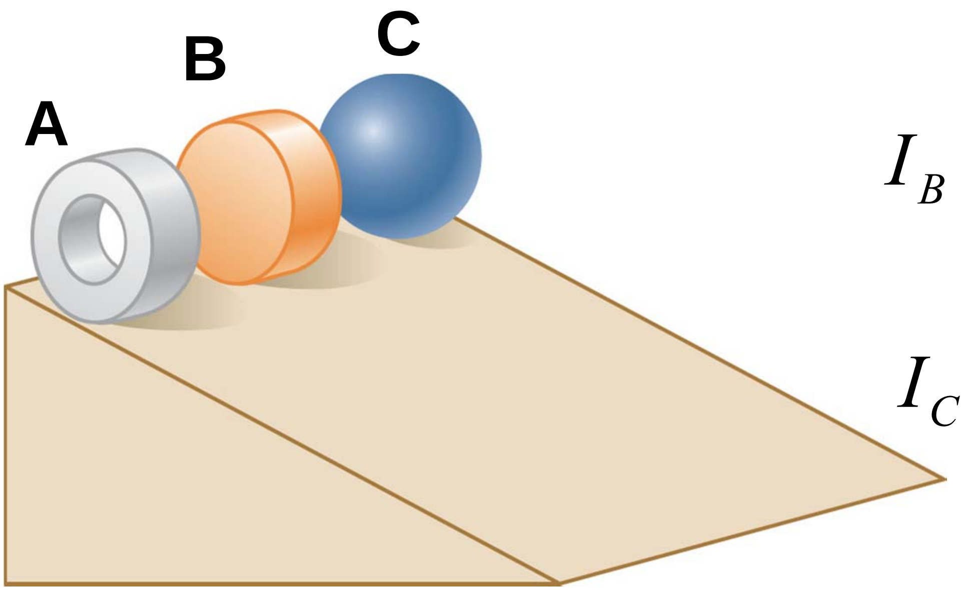
$$K_{total} = K_{rot} + K_{CM}$$

$$= \frac{1}{2}\frac{I}{R^2}(R\omega)^2 + \frac{1}{2} M v_{CM}{}^2$$

$$= \frac{1}{2}\left(\frac{I}{R^2} + M\right) v_{CM}{}^2$$

***iclicker exercise:***

**Three round balls, each having a mass M and radius R, start from rest at the top of the incline. After they are released, they roll without slipping down the incline. Which ball will reach the bottom first?**




$$I_A = MR^2$$

$$I_B = \frac{1}{2}MR^2 = 0.5MR^2$$

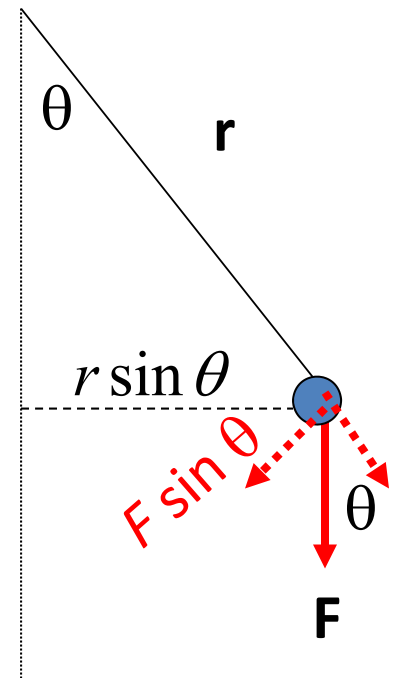
$$I_C = \frac{2}{5}MR^2 = 0.4MR^2$$

How to make objects rotate.

Define torque:

$\tau$ = **r** x **F**

$\tau$ = rF sin $\theta$

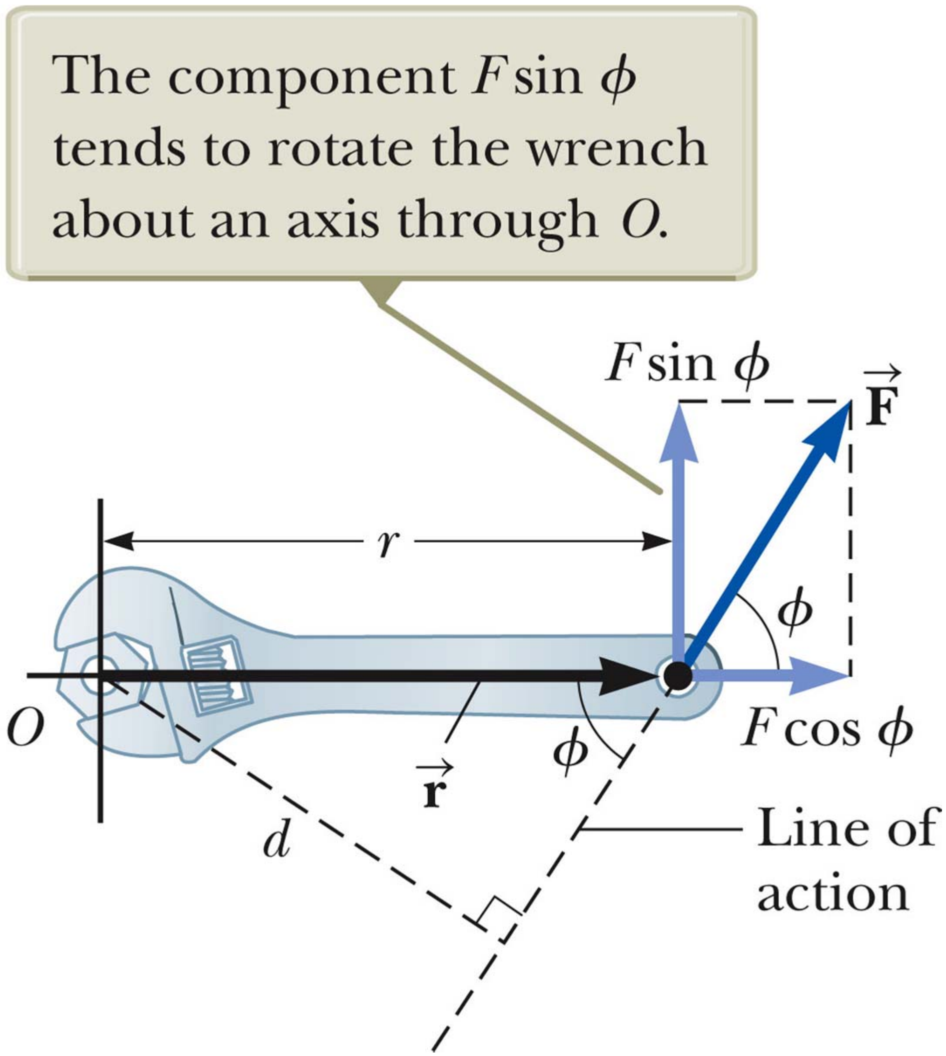


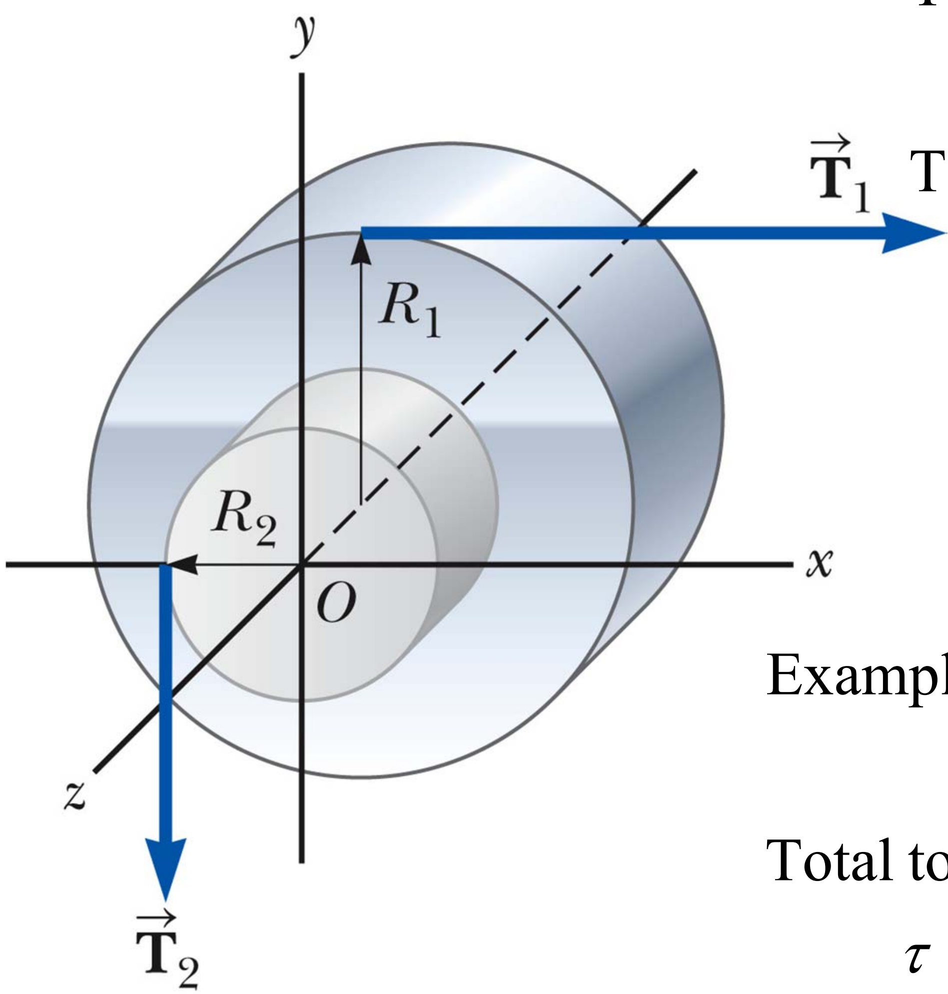

$$\mathbf{F} = m\mathbf{a}$$

$$\mathbf{r} \times \mathbf{F} \equiv \boldsymbol{\tau} = \mathbf{r} \times m\mathbf{a} = I\boldsymbol{\alpha}$$

**Note: We will define and use the "vector cross product" next time. For now, we focus on the fact that the direction of the torque determines the direction of rotation.**

## Another example of torque:






Torque from $T_1$ :

$$\tau_1 = -R_1 T_1 \quad \text{(clockwise)}$$

Torque from $T_2$ :

$$\tau_2 = R_2 T_2 \quad \text{(counter clockwise)}$$

Example: $R_1 = 1\text{m},\ T_1 = 5\text{N}$

$R_2 = 0.5\text{m},\ T_2 = 15\text{N}$

Total torque:

$$\tau = R_2 T_2 - R_1 T_1$$

$$= \big((0.5)(15) - (1)(5)\big) Nm$$

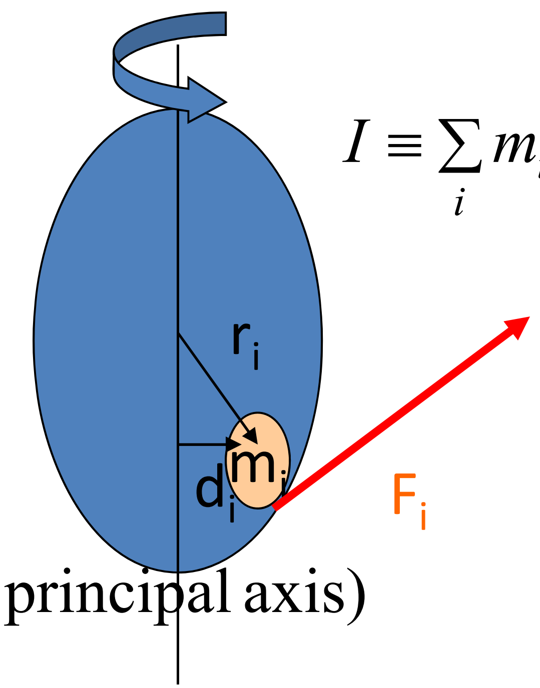
$$= 2.5\, Nm \quad \text{(counter clockwise)}$$

## Newton's second law applied to center-of-mass motion

$$\sum_i \mathbf{F}_i = \sum_i m_i \frac{d\mathbf{v}_i}{dt} \Rightarrow \mathbf{F}_{total} = M\frac{d\mathbf{v}_{CM}}{dt}$$

## Newton's second law applied to rotational motion

$$\mathbf{F}_i = m_i \frac{d\mathbf{v}_i}{dt} \Rightarrow \mathbf{r}_i \times \mathbf{F}_i = \mathbf{r}_i \times m_i \frac{d\mathbf{v}_i}{dt}$$




$$I \equiv \sum_i m_i d_i^2$$

$$\boldsymbol{\tau}_i = \mathbf{r}_i \times \mathbf{F}_i$$

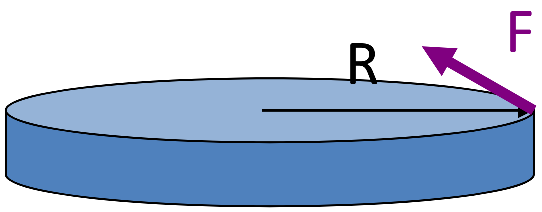
$$\mathbf{v}_i = \boldsymbol{\omega} \times \mathbf{r}_i$$

$$\Rightarrow \boldsymbol{\tau}_i = m_i \mathbf{r}_i \times \frac{d(\boldsymbol{\omega} \times \mathbf{r}_i)}{dt}$$

$$\Rightarrow \boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = I\boldsymbol{\alpha} \quad \text{(for rotating about principal axis)}$$

An example:

A horizontal 800 N merry-go-round is a solid disc of radius 1.50 m and is started from rest by a constant horizontal force of 50 N applied tangentially to the cylinder. Find the kinetic energy of solid cylinder after 3 s.




$K = ½ I \omega^2$ $\tau = I \alpha$ $\omega = \omega_i + \alpha t = \alpha t$

In this case $I = ½ m R^2$ and $\tau = FR$

$$FR = I\alpha \qquad \omega = \alpha t = \frac{FR}{I}t \qquad I = \frac{1}{2}\frac{mg}{g}R^2$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{FR}{I}t\right)^2 = \frac{1}{2}\frac{F^2t^2}{I/R^2} = g\frac{F^2t^2}{mg} = 9.8\text{m/s}^2\,\frac{(50N)^2}{800N}(3s)^2 = 275.625J$$

# Re-examination of "Atwood's" machine

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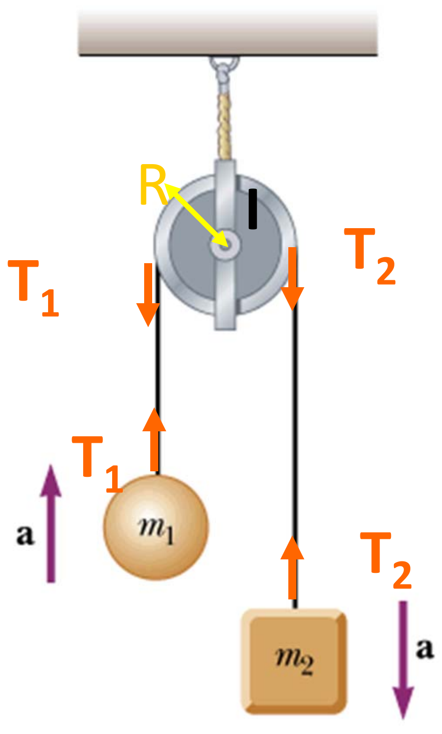
ire 5.15



(a)

$T_1$-$m_1g = m_1a$

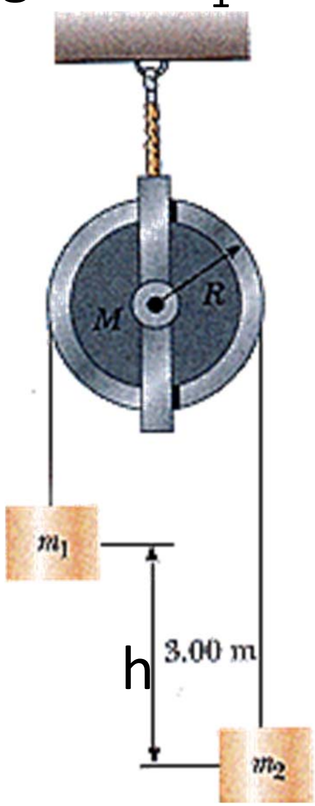
$T_2$-$m_2g = -m_2a$

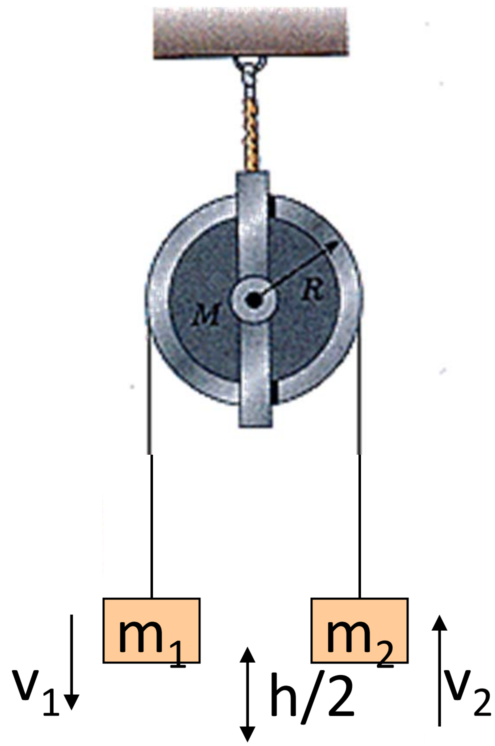
$\tau = T_2R - T_1R = I\,\alpha = I\,a/R$

$$a = g\left(\frac{m_2 - m_1}{m_2 + m_1 + I/R^2}\right)$$

$$\tau = \frac{Ig}{R}\left(\frac{m_2 - m_1}{m_2 + m_1 + I/R^2}\right)$$

Another example: Two masses connect by a frictionless pulley having moment of inertia I and radius R, are initially separated by h=3m. What is the velocity $v=v_2= -v_1$ when the masses are at the same height? $m_1$=2kg; $m_2$=1kg; I=1kg $m^2$; R=0.2m .




Conservation of energy:

$$K_i + U_i = K_f + U_f$$

$$0 + m_1 g h = \tfrac{1}{2} m_1 v^2 + \tfrac{1}{2} m_2 v^2 + \tfrac{1}{2} \tfrac{I}{R^2} v^2 + m_1 g\left(\tfrac{1}{2} h\right) + m_2 g\left(\tfrac{1}{2} h\right)$$

$$v = \sqrt{\frac{m_1 - m_2}{m_1 + m_2 + I/R^2}}$$

$$= \sqrt{\frac{(2)-(1)}{(2)+(1)+\left(1/(0.2)^2\right)}} = 0.19 m/s$$

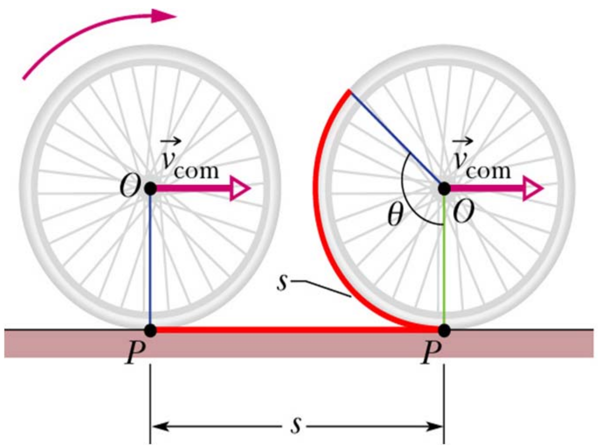
# Rolling motion reconsidered:

Kinetic energy associated with rotation:

$$K_{rot} = \tfrac{1}{2} I\omega^2 \qquad I \equiv \sum_i m_i r_i^2$$

Distance to axis of rotation

Rolling:




$$K_{tot} = K_{com} + K_{rot}$$

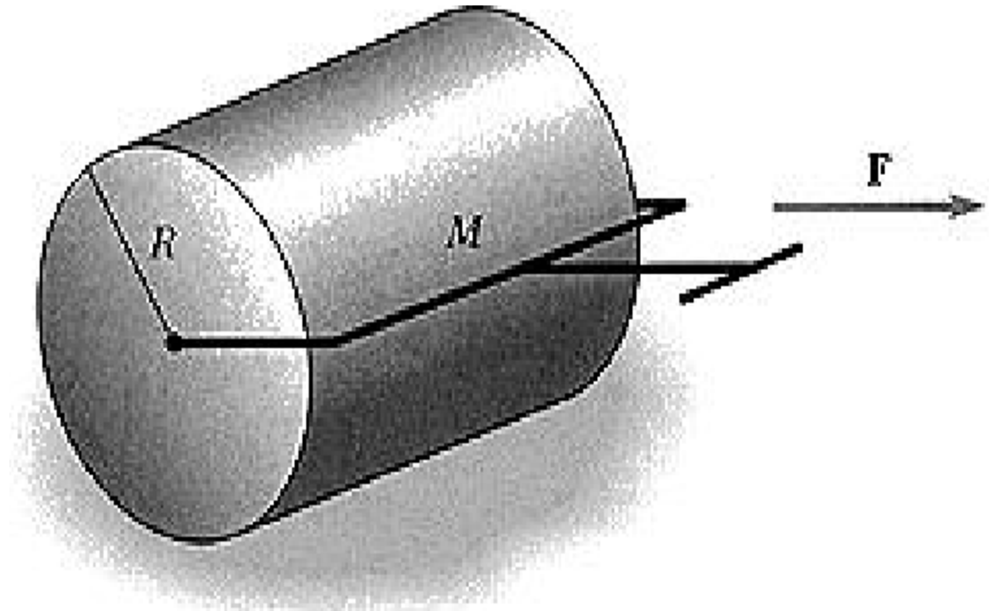
If there is no slipping : $\quad v_{com} = R\omega$

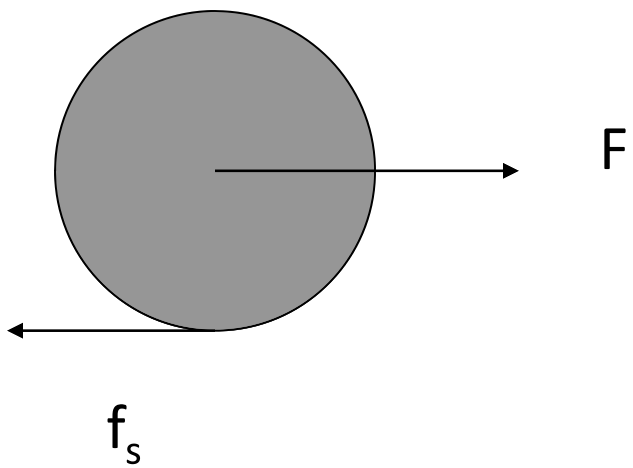
$$\Rightarrow K_{tot} = \tfrac{1}{2} M\left(1 + \frac{I}{MR^2}\right) v_{com}^2$$

## Note that rolling motion is caused by the torque of friction:

Newton’s law for torque:

$$\boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = I\boldsymbol{\alpha}$$



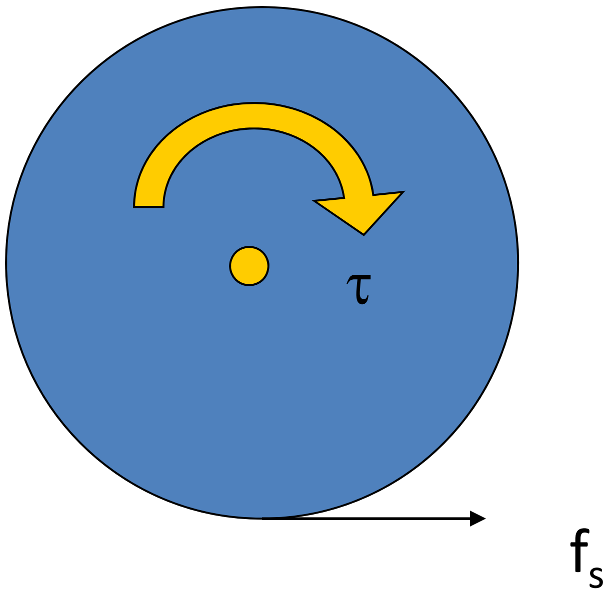

$$F - f_s = Ma_{CM}$$

$$f_s R = I\alpha = Ia_{CM}/R \qquad \Rightarrow a_{CM} = \frac{f_s R^2}{I}$$

$$f_s = F\left(\frac{1}{1+\left(MR^2\right)/I}\right)$$

For a solid cylinder, $I = \frac{1}{2}MR^2 \qquad \Rightarrow f_s = \frac{1}{3}F$

Bicycle or automobile wheel:




$$f_s = Ma_{CM}$$

$$\tau - Rf_s = I\alpha = Ia_{CM} / R$$

$$f_s = \frac{\tau/R}{\left(1 + I/MR^2\right)}$$

For $I = MR^2$

$$f_s = \frac{1}{2}\tau/R$$

***iclicker exercise:***

**What happens when the bicycle skids?**

A. **Too much torque is applied**
B. **Too little torque is applied**
C. **The coefficient of kinetic friction is too small**
D. **The coefficient of static friction is too small**
E. **More than one of these**