PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 22:

Chapter 15 – Simple harmonic motion

1. **Object attached to a spring and pendulum motion**
2. **Resonance phenomena**
3. **Note: We will not thoroughly cover damping and the analogy to circular motion.**

14	10/03/2012	Momentum and collisions	9.5-9.9	9.29,9.37	10/05/2012
	10/05/2012	Review	6-9		
	10/08/2012	Exam	6-9		
15	10/10/2012	Rotational motion	10.1-10.5	10.6, 10.13, 10.25	10/12/2012
16	10/12/2012	Torque	10.6-10.9	10.37, 10.55	10/15/2012
17	10/15/2012	Angular momentum	11.1-11.5	11.11, 11.34	10/17/2012
18	10/17/2012	Equilibrium	12.1-12.4	12.11, 12.39	10/22/2012
	10/19/2012	*Fall Break*			
19	10/22/2012	Simple harmonic motion	15.1-15.3	15.4, 15.20	10/24/2012
20	10/24/2012	Resonance	15.4-15.7	15.43, 15.43, 15.52	10/26/2012
21	10/26/2012	Gravitational force	13.1-13.3	13.6, 13.10, 13.13	10/29/2012
22	10/29/2012	Kepler's laws and satellite motion	13.4-13.6	13.28, 13.34	10/31/2012
	10/31/2012	Review	10-13,15		
	11/02/2012	Exam	10-13,15		
23	11/05/2012	Fluid mechanics	14.1-14.4		11/07/2012
24	11/07/2012	Fluid mechanics	14.5-14.7		11/09/2012
25	11/09/2012	Temperature	19.1-19.5		11/12/2012

Comment on final exam

Examination Schedule: Fall, 2012		
9 a.m. exam time **If your class meets:**	**2 p.m. exam time** **If your class meets:**	**7 p.m. exam time** **If your class meets:**
Dec. 10: 9:30 WF; 10:00 MWF	9:30 TR	8:00 TR; 5:00 MWF, MW, WF, MF
Dec. 11: 2:00 TR	2:00 MWF, MW, WF, MF	ACC 221 block/Alternate ACC 111
Dec. 12: 12:30 TR	12:30 MW, WF, MF; 1:00 MWF	8:00 MWF/WF; 5:00 TR
Dec. 13: 9:00 MWF	3:30 TR or WF	ACC 111 block/Alternate ACC
Dec. 14: MTH/BUS block	12:00 MWF	
Dec. 15: 11:00 TR	11:00 MWF; WF	

Review : Newton's law for mass - spring system :

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Solution for $x(t)$:

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}t + \varphi\right)$$ where A and φ are unknown constants

Review : $\Rightarrow$ mass - spring system has a characteristic frequency :

$$\omega = \sqrt{\frac{k}{m}} \text{ rad/s}$$

Energy associated with simple harmonic motion

$$\text{Energy}:$$

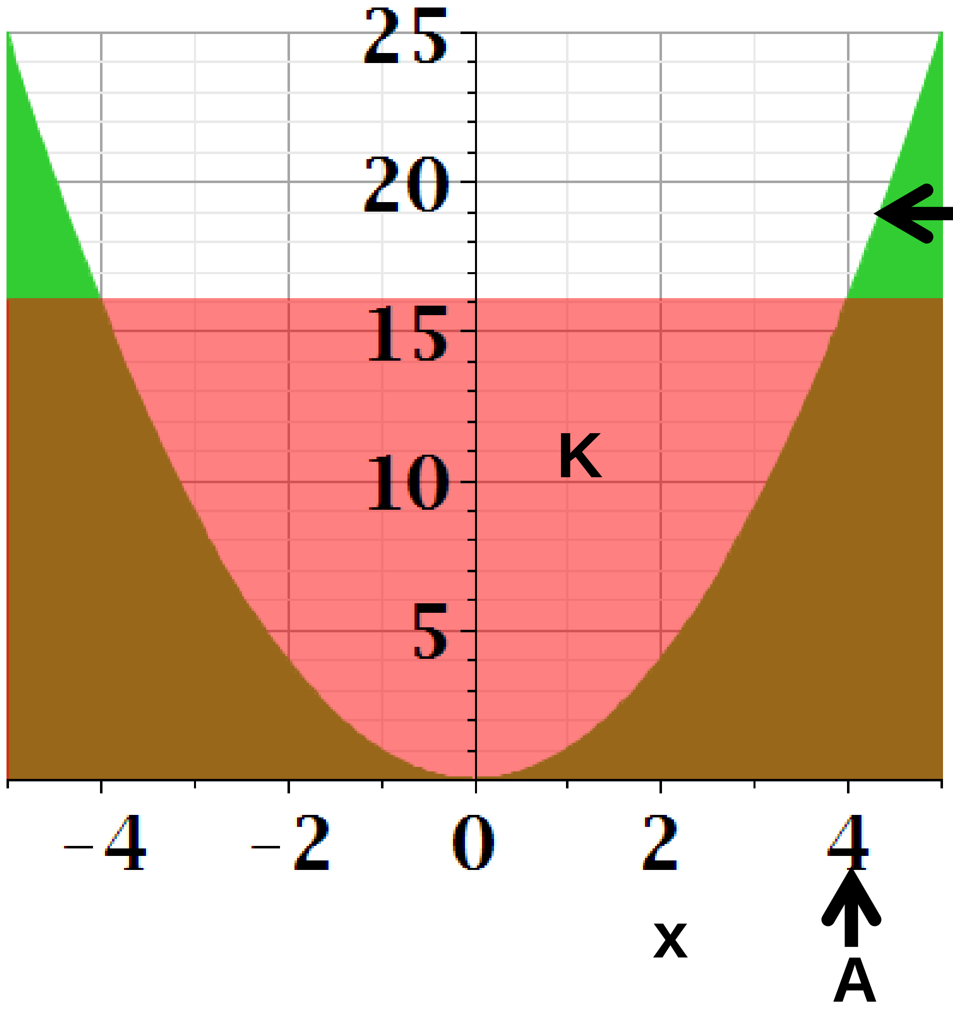
$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}m\omega^2 A^2\left(\sin(\omega t + \varphi)\right)^2 + \frac{1}{2}kA^2\left(\cos(\omega t + \varphi)\right)^2$$

$$\text{But} \quad \omega^2 = \frac{k}{m}$$

$$\Rightarrow E = \frac{1}{2}kA^2\left[\left(\sin(\omega t + \varphi)\right)^2 + \left(\cos(\omega t + \varphi)\right)^2\right] = \frac{1}{2}kA^2$$

Energy diagram:

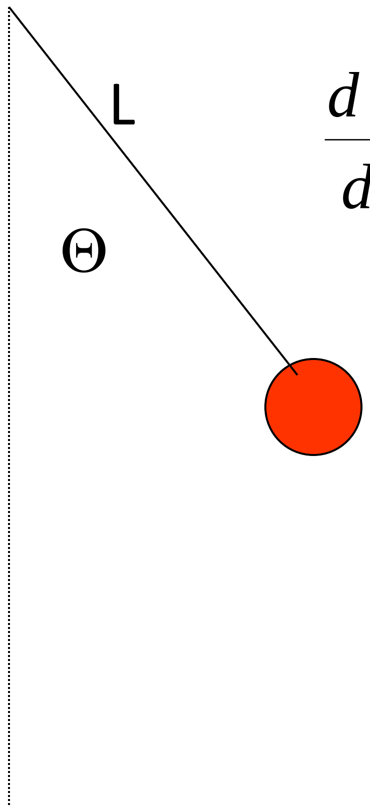


$U(x)=1/2\ k\ x^2$

$E=1/2\ k\ A^2$

Simple harmonic motion for a pendulum:

$$\tau = mgL\sin\Theta = -I\alpha = -I\,\frac{d^2\Theta}{dt^2}$$



$$\frac{d^2\Theta}{dt^2} = -\frac{mgL}{I}\sin\Theta = -\frac{g}{L}\sin\Theta \quad (\text{since } I = mL^2)$$

Approximation for small Θ:

$$\sin\Theta \approx \Theta$$

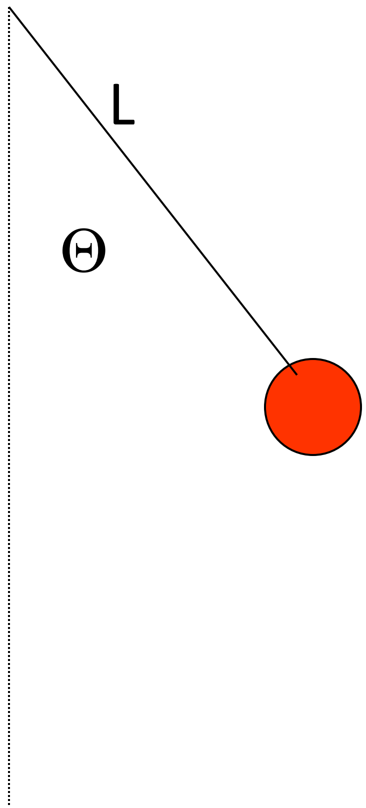
$$\Rightarrow \frac{d^2\Theta}{dt^2} = -\frac{g}{L}\Theta$$

Solution :

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{\mathrm{g}}{\mathrm{L}}}$$

Pendulum example:

Suppose L=2m, what is the period of the pendulum?



$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\text{m/s}^2}{2\text{m}}} = 2.2135\,\text{rad/s} = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega} = 2.84\,\text{s}$$

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{g}{L}}$$

iclicker exercise:

What happens if Θ is too large?

A. **The frequency of oscillation will no longer be constant**
B. **The pendulum will no longer oscillate**
C. **Energy will be lost even if air friction is negligible**

The notion of resonance:

Suppose $F=-kx+F_0 \sin(\Omega t)$

According to Newton:

$$-kx + F_0 \sin(\Omega t) = m\frac{d^2x}{dt^2}$$

Differential equation ("inhomogeneous"):

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x + \frac{F_0}{m}\sin(\Omega t)$$

Solution:

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) \equiv \frac{F_0/m}{\omega^2 - \Omega^2}\sin(\Omega t)$$

Physics of a "driven" harmonic oscillator:

$$-kx + F_0 \sin(\Omega t) = m\frac{d^2x}{dt^2}$$

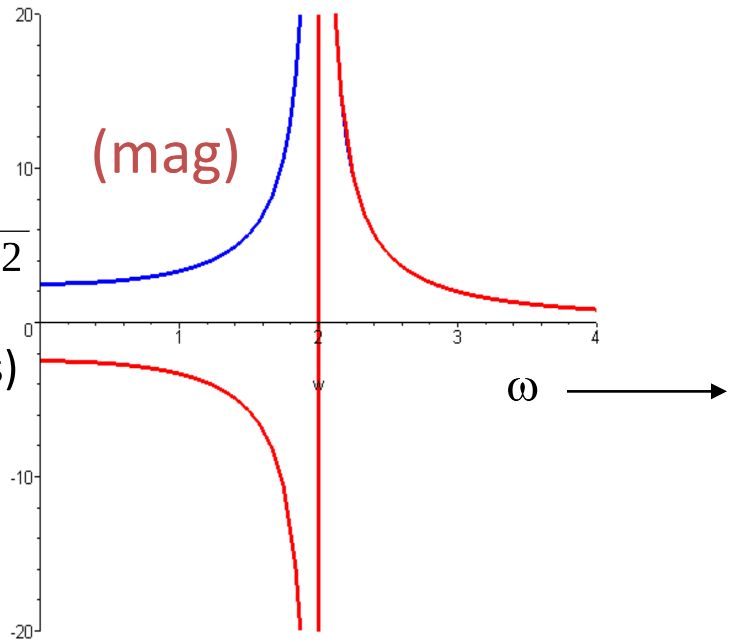
"driving" frequency

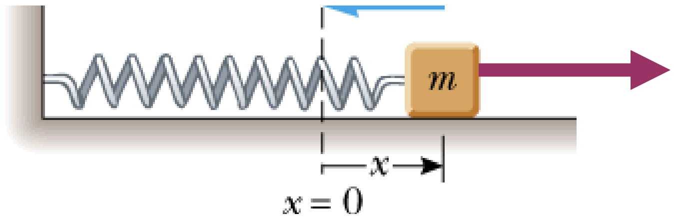
$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) \equiv \frac{F_0/m}{\omega^2 - \Omega^2}\sin(\Omega t)$$

"natural" frequency

$$\frac{F_0/m}{\omega^2 - \Omega^2}$$

(Ω=2rad/s)





F(t)=1 N sin (3t)

Examples:

Suppose a mass m=0.2 kg is attached to a spring with k=1.81N/m and an oscillating driving force as shown above. Find the steady-state displacement x(t).

$$x(t) = \frac{F_0 / m}{k/m - \Omega^2}\sin(\Omega t) = \frac{1/0.2}{1.81/0.2 - 3^2}\sin(3t)\ \mathrm{m} = 100\sin(3t)\ \mathrm{m}$$

Note: If k=1.90 N/m then:

$$x(t) = \frac{F_0 / m}{k/m - \Omega^2}\sin(\Omega t) = \frac{1/0.2}{1.90/0.2 - 3^2}\sin(3t)\ \mathrm{m} = 10\sin(3t)\ \mathrm{m}$$

Examples:
From Assignment 19

4. –/0.5 points | My Notes | SerPSE8 15.P.020.

A 1.50-kg object is attached to a spring and placed on frictionless, horizontal surface. A horizontal force of 11.0 N is required to hold the object at rest when it is pulled 0.200 m from its equilibrium position (the origin of the x axis). The object is now released from rest from this stretched position, and it subsequently undergoes simple harmonic oscillations.

(a) Find the force constant of the spring.
____ N/m

(b) Find the frequency of the oscillations.
____ Hz

(c) Find the maximum speed of the object.
____ m/s

(d) Where does this maximum speed occur?
$x = \pm$ ____ m

(e) Find the maximum acceleration of the object.
____ m/s^2

(f) Where does the maximum acceleration occur?
$x = \pm$ ____ m

4. –/0.5 points My Notes | SerPSE8 15.P.020.

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(a) Find the force constant of the spring.

____ N/m

(b) Find the frequency of the oscillations.

____ Hz

(e) Find the maximum acceleration of the object.

____ m/s^2

(f) Where does the maximum acceleration occur?

$x = \pm$ ____ m

(g) Find the total energy of the oscillating system.

____ J

(h) Find the speed of the object when its position is equal to one-third of the maximum value.

____ m/s

(i) Find the acceleration of the object when its position is equal to one-third of the maximum value.

____ m/s^2

5. –/0 points My Notes | SerPSE8 15.P.027.

A simple pendulum makes 143 complete oscillations in 3.60 min at a location where $g = 9.80\ \text{m/s}^2$.

(a) Find the period of the pendulum.

_____ s

(b) Find the length of the pendulum.

_____ m

More examples:
From Assignment 20

4. ⊕ –/0 points | My Notes | SerPSE8 15.P.060.WI.

A simple pendulum with a length of 2.93 m and a mass of 6.94 kg is given an initial speed of 1.46 m/s at its equilibrium position.

(a) Assuming it undergoes simple harmonic motion, determine its period.

_____ s

(b) Determine its total energy.

_____ J

(c) Determine its maximum angular displacement. (For large *v*, and/or small *l*, the small angle approximation may not be good enough here.)

_____ °

1. –/0.334 points My Notes | SerPSE8 15.P.043.

A 1.90-kg object attached to a spring moves without friction (b = 0) and is driven by an external force given by the expression F = 5.10sin($2\pi t$), where F is in newtons and t is in seconds. The force constant of the spring is 34.0 N/m.

(a) Find the resonance angular frequency of the system.

______ s^{-1}

(b) Find the angular frequency of the driven system.

______ s^{-1}

(c) Find the amplitude of the motion.

______ cm

Tacoma Narrows bridge resonance
Images from Wikipedia

Rebuilt bridge in Tacoma WA

Bridge in July, 1940

Collapse of bridge on Nov. 7, 1940

http://en.wikipedia.org/wiki/Tacoma_Narrows_Bridge_%281940%29