PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 19:

Chapter 11 – angular momentum

1. **Vector cross product**
2. **Angular momentum of a rotating rigid object**
3. **Conservation of angular momentum**

13	10/01/2012	Momentum and collisions	9.1-9.4	9.15,9.18	10/03/2012
14	10/03/2012	Momentum and collisions	9.5-9.9	9.29,9.37	10/05/2012
	10/05/2012	Review	6-9		
	10/08/2012	Exam	6-9		
15	10/10/2012	Rotational motion	10.1-10.5	10.6, 10.13, 10.25	10/12/2012
16	10/12/2012	Torque	10.6-10.9	10.37, 10.55	10/15/2012
17	10/15/2012	Angular momentum	11.1-11.5	11.11, 11.34	10/17/2012
18	10/17/2012	Equilibrium	12.1-12.4		10/22/2012
	10/19/2012	*Fall Break*			
19	10/22/2012	Simple harmonic motion	15.1-15.3		10/24/2012
20	10/24/2012	Resonance	15.4-15.7		10/26/2012
21	10/26/2012	Gravitational force	13.1-13.3		10/29/2012
22	10/29/2012	Kepler's laws and satellite motion	13.4-13.6		10/31/2012
	10/31/2012	Review	10-13,15		
	11/02/2012	Exam	10-13,15		
23	11/05/2012	Fluid mechanics	14.1-14.4		11/07/2012

Previously:

How to make objects rotate.

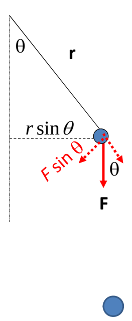
Define torque:

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\tau = rF \sin\theta$

$\mathbf{F} = m\mathbf{a}$

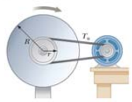
$\mathbf{r} \times \mathbf{F} \equiv \boldsymbol{\tau} = \mathbf{r} \times m\mathbf{a} = I\boldsymbol{\alpha}$



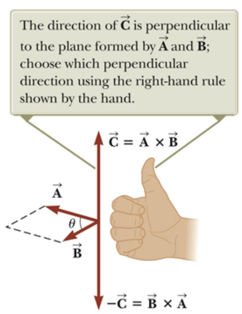
2. –/0.5 points My Notes | SerPSE8 10.P.037.

An electric motor turns a flywheel through a drive belt that joins a pulley on the motor and a pulley that is rigidly attached to the flywheel as shown in the figure below. The flywheel is a solid disk with a mass of 84.0 kg and a radius $R = 0.625$ m. It turns on a frictionless axle. Its pulley has much smaller mass and a radius of 0.230 m. The tension T_u in the upper (taut) segment of the belt is 152 N, and the flywheel has a clockwise angular acceleration of 1.67 rad/s^2. Find the tension in the lower (slack) segment of the belt.

____ N



Vector cross product; right hand rule



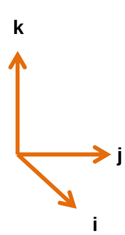
$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

$$|\mathbf{C}| = |\mathbf{A}||\mathbf{B}|\sin\theta$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$
$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$$
$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$
$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$$

For unit vectors:

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$
$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$$
$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$
$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$$



More details of vector cross products:

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}=\begin{vmatrix}A_y & A_z\\ B_y & B_z\end{vmatrix}\hat{\mathbf{i}}+\begin{vmatrix}A_x & A_z\\ B_x & B_z\end{vmatrix}\hat{\mathbf{j}}+\begin{vmatrix}A_x & A_y\\ B_x & B_y\end{vmatrix}\hat{\mathbf{k}}$$

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}}=\left(A_yB_z-A_zB_y\right)\hat{\mathbf{i}}+\left(A_xB_z-A_zB_x\right)\hat{\mathbf{j}}+\left(A_xB_y-A_yB_x\right)\hat{\mathbf{k}}$$

iclicker exercise:

What is the point of vector products

A. To terrify physics students
B. To exercise your right hand
C. To define an axial vector
D. To keep track of the direction of rotation

From Newton's second law:

$$\mathbf{F}=m\mathbf{a}$$

$$\mathbf{r}\times\mathbf{F}\equiv\boldsymbol{\tau}=\mathbf{r}\times m\mathbf{a}=\mathbf{r}\times m\frac{d\mathbf{v}}{dt}=\mathbf{r}\times\frac{d(m\mathbf{v})}{dt}=\frac{d}{dt}(\mathbf{r}\times\mathbf{p})$$

iclicker exercise:

Consider

$$\mathbf{r}\times\frac{d(m\mathbf{v})}{dt}=\frac{d}{dt}(\mathbf{r}\times\mathbf{p})$$

Is this

A. Wrong?
B. Approximately right?
C. Exactly right?

From Newton's second law – continued – conservation of angular momentum:

$$\mathbf{r}\times\mathbf{F}=\boldsymbol{\tau}=\frac{d}{dt}(\mathbf{r}\times\mathbf{p})$$

$$\text{Define:}\quad \mathbf{L}\equiv\mathbf{r}\times\mathbf{p}$$

$$\text{If}\quad \boldsymbol{\tau}=0 \qquad \frac{d\mathbf{L}}{dt}=0$$

$$\Rightarrow \mathbf{L}=(\text{constant})$$

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Torque and angular momentum

Define angular momentum: $\mathbf{L}\equiv\mathbf{r}\times\mathbf{p}$

For composite object: $L = I\omega$

Newton's law for torque:

$$\boldsymbol{\tau}_{total}=I\frac{d\boldsymbol{\omega}}{dt}=\frac{d\mathbf{L}}{dt}$$

In the absence of a net torque on a system, angular momentum is conserved.

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iclicker exercise:

A student sits on a rotatable stool holding a spinning bicycle wheel with angular momentum $\mathbf{L}_i$. What happens when the wheel is inverted?

(a) The student will remain at rest.

(b) The student will rotate counterclockwise.

(c) The student will rotate clockwise.

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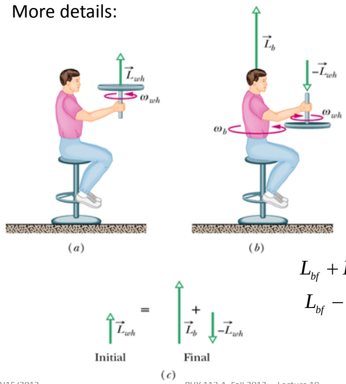
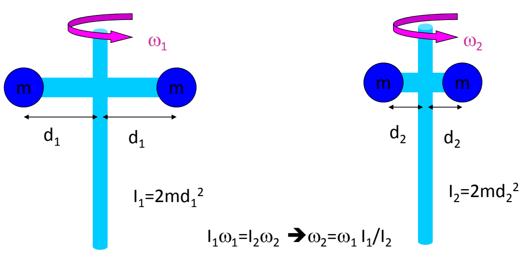
More details:



$$L_{bf} + L_{wheelf} = L_{bi} + L_{wheeli}$$
$$L_{bf} - L_{wheel} = 0 + L_{wheel}$$
$$L_{bf} = 2L_{wheel}$$

Other examples of conservation of angular momentum

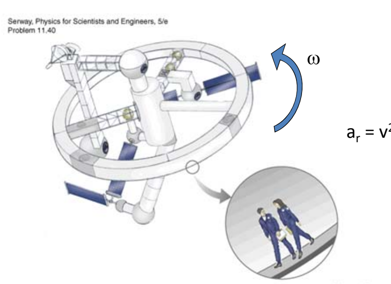


$I_1=2md_1^2$

$I_2=2md_2^2$

$I_1\omega_1=I_2\omega_2 \rightarrow \omega_2=\omega_1\, I_1/I_2$

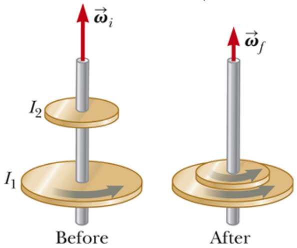
What about centripetal acceleration?



$a_r = v^2/R = \omega^2 R$

Webassign problem:

A disk with moment of inertia I_1 is initially rotating at angular velocity ω_i. A second disk having angular momentum I_2, initially is not rotating, but suddenly drops and sticks to the second disk. Assuming angular moment to be conserved, what would be the final angular velocity ω_f?



$$I_1\omega_i = (I_1 + I_2)\omega_f$$

$$\omega_f = \omega_i \frac{I_1}{I_1 + I_2}$$

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