

**PHY 113 A General Physics I**  
**9-9:50 AM MWF Olin 101**

**Plan for Lecture 20:**

**Chapter 12 – Static equilibrium**

- 1. Balancing forces and torques; stability**
- 2. Center of gravity**
- 3. Note: May not have time to cover elastic properties of materials**

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	10/05/2012	Review	6-9	
	10/09/2012	Exam	6-9	
15	10/10/2012	Rotational motion	10.1-10.6 10.8 10.13 10.21	10/12/2012
16	10/12/2012	Torque	10.6-10.9 10.37 10.55	10/16/2012
17	10/15/2012	Angular momentum	11.1-11.5 11.11 11.34	10/17/2012
18	10/17/2012	Equilibrium	12.1-12.4 12.11 12.39	10/22/2012
	10/19/2012	Fall Break		
19	10/22/2012	Simple harmonic motion	15.1-15.3	10/24/2012
20	10/24/2012	Resonance	15.4-15.7	10/26/2012
21	10/26/2012	Gravitational force	13.1-13.3	10/29/2012
22	10/29/2012	Kepler's laws and satellite motion	13.4-13.6	10/31/2012
	10/31/2012	Review	10-13 35	
	11/02/2012	Exam	10-13, 15	
23	11/05/2012	Fluid mechanics	14.1-14.4	11/07/2012
24	11/07/2012	Fluid mechanics	14.5-14.7	11/09/2012
25	11/09/2012	Temperature	19.1-19.5	11/12/2012
26	11/12/2012	Heat	20.1-20.4	11/14/2012
27	11/14/2012	First law of thermodynamics	20.5-20.7	11/16/2012

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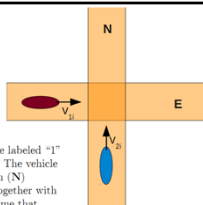
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**Comment on exam question #4**



The diagram above shows two vehicles approaching an intersection. The vehicle labeled "1" has mass  $m_1 = 1500\text{kg}$  and a velocity of  $v_{1i} = 20\text{m/s}$  in the east (E) direction. The vehicle labeled "2" has a mass  $m_2 = 2000\text{kg}$  and a velocity of  $v_{2i} = 20\text{m/s}$  in the north (N) direction. The two vehicles collide in the middle of the intersection and stick together with the velocity of the combined system of  $v_f$  immediately after the collision. Assume that because of road conditions (small friction due to ice), it is a good approximation to assume that momentum is conserved immediately before and after the collision.

- (a) What is the momentum of the two vehicle system before the collision?
- (b) What is the momentum of the two vehicle system immediately after the collision?
- (c) What is the velocity of the combined system of  $v_f$  (magnitude and direction) immediately after the collision?

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Solution:

(a)

$$\mathbf{p}_i = m_1 v_{1i} \hat{\mathbf{E}} + m_2 v_{2i} \hat{\mathbf{N}},$$

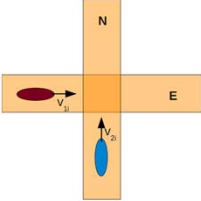
$$\mathbf{p}_i = (1500)(20) \text{ kg} \cdot \text{m/s} \hat{\mathbf{E}} + (2000)(20) \text{ kg} \cdot \text{m/s} \hat{\mathbf{N}} = 30000 \text{ kg} \cdot \text{m/s} \hat{\mathbf{E}} + 40000 \text{ kg} \cdot \text{m/s} \hat{\mathbf{N}}.$$

(b)

$$\mathbf{p}_i = \mathbf{p}_f.$$

(c)

$$\mathbf{v}_f = \frac{\mathbf{p}_f}{m_1 + m_2} = \frac{30000 \text{ kg} \cdot \text{m/s} \hat{\mathbf{E}} + 40000 \text{ kg} \cdot \text{m/s} \hat{\mathbf{N}}}{3500 \text{ kg}} = 8.5714 \text{ m/s} \hat{\mathbf{E}} + 11.4286 \text{ m/s} \hat{\mathbf{N}}.$$

$$v_f = 14.29 \text{ m/s}; \text{ angle north of east: } \theta = 53.13^\circ.$$


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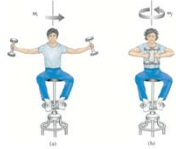
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**From Webassign #17 (11.34)**

A student sits on a freely rotating stool holding two dumbbells, each of mass 2.99 kg (see figure below). When his arms are extended horizontally (figure a), the dumbbells are 1.07 m from the axis of rotation and the student rotates with an angular speed of 0.757 rad/s. The moment of inertia of the student plus stool is 2.54 kg · m<sup>2</sup> and is assumed to be constant. The student pulls the dumbbells inward horizontally to a position 0.301 m from the rotation axis (figure b).



(a) Find the new angular speed of the student, rad/s.

(b) Find the kinetic energy of the rotating system before and after he pulls the dumbbells inward.

$K_{\text{before}} =$  J

$K_{\text{after}} =$  J

**Conservation of angular momentum:**

$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

$$I_i = I_{\text{Student}} + 2md_i^2$$

$$I_f = I_{\text{Student}} + 2md_f^2$$

$$\omega_f = \omega_i \frac{I_i}{I_f}$$

**Rotational kinetic energy:**

$$K_i = \frac{1}{2} I_i \omega_i^2 \quad K_f = \frac{1}{2} I_f \omega_f^2$$

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**Conditions for stable equilibrium**

Balance of force:  $\sum_i \mathbf{F}_i = 0$

Balance of torque:  $\sum_i \boldsymbol{\tau}_i = 0$

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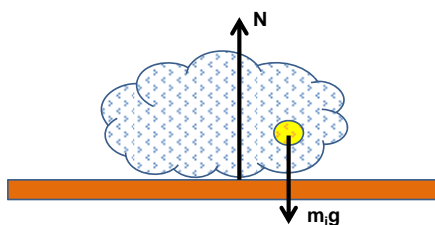
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## Stability of "rigid bodies"



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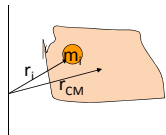
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Center-of-mass  $\mathbf{r}_{CM} \equiv \frac{\sum_i m_i \mathbf{r}_i}{\sum_i m_i}$

Torque on an extended object due to gravity (near surface of the earth) is the same as the torque on a point mass  $M$  located at the center of mass.



$$\boldsymbol{\tau} = \sum_i \mathbf{r}_i \times \{m_i g(-\mathbf{j})\} = \mathbf{r}_{CM} \times \{Mg(-\mathbf{j})\}$$

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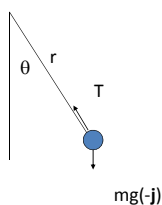
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Notion of equilibrium:  $\sum_i \mathbf{F}_i = 0 \quad \sum_i \boldsymbol{\tau}_i = 0$

Notion of stability:



$$\mathbf{F} = m\mathbf{a} \rightarrow \begin{aligned} T - mg \cos \theta &= 0 \\ -mg \sin \theta &= -ma_\theta \end{aligned}$$

$$\boldsymbol{\tau} = I \boldsymbol{\alpha} \rightarrow r mg \sin \theta = mr^2 \alpha = mra_\theta$$

Example of stable equilibrium.

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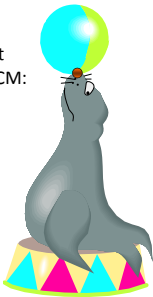
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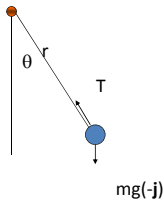
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Unstable equilibrium:

Support **below** CM:



Support **above** CM:



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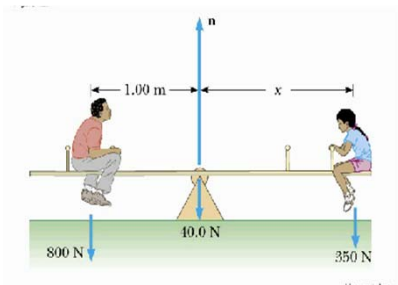
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Analysis of stability:  $\sum_i \mathbf{F}_i = 0$   $\sum_i \boldsymbol{\tau}_i = 0$



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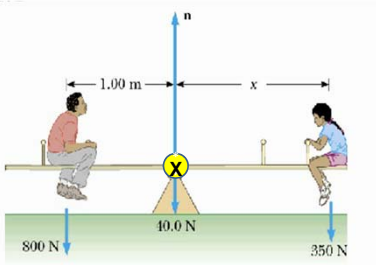
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Forces :  $n - M_D g - m_c g - m_P g = 0$

Torques :  $M_D g (1m) - m_c g x = 0$

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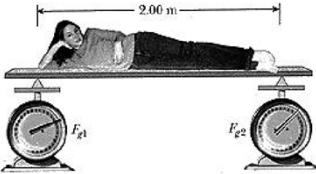
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A student takes a nap on a massless plank which is supported by two scales as shown. If the left and right scale readings are  $F_{g1} = 350 \text{ N}$  and  $F_{g2} = 300 \text{ N}$ , respectively, what is her total weight and where is her center of mass located? (Please indicate whether you are measuring her center of mass from her feet or head.)

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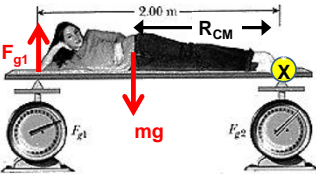
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Torques :  $-F_{g1}(2m) + mg(R_{CM}) = 0$

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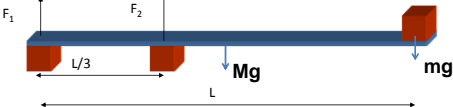
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**iclicker question:**



Consider the above drawing of the two supports for a uniform plank which has a total weight  $Mg$  and has a weight  $mg$  at its end. What can you say about  $F_1$  and  $F_2$ ?

(a)  $F_1$  and  $F_2$  are both up as shown.  
 (b)  $F_1$  is up but  $F_2$  is down.  
 (c)  $F_1$  is down but  $F_2$  is up.

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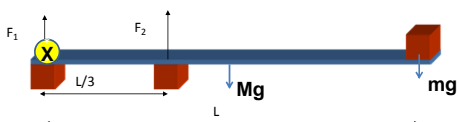
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Forces :  $F_1 + F_2 - Mg - mg = 0$

Torques :  $F_2 \frac{L}{3} - Mg \frac{L}{2} - mgL = 0$

$$F_2 = \frac{3}{2}Mg + 3mg \quad F_1 = -\frac{1}{2}Mg - 2mg$$

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**iclicker question:**

The fact that we found  $F_1 < 0$  means:

- A. We set up the problem incorrectly
- B. The analysis is correct, but the direction of  $F_1$  is opposite to the arrow
- C. Physics makes no sense

**iclicker question:**

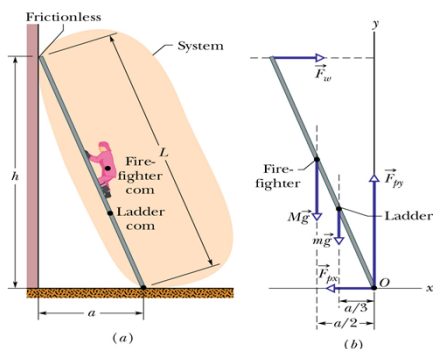
What would happen if we analyzed this problem by placing the pivot point at  $F_1$  ?:

- A. The answer would be the same.
- B. The answer would be different.
- C. Physics makes no sense

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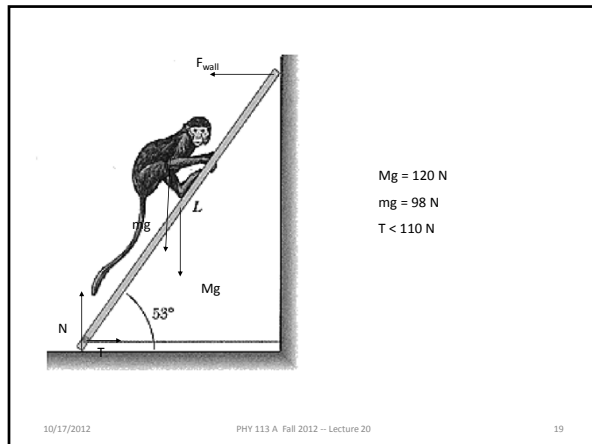
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