PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 21:

Chapter 15 – Simple harmonic motion

1. **Object attached to a spring**
   - **Displacement as a function of time**
   - **Kinetic and potential energy**
2. **Pendulum motion**

14	10/03/2012	Momentum and collisions	9.5-9.9	9.29,9.37	10/05/2012
	10/05/2012	Review	6-9		
	10/08/2012	Exam	6-9		
15	10/10/2012	Rotational motion	10.1-10.5	10.6, 10.13, 10.25	10/12/2012
16	10/12/2012	Torque	10.6-10.9	10.37, 10.55	10/15/2012
17	10/15/2012	Angular momentum	11.1-11.5	11.11, 11.34	10/17/2012
18	10/17/2012	Equilibrium	12.1-12.4	12.11, 12.39	10/22/2012
	10/19/2012	*Fall Break*			
19	10/22/2012	Simple harmonic motion	15.1-15.3	15.4, 15.20	10/24/2012
20	10/24/2012	Resonance	15.4-15.7	15.43, 15.43, 15.52	10/26/2012
21	10/26/2012	Gravitational force	13.1-13.3	13.6, 13.10, 13.13	10/29/2012
22	10/29/2012	Kepler's laws and satellite motion	13.4-13.6	13.28, 13.34	10/31/2012
	10/31/2012	Review	10-13,15		
	11/02/2012	Exam	10-13,15		
23	11/05/2012	Fluid mechanics	14.1-14.4		11/07/2012
24	11/07/2012	Fluid mechanics	14.5-14.7		11/09/2012
25	11/09/2012	Temperature	19.1-19.5		11/12/2012

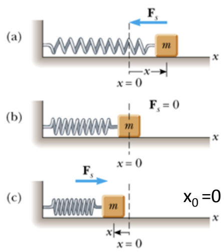
iclicker exercise:
Why did your textbook develop a whole chapter on oscillatory motion?
- **A. Because the authors like to torture students.**
- **B. Because it is different from Newton's laws and needs many pages to explain.**
- **C. Because it is an example of Newton's laws and needs many pages to explain.**

iclicker exercise:
Simple harmonic motion has a characteristic time period T. What determines T?
- **A. The characteristics of the physical system.**
- **B. The initial displacements or velocities.**
- **C. It is not possible to know T.**

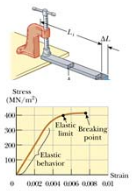
Hooke's law

$F_s = -k(x-x_0)$

Serway, Physics for Scientists and Engineers, 5/e Figure 13.1



Behavior of materials:



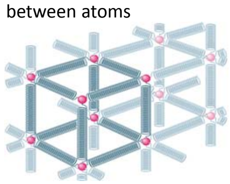
Young's modulus

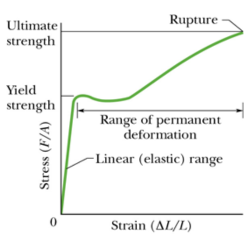
$$E = \frac{F_{applied}/A}{\Delta L/L}$$

$$F_{material} = -F_{applied}$$

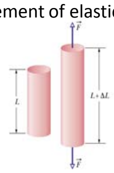
$$\Rightarrow F_{material} = -\left(\frac{EA}{L}\right)\Delta L$$

Microscopic picture of material with springs representing bonds between atoms





Measurement of elastic response:



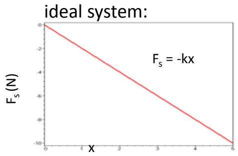
$$E = \frac{F_{applied}/A}{\Delta L/L}$$

$$F_{material} = -F_{applied}$$

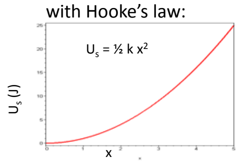
$$\Rightarrow F_{material} = -\left(\frac{EA}{L}\right)\Delta L$$

k

Form of Hooke's law for ideal system:

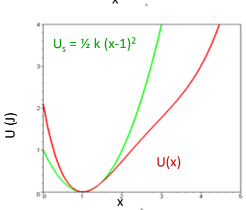


Potential energy associated with Hooke's law:



General potential energy curve:

$$k = \frac{d^2U}{dx^2}(x=1)$$



In addition to the spring-mass system, Hooke's law approximates many physical systems near equilibrium.

Motion associated with Hooke's law forces

Newton's second law:

F = -k x = m a

$$F = -kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$ ➔ "second-order" linear differential equation

How to solve a second order linear differential equation:

Earlier example – constant force F_0 ➔ acceleration a_0

$$\frac{d^2x}{dt^2} = \frac{F_0}{m} \equiv a_0$$

$x(t) = x_0 + v_0 t + ½ a_0 t^2$

2 constants (initial values)

Hooke's law motion:

$$F = -kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Forms of solution:

$$x(t) = A\cos(\omega t + \varphi)$$
$$x(t) = A\cos(\omega t) + B\sin(\omega t)$$

where: $\omega \equiv \sqrt{\frac{k}{m}}$

2 constants (initial values)

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Verification:

Differential relations:

$$\frac{d\sin(\omega t + \varphi)}{dt} = \omega\cos(\omega t + \varphi)$$

$$\frac{d\cos(\omega t + \varphi)}{dt} = -\omega\sin(\omega t + \varphi)$$

Therefore: $$\frac{d^2A\cos(\omega t + \varphi)}{dt^2} = -\omega^2 A\cos(\omega t + \varphi)$$

$\Rightarrow x(t) = A\cos(\omega t + \varphi)$ *satisfies*

$\frac{d^2x}{dt^2} = -\frac{k}{m}x$ *provided that* $\omega^2 = \frac{k}{m}$

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Recap: Newton's law for mass-spring system:

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Guess that solution for $x(t)$ has the form:

$x(t) = A\cos(\omega t + \varphi)$ where A and φ and ω are unknown constants

Condition that guess satisfies the equation:

$$\frac{d^2[A\cos(\omega t + \varphi)]}{dt^2} = -\omega^2[A\cos(\omega t + \varphi)] = -\frac{k}{m}[A\cos(\omega t + \varphi)]$$

$\Rightarrow \omega^2 = \frac{k}{m}$ (determines ω)

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iclicker exercise:

Which of the following other possible guesses would provide a solution to Newton's law for the mass-spring system:

A. $A\sin(\omega t+\varphi)$
B. $A\cos(\omega t)+B\sin(\omega t)$
C. $v_0 t+\frac{1}{2}gt^2$
D. $\exp(\omega t+\varphi)$
E. More than one of the above.

Note that: $A\sin(\omega t+\varphi)=A\cos(\varphi)\sin(\omega t)+A\sin(\varphi)\cos(\omega t)$
$A\cos(\omega t+\varphi)=A\cos(\varphi)\cos(\omega t)-A\sin(\varphi)\sin(\omega t)$

Complete solution for Newton's law for mass-spring system:

$$\frac{d^2x}{dt^2}=-\frac{k}{m}x$$

We now know that the solution for $x(t)$ has the form:

$$x(t)=A\cos\left(\sqrt{\frac{k}{m}}t+\varphi\right)$$ where A and φ are unknown constants

Finding A and φ from initial conditions:

Suppose we know that

$$x(t=0)=x_0 \quad \text{and} \quad \frac{dx}{dt}(t=0)=0$$

$$x(t=0)=x_0=A\cos(\varphi) \qquad \frac{dx}{dt}(t=0)=0=-A\sin(\varphi)$$

$$\Rightarrow \varphi=0 \quad \text{and} \quad A=x_0 \qquad \Rightarrow x(t)=x_0\cos\left(\sqrt{\frac{k}{m}}t\right)$$

Complete solution for Newton's law for mass-spring system:

$$\frac{d^2x}{dt^2}=-\frac{k}{m}x$$

We now know that the solution for $x(t)$ has the form:

$$x(t)=A\cos\left(\sqrt{\frac{k}{m}}t+\varphi\right)$$ where A and φ are unknown constants

Finding A and φ from initial conditions:

Suppose we know that

$$x(t=0)=0 \quad \text{and} \quad \frac{dx}{dt}(t=0)=v_0$$

$$x(t=0)=0=A\cos(\varphi) \qquad \frac{dx}{dt}(t=0)=v_0=-A\sqrt{\frac{k}{m}}\sin(\varphi)$$

$$\Rightarrow \varphi=\frac{\pi}{2} \quad \text{and} \quad A=-\frac{v_0}{\sqrt{\frac{k}{m}}} \qquad \Rightarrow x(t)=\frac{v_0}{\sqrt{\frac{k}{m}}}\sin\left(\sqrt{\frac{k}{m}}t\right)$$

"Simple harmonic motion" in practice

A block with a mass of 0.2 kg is connected to a light spring for which the force constant is 5 N/m and is free to oscillate on a horizontal, frictionless surface. The block is displaced 0.05 m from equilibrium and released from rest. Find its subsequent motion.

$\omega = \sqrt{k/m} = \sqrt{5/0.2}\ \text{rad/s} = 5\ \text{rad/s}$

$x(t) = A\cos(\omega t+\phi)$ $\quad x(0) = A\cos(\phi) = 0.05$ m

$v(t) = -A\omega\sin(\omega t+\phi)$ $\quad v(0) = -A\omega\sin(\phi) = 0$ m/s

➔ $\phi = 0$ and A = 0.05 m

iclicker exercise:

A certain mass m on a spring oscillates with a characteristic frequency of 2 cycles per second. Which of the following changes to the mass would increase the frequency to 4 cycles per second?

(a) 2m (b) 4m (c) m/2 (d) m/4

Summary --

Simple harmonic motion:

$$F = -kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$x(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{k}{m}}$$

Conveniently evaluated in *radians*

Constants

Note that:

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi)$$

$$a(t) = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \varphi)$$

Energy associated with simple harmonic motion

Form of displacement :

$x(t) = A\cos(\omega t + \varphi)$ where $\omega = \sqrt{\frac{k}{m}}$ and A and φ are constants

Energy :

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi)$$

$$\Rightarrow E = \frac{1}{2}m\omega^2 A^2(\sin(\omega t + \varphi))^2 + \frac{1}{2}kA^2(\cos(\omega t + \varphi))^2$$

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Energy associated with simple harmonic motion

Energy :

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}m\omega^2 A^2(\sin(\omega t + \varphi))^2 + \frac{1}{2}kA^2(\cos(\omega t + \varphi))^2$$

But $\omega^2 = \frac{k}{m}$

$$\Rightarrow E = \frac{1}{2}kA^2\left[(\sin(\omega t + \varphi))^2 + (\cos(\omega t + \varphi))^2\right] = \frac{1}{2}kA^2$$

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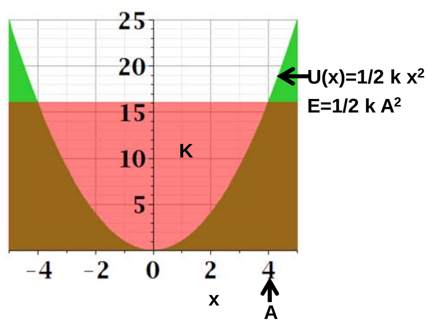
Energy diagram:



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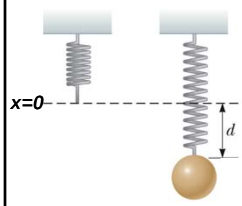
Effects of gravity on spring motion



$$\text{At equilibrium}: \quad kd - mg = 0$$

$$d = \frac{mg}{k}$$

$$F = -kx - mg = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x - g$$

Rewriting :

$$\frac{d^2(x+d)}{dt^2} = -\frac{k}{m}\left(x + \frac{mg}{k}\right) = -\frac{k}{m}(x+d)$$

$$\Rightarrow x(t) = -d + A\cos\left(\sqrt{\frac{k}{m}}t + \varphi\right)$$

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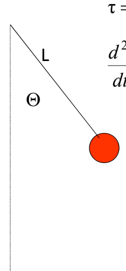
Simple harmonic motion for a pendulum:

$$\tau = mgL\sin\Theta = -I\alpha = -I\frac{d^2\Theta}{dt^2}$$



$$\frac{d^2\Theta}{dt^2} = -\frac{mgL}{I}\sin\Theta = -\frac{g}{L}\sin\Theta \quad (\text{since } I = mL^2)$$

Approximation for small Θ:

$$\sin\Theta \approx \Theta$$

$$\Rightarrow \frac{d^2\Theta}{dt^2} = -\frac{g}{L}\Theta$$

Solution :

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{g}{L}}$$

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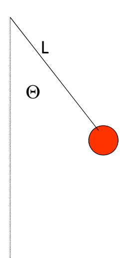
Pendulum example:

Suppose L=2m, what is the period of the pendulum?



$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\text{m/s}^2}{2\text{m}}} = 2.2135\ \text{rad/s} = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = 2.84\ \text{s}$$

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{g}{L}}$$

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