PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 22:

Chapter 15 – Simple harmonic motion

1. **Object attached to a spring and pendulum motion**
2. **Resonance phenomena**
3. **Note: We will not thoroughly cover damping and the analogy to circular motion.**

14	10/03/2012	Momentum and collisions	9.5-9.9	9.29,9.37	10/05/2012
	10/05/2012	Review	6-9		
	10/08/2012	Exam	6-9		
15	10/10/2012	Rotational motion	10.1-10.5	10.6, 10.13, 10.25	10/12/2012
16	10/12/2012	Torque	10.6-10.9	10.37, 10.55	10/15/2012
17	10/15/2012	Angular momentum	11.1-11.5	11.11, 11.34	10/17/2012
18	10/17/2012	Equilibrium	12.1-12.4	12.11, 12.39	10/22/2012
	10/19/2012	*Fall Break*			
19	10/22/2012	Simple harmonic motion	15.1-15.3	15.4, 15.20	10/24/2012
20	10/24/2012	Resonance	15.4-15.7	15.43, 15.43, 15.52	10/26/2012
21	10/26/2012	Gravitational force	13.1-13.3	13.6, 13.10, 13.13	10/29/2012
22	10/29/2012	Kepler's laws and satellite motion	13.4-13.6	13.28, 13.34	10/31/2012
	10/31/2012	Review	10-13,15		
	11/02/2012	Exam	10-13,15		
23	11/05/2012	Fluid mechanics	14.1-14.4		11/07/2012
24	11/07/2012	Fluid mechanics	14.5-14.7		11/09/2012
25	11/09/2012	Temperature	19.1-19.5		11/12/2012

Comment on final exam

Examination Schedule: Fall, 2012		
9 a.m. exam time **If your class meets:**	**2 p.m. exam time** **If your class meets:**	**7 p.m. exam time** **If your class meets:**
Dec. 10: 9:30 WF; 10:00 MWF	9:30 TR	8:00 TR; 5:00 MWF, MW, WF, MF
Dec. 11: 2:00 TR	2:00 MWF, MW, WF, MF	ACC 221 block/Alternate ACC 111
Dec. 12: 12:30 TR	12:30 MW, WF, MF; 1:00 MWF	8:00 MWF/WF; 5:00 TR
Dec. 13: 9:00 MWF	3:30 TR or WF	ACC 111 block/Alternate ACC
Dec. 14: MTH/BUS block	12:00 MWF	
Dec. 15: 11:00 TR	11:00 MWF; WF	

Review : Newton's law for mass-spring system :

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Solution for $x(t)$:

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}t + \varphi\right)$$ where A and φ are unknown constants

Review : $\Rightarrow$ mass-spring system has a characteristic frequency :

$$\omega = \sqrt{\frac{k}{m}} \text{ rad/s}$$

Energy associated with simple harmonic motion

Energy :

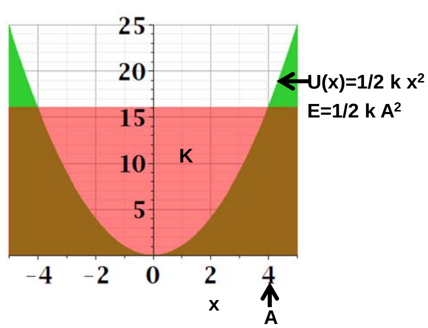
$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}m\omega^2 A^2(\sin(\omega t + \varphi))^2 + \frac{1}{2}kA^2(\cos(\omega t + \varphi))^2$$

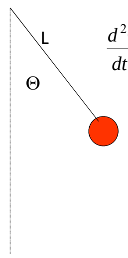
But $\omega^2 = \frac{k}{m}$

$$\Rightarrow E = \frac{1}{2}kA^2\left[(\sin(\omega t + \varphi))^2 + (\cos(\omega t + \varphi))^2\right] = \frac{1}{2}kA^2$$

Energy diagram:



Simple harmonic motion for a pendulum:



$$\tau = mgL\sin\Theta = -I\alpha = -I\frac{d^2\Theta}{dt^2}$$

$$\frac{d^2\Theta}{dt^2} = -\frac{mgL}{I}\sin\Theta = -\frac{g}{L}\sin\Theta \quad (\text{since } I = mL^2)$$

Approximation for small Θ:

$$\sin\Theta \approx \Theta$$

$$\Rightarrow \frac{d^2\Theta}{dt^2} = -\frac{g}{L}\Theta$$

Solution:

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{\mathrm{g}}{\mathrm{L}}}$$

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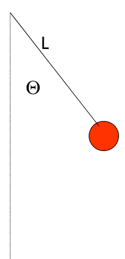
Pendulum example:

Suppose L=2m, what is the period of the pendulum?



$$\omega = \sqrt{\frac{\mathrm{g}}{\mathrm{L}}} = \sqrt{\frac{9.8\mathrm{m/s}^2}{2\mathrm{m}}} = 2.2135\ \mathrm{rad/s} = \frac{2\pi}{\mathrm{T}}$$

$$\mathrm{T} = \frac{2\pi}{\omega} = 2.84\ \mathrm{s}$$

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{\mathrm{g}}{\mathrm{L}}}$$

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iclicker exercise:

What happens if Θ is too large?

- **A. The frequency of oscillation will no longer be constant**
- **B. The pendulum will no longer oscillate**
- **C. Energy will be lost even if air friction is negligible**

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The notion of resonance:

Suppose F=-kx+F_0 sin(Ωt)

According to Newton:

$$-kx + F_0 \sin(\Omega t) = m\frac{d^2x}{dt^2}$$

Differential equation ("inhomogeneous"):

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x + \frac{F_0}{m}\sin(\Omega t)$$

Solution:

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) \equiv \frac{F_0/m}{\omega^2 - \Omega^2}\sin(\Omega t)$$

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Physics of a "driven" harmonic oscillator:

$$-kx + F_0 \sin(\Omega t) = m\frac{d^2x}{dt^2}$$

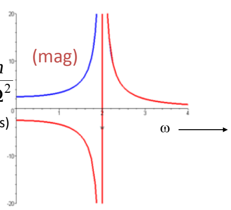
"driving" frequency

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) \equiv \frac{F_0/m}{\omega^2 - \Omega^2}\sin(\Omega t)$$

"natural" frequency

$\frac{F_0/m}{\omega^2 - \Omega^2}$

(Ω=2rad/s)



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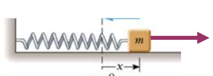

F(t)=1 N sin (3t)

Examples:

Suppose a mass m=0.2 kg is attached to a spring with k=1.81N/m and an oscillating driving force as shown above. Find the steady-state displacement x(t).

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) = \frac{1/0.2}{1.81/0.2 - 3^2}\sin(3t)\text{ m} = 100\sin(3t)\text{ m}$$

Note: If k=1.90 N/m then:

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) = \frac{1/0.2}{1.90/0.2 - 3^2}\sin(3t)\text{ m} = 10\sin(3t)\text{ m}$$

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Examples:
From Assignment 19

4. –/0.5 points My Notes | SerPSE8 15.P.020.

A 1.50-kg object is attached to a spring and placed on frictionless, horizontal surface. A horizontal force of 11.0 N is required to hold the object at rest when it is pulled 0.200 m from its equilibrium position (the origin of the x axis). The object is now released from rest from this stretched position, and it subsequently undergoes simple harmonic oscillations.

(a) Find the force constant of the spring.
N/m

(b) Find the frequency of the oscillations.
Hz

(c) Find the maximum speed of the object.
m/s

(d) Where does this maximum speed occur?
$x = \pm$ m

(e) Find the maximum acceleration of the object.
m/s^2

(f) Where does the maximum acceleration occur?
$x = \pm$ m

4. –/0.5 points My Notes | SerPSE8 15.P.020.

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(a) Find the force constant of the spring.
N/m

(b) Find the frequency of the oscillations.
Hz

(e) Find the maximum acceleration of the object.
m/s^2

(f) Where does the maximum acceleration occur?
$x = \pm$ m

(g) Find the total energy of the oscillating system.
J

(h) Find the speed of the object when its position is equal to one-third of the maximum value.
m/s

(i) Find the acceleration of the object when its position is equal to one-third of the maximum value.
m/s^2

5. –/0 points My Notes | SerPSE8 15.P.027.

A simple pendulum makes 143 complete oscillations in 3.60 min at a location where $g = 9.80\ m/s^2$.

(a) Find the period of the pendulum.
s

(b) Find the length of the pendulum.
m

More examples:
From Assignment 20

4. –/0 points | My Notes | SerPSE8 15.P.060.WI.

A simple pendulum with a length of 2.93 m and a mass of 6.94 kg is given an initial speed of 1.46 m/s at its equilibrium position.

(a) Assuming it undergoes simple harmonic motion, determine its period.
s

(b) Determine its total energy.
J

(c) Determine its maximum angular displacement. (For large v, and/or small l, the small angle approximation may not be good enough here.)
°

1. –/0.334 points | My Notes | SerPSE8 15.P.043.

A 1.90-kg object attached to a spring moves without friction ($b = 0$) and is driven by an external force given by the expression $F = 5.10\sin(2\pi t)$, where F is in newtons and t is in seconds. The force constant of the spring is 34.0 N/m.

(a) Find the resonance angular frequency of the system.
s^{-1}

(b) Find the angular frequency of the driven system.
s^{-1}

(c) Find the amplitude of the motion.
cm

Tacoma Narrows bridge resonance
Images from Wikipedia

Rebuilt bridge in Tacoma WA

Bridge in July, 1940

Collapse of bridge on Nov. 7, 1940

http://en.wikipedia.org/wiki/Tacoma_Narrows_Bridge_%281940%29