PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 18:

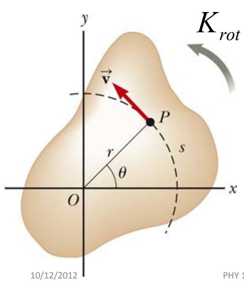
Chapter 10 – rotational motion

1. **Torque**
2. **Conservation of energy including both translational and rotational motion**

13	10/01/2012	Momentum and collisions	9.1-9.4	9.15,9.18	10/03/2012
14	10/03/2012	Momentum and collisions	9.5-9.9	9.29,9.37	10/05/2012
	10/05/2012	Review	6-9		
	10/08/2012	Exam	6-9		
15	10/10/2012	Rotational motion	10.1-10.5	10.6, 10.13, 10.25	10/12/2012
16	10/12/2012	Torque	10.6-10.9	10.37, 10.55	10/15/2012
17	10/15/2012	Angular momentum	11.1-11.5	11.11, 11.34	10/17/2012
18	10/17/2012	Equilibrium	12.1-12.4		10/22/2012
	10/19/2012	*Fall Break*			
19	10/22/2012	Simple harmonic motion	15.1-15.3		10/24/2012
20	10/24/2012	Resonance	15.4-15.7		10/26/2012
21	10/26/2012	Gravitational force	13.1-13.3		10/29/2012
22	10/29/2012	Kepler's laws and satellite motion	13.4-13.6		10/31/2012
	10/31/2012	Review	10-13,15		
	11/02/2012	Exam	10-13,15		
23	11/05/2012	Fluid mechanics	14.1-14.4		11/07/2012

Review of rotational energy associated with a rigid body

Rotational energy:

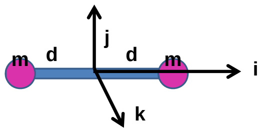


$$K_{rot} \equiv \frac{1}{2}\sum_i m_i v_i^2 = \frac{1}{2}\sum_i m_i (\omega r_i)^2$$

$$= \frac{1}{2}\sum_i m_i r_i^2 \omega^2 \equiv \frac{1}{2} I \omega^2$$

$$\text{where } I \equiv \sum_i m_i r_i^2$$

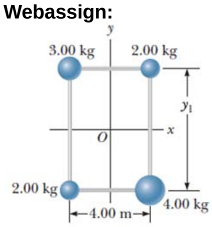
Note that for a given center of rotation, any solid object has 3 moments of inertia; some times two or more can be equal



iclicker exercise:
Which moment of inertia is the smallest?
(A) i (B) j (C) k

$I_A=0$ $I_B=2md^2$ $I_C=2md^2$

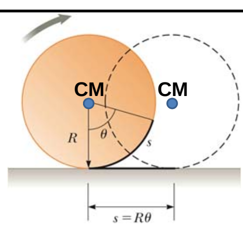
From Webassign:



$$I_{yy} = \sum_i m_i x_i^2 = \left((3)(2)^2 + (2)(2)^2 + (2)(2)^2 + (4)(2)^2\right) kg \cdot m^2$$

Total kinetic energy of rolling object:

$$K_{total} = K_{rot} + K_{CM}$$
$$= \frac{1}{2} I \omega^2 + \frac{1}{2} M v_{CM}{}^2$$



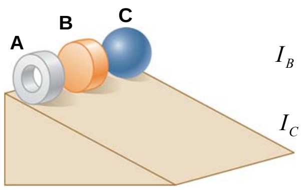
Note that:

$$\omega = \frac{d\theta}{dt}$$
$$\frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega = v_{CM}$$

$$K_{total} = K_{rot} + K_{CM}$$
$$= \frac{1}{2}\frac{I}{R^2}(R\omega)^2 + \frac{1}{2} M v_{CM}{}^2$$
$$= \frac{1}{2}\left(\frac{I}{R^2} + M\right) v_{CM}{}^2$$

iclicker exercise:

Three round balls, each having a mass M and radius R, start from rest at the top of the incline. After they are released, they roll without slipping down the incline. Which ball will reach the bottom first?



$$I_A = MR^2$$

$$I_B = \frac{1}{2}MR^2 = 0.5MR^2$$

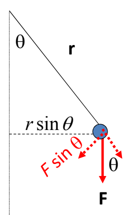
$$I_C = \frac{2}{5}MR^2 = 0.4MR^2$$

How to make objects rotate.

Define torque:

$\tau = \mathbf{r} \times \mathbf{F}$

$\tau = rF \sin \theta$

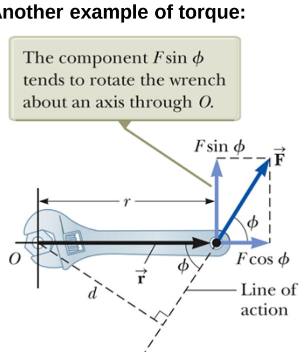


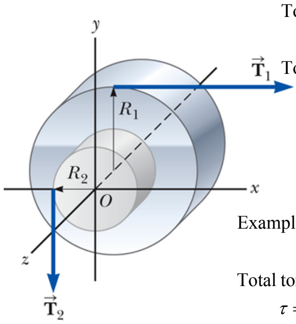
$$\mathbf{F} = m\mathbf{a}$$

$$\mathbf{r} \times \mathbf{F} \equiv \boldsymbol{\tau} = \mathbf{r} \times m\mathbf{a} = I\boldsymbol{\alpha}$$

Note: We will define and use the "vector cross product" next time. For now, we focus on the fact that the direction of the torque determines the direction of rotation.

Another example of torque:





Torque from T_1 :

$$\tau_1 = -R_1 T_1 \quad \text{(clockwise)}$$

Torque from T_2 :

$$\tau_2 = R_2 T_2 \quad \text{(counter clockwise)}$$

Example: $R_1 = 1\text{m},\ T_1 = 5\text{N}$

$R_2 = 0.5\text{m},\ T_2 = 15\text{N}$

Total torque :

$$\begin{aligned} \tau &= R_2 T_2 - R_1 T_1 \\ &= \big((0.5)(15) - (1)(5)\big) Nm \\ &= 2.5 Nm \quad \text{(counter clockwise)} \end{aligned}$$

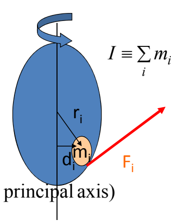
Newton's second law applied to center-of-mass motion

$$\sum_i \mathbf{F}_i = \sum_i m_i \frac{d\mathbf{v}_i}{dt} \Rightarrow \mathbf{F}_{total} = M\frac{d\mathbf{v}_{CM}}{dt}$$

Newton's second law applied to rotational motion

$$\mathbf{F}_i = m_i \frac{d\mathbf{v}_i}{dt} \Rightarrow \mathbf{r}_i \times \mathbf{F}_i = \mathbf{r}_i \times m_i \frac{d\mathbf{v}_i}{dt}$$

$$I \equiv \sum_i m_i d_i^2$$



$$\boldsymbol{\tau}_i = \mathbf{r}_i \times \mathbf{F}_i$$

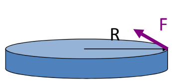
$$\mathbf{v}_i = \boldsymbol{\omega} \times \mathbf{r}_i$$

$$\Rightarrow \boldsymbol{\tau}_i = m_i \mathbf{r}_i \times \frac{d(\boldsymbol{\omega} \times \mathbf{r}_i)}{dt}$$

$$\Rightarrow \boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = I\boldsymbol{\alpha} \quad \text{(for rotating about principal axis)}$$

An example:

A horizontal 800 N merry-go-round is a solid disc of radius 1.50 m and is started from rest by a constant horizontal force of 50 N applied tangentially to the cylinder. Find the kinetic energy of solid cylinder after 3 s.



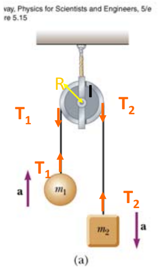
K = ½ I ω^2 τ = I α $\omega = \omega_i + \alpha t = \alpha t$

In this case I = ½ m R^2 and τ = FR

$$FR = I\alpha \qquad \omega = \alpha t = \frac{FR}{I}t \qquad I = \frac{1}{2}\frac{mg}{g}R^2$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{FR}{I}t\right)^2 = \frac{1}{2}\frac{F^2t^2}{I/R^2} = g\frac{F^2t^2}{mg} = 9.8\text{m/s}^2\frac{(50N)^2}{800N}(3s)^2 = 275.625J$$

Re-examination of "Atwood's" machine



$T_1 - m_1 g = m_1 a$

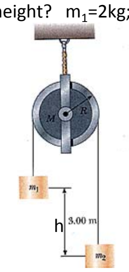
$T_2 - m_2 g = -m_2 a$

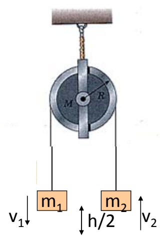
$\tau = T_2 R - T_1 R = I\,\alpha = I\,a/R$

$$a = g\left(\frac{m_2 - m_1}{m_2 + m_1 + I/R^2}\right)$$

$$\tau = \frac{Ig}{R}\left(\frac{m_2 - m_1}{m_2 + m_1 + I/R^2}\right)$$

Another example: Two masses connect by a frictionless pulley having moment of inertia I and radius R, are initially separated by h=3m. What is the velocity $v=v_2=-v_1$ when the masses are at the same height? m_1=2kg; m_2=1kg; I=1kg m^2; R=0.2m .





Conservation of energy:

$$K_i + U_i = K_f + U_f$$

$$0 + m_1 g h = \tfrac{1}{2} m_1 v^2 + \tfrac{1}{2} m_2 v^2 + \tfrac{1}{2}\tfrac{I}{R^2} v^2 + m_1 g\left(\tfrac{1}{2}h\right) + m_2 g\left(\tfrac{1}{2}h\right)$$

$$v = \sqrt{\frac{m_1 - m_2}{m_1 + m_2 + I/R^2}}$$

$$= \sqrt{\frac{(2)-(1)}{(2)+(1)+\left(1/(0.2)^2\right)}} = 0.19\,m/s$$

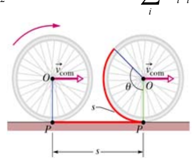
Rolling motion reconsidered:

Kinetic energy associated with rotation:

$K_{rot} = \tfrac{1}{2} I \omega^2$ $\quad I \equiv \sum_i m_i r_i^2$

Distance to axis of rotation

Rolling:



$K_{tot} = K_{com} + K_{rot}$

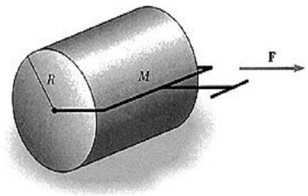
If there is no slipping: $v_{com} = R\omega$

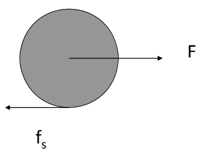
$$\Rightarrow K_{tot} = \tfrac{1}{2} M\left(1 + \frac{I}{MR^2}\right) v_{com}^2$$

Note that rolling motion is caused by the torque of friction:

Newton's law for torque:

$$\boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = I\boldsymbol{\alpha}$$





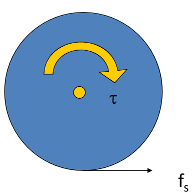
$$F - f_s = Ma_{CM}$$

$$f_s R = I\alpha = Ia_{CM}/R \qquad \Rightarrow a_{CM} = \frac{f_s R^2}{I}$$

$$f_s = F\left(\frac{1}{1+(MR^2)/I}\right)$$

$$\text{For a solid cylinder, } I = \tfrac{1}{2}MR^2 \qquad \Rightarrow f_s = \tfrac{1}{3}F$$

Bicycle or automobile wheel:



$$f_s = Ma_{CM}$$

$$\tau - Rf_s = I\alpha = Ia_{CM}/R$$

$$f_s = \frac{\tau/R}{(1 + I/MR^2)}$$

$$\text{For } I = MR^2$$

$$f_s = \frac{1}{2}\tau/R$$

iclicker exercise:

What happens when the bicycle skids?

A. **Too much torque is applied**
B. **Too little torque is applied**
C. **The coefficient of kinetic friction is too small**
D. **The coefficient of static friction is too small**
E. **More than one of these**