PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

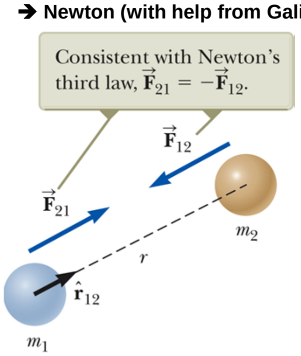
Plan for Lecture 23:

Chapter 13 – Fundamental force of gravity

1. Relationship with g near earth's surface
2. Orbital motion due to gravity
3. Kepler's orbital equation
4. Note: We will probably not emphasize elliptical orbits in this chapter.

14	10/03/2012	Momentum and collisions	9.5-9.9	9.29,9.37	10/05/2012
	10/05/2012	Review	6-9		
	10/08/2012	Exam	6-9		
15	10/10/2012	Rotational motion	10.1-10.5	10.6, 10.13, 10.25	10/12/2012
16	10/12/2012	Torque	10.6-10.9	10.37, 10.55	10/15/2012
17	10/15/2012	Angular momentum	11.1-11.5	11.11, 11.34	10/17/2012
18	10/17/2012	Equilibrium	12.1-12.4	12.11, 12.39	10/22/2012
	10/19/2012	*Fall Break*			
19	10/22/2012	Simple harmonic motion	15.1-15.3	15.4, 15.20	10/24/2012
20	10/24/2012	Resonance	15.4-15.7	15.43, 15.43, 15.52	10/26/2012
21	10/26/2012	Gravitational force	13.1-13.3	13.6, 13.10, 13.13	10/29/2012
22	10/29/2012	Kepler's laws and satellite motion	13.4-13.6	13.28, 13.34	10/31/2012
	10/31/2012	Review	10-13,15		
	11/02/2012	Exam	10-13,15		
23	11/05/2012	Fluid mechanics	14.1-14.4		11/07/2012
24	11/07/2012	Fluid mechanics	14.5-14.7		11/09/2012
25	11/09/2012	Temperature	19.1-19.5		11/12/2012

Universal law of gravitation
➔ Newton (with help from Galileo, Kepler, etc.) 1687

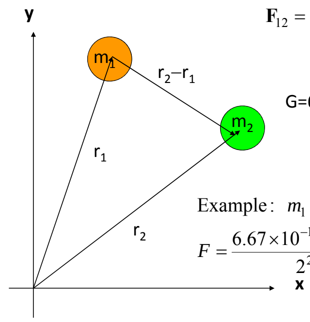


$$\mathbf{F}_{12} = \frac{G m_1 m_2 \hat{\mathbf{r}}_{12}}{r_{12}^2}$$

$$G = 6.674 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$$

Newton's law of gravitation: m_2 *attracts* m_1 according to:

$$\mathbf{F}_{12} = \frac{G m_1 m_2 \hat{\mathbf{r}}_{12}}{r_{12}^2}$$

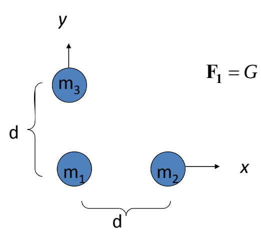


G=6.67 x 10^{-11} N m^2/kg^2

$$\text{Example: } m_1 = m_2 = 70\,\text{kg};\quad r_{12} = 2\,\text{m}:$$

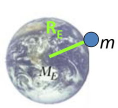
$$F = \frac{6.67\times10^{-11}\cdot 70\cdot 70}{2^2} N = 8.17\times10^{-8} N$$

Vector nature of Gravitational law:



$$\mathbf{F}_1 = G\frac{m_1}{d^2}(m_2\mathbf{i} + m_3\mathbf{j})$$

Gravitational force of the Earth



$$F = \frac{G M_E m}{R_E^2}$$

$$\Rightarrow g = \frac{G M_E}{R_E^2} = \frac{6.67\times10^{-11}\cdot 5.98\times10^{24}}{(6.37\times10^6)^2}\,\text{m/s}^2 = 9.8\text{m/s}^2$$

Question:

Suppose you are flying in an airplane at an altitude of 35000ft~11km above the Earth's surface. What is the acceleration due to Earth's gravity?

$$F = \frac{GM_E m}{(R_E + h)^2} = ma$$

$$a = \frac{GM_E}{(R_E + h)^2} = \frac{6.67\times10^{-11}\cdot 5.98\times10^{24}}{((6.37+0.011)\times10^6)^2}\,\text{m/s}^2 = 9.796\,\text{m/s}^2$$

a/g = 0.997

Attraction of moon to the Earth:

$$F = \frac{GM_E M_M}{R_{EM}^2} = \frac{6.67\times10^{-11}\cdot 5.98\times10^{24}\cdot 7.36\times10^{22}}{(3.84\times10^8)^2}\,\text{N} = 1.99\times10^{20}\,\text{N}$$

Acceleration of moon toward the Earth:

$F = M_M a$ ➔ a = $1.99\text{x}20^{20}$ N/$7.36\text{x}10^{22}$kg =0.0027 m/s^2

TABLE 13.2 *Useful Planetary Data*

Body	Mass (kg)	Mean Radius (m)	Period of Revolution (s)	Mean Distance from the Sun (m)	$\frac{T^2}{r^3}$ (s^2/m^3)
Mercury	3.30×10^{23}	2.44×10^6	7.60×10^6	5.79×10^{10}	2.98×10^{-19}
Venus	4.87×10^{24}	6.05×10^6	1.94×10^7	1.08×10^{11}	2.99×10^{-19}
Earth	5.97×10^{24}	6.37×10^6	3.156×10^7	1.496×10^{11}	2.97×10^{-19}
Mars	6.42×10^{23}	3.39×10^6	5.94×10^7	2.28×10^{11}	2.98×10^{-19}
Jupiter	1.90×10^{27}	6.99×10^7	3.74×10^8	7.78×10^{11}	2.97×10^{-19}
Saturn	5.68×10^{26}	5.82×10^7	9.29×10^8	1.43×10^{12}	2.95×10^{-19}
Uranus	8.68×10^{25}	2.54×10^7	2.65×10^9	2.87×10^{12}	2.97×10^{-19}
Neptune	1.02×10^{26}	2.46×10^7	5.18×10^9	4.50×10^{12}	2.94×10^{-19}
Pluto[a]	1.25×10^{22}	1.20×10^6	7.82×10^9	5.91×10^{12}	2.96×10^{-19}
Moon	7.35×10^{22}	1.74×10^6	—	—	—
Sun	1.989×10^{30}	6.96×10^8	—	—	—

[a]In August 2006, the International Astronomical Union adopted a definition of a planet that separates Pluto from the other eight planets. Pluto is now defined as a "dwarf planet" like the asteroid Ceres.

Gravity on the surface of the moon

$$F = \frac{GmM_M}{R_M^2} = m\frac{6.67\times10^{-11}\cdot 7.35\times10^{22}}{(1.74\times10^6)^2}\,\mathrm{N} \equiv mg_M$$

$$g_M = \frac{6.67\times10^{-11}\cdot 7.35\times10^{22}}{(1.74\times10^6)^2} = 1.62\ \mathrm{m/s^2}$$

$$g_M \approx 0.165 g_E$$

Gravity on the surface of mars

$$g_{Mars} = \frac{GM_{Mars}}{R_{Mars}^2} = \frac{6.67\times10^{-11}\cdot 6.42\times10^{23}}{(3.39\times10^6)^2} = 3.73\ \mathrm{m/s^2}$$

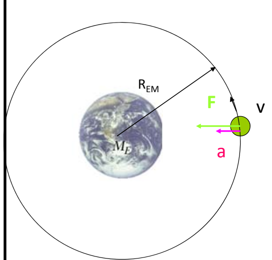
$$g_{Mars} \approx 0.380 g_E$$

iclicker question:

In estimating the gravitational acceleration near the surfaces of the Earth, Moon, or Mars, we used the full mass of the planet or moon, ignoring the shape of its distribution. This is a reasonable approximation because:

- **A. The special form of the gravitational force law makes this mathematically correct.**
- **B. Most of the mass of the planets/moon is actually concentrated near the center of the planet/moon.**
- **C. It is a very crude approximation.**

Stable circular orbit of two gravitationally attracted objects (such as the moon and the Earth)



$$a = \frac{v^2}{R_{EM}} = \frac{GM_E}{R_{EM}^2}$$

$$v = \omega R_{EM} = \frac{2\pi}{T}R_{EM}$$

$$T = 2\pi\sqrt{\frac{R_{EM}^3}{GM_E}}$$

$$= 2\pi\sqrt{\frac{(3.84\times10^8)^3}{6.67\times10^{-11}\cdot 5.98\times10^{24}}}$$

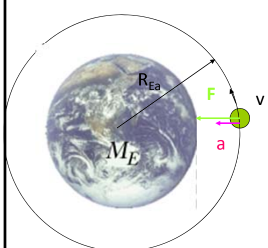
$$= 2367353.953\,\mathrm{s} = 27.4\ \mathrm{days}$$

iclicker question:

In the previous discussion, we saw how the moon orbits the Earth in a stable circular orbit because of the radial gravitational attraction of the moon and Newton's second law: F=ma, where a is the centripetal acceleration of the moon in its circular orbit. Is this the same mechanism which stabilizes airplane travel? Assume that a typical cruising altitude of an airplane is 11 km above the Earth's surface and that the Earth's radius is 6370 km.

(a) Yes (b) No

Stable (??) circular orbit of two gravitationally attracted objects (such as the airplane and the Earth)



$$a = \frac{v^2}{R_{Ea}} = \frac{GM_E}{R_{Ea}^2}$$

$$v = \omega R_{Ea} = \frac{2\pi}{T} R_{Ea}$$

$$T = 2\pi\sqrt{\frac{R_{Ea}^3}{GM_E}}$$

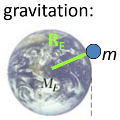
$$= 2\pi\sqrt{\frac{(6.38\times10^6)^3}{6.67\times10^{-11}\cdot 5.98\times10^{24}}}$$

$$= 5070\text{ s} = 1.4\text{ hours}$$

$$\Rightarrow \text{v} = 7.9\text{ km/s} \approx 18000\text{ mi/h!!}$$

Newton's law of gravitation: $\mathbf{F}_{12} = \frac{Gm_1m_2\hat{\mathbf{r}}_{12}}{r_{12}^2}$

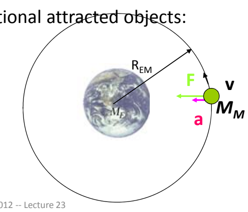
Earth's gravity:



$$F = \frac{GM_E m}{R_E^2}$$

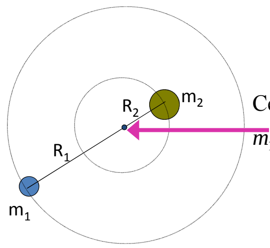
$$\Rightarrow g = \frac{GM_E}{R_E^2} = \frac{6.67\times10^{-11}\cdot 5.98\times10^{24}}{(6.37\times10^6)^2}\text{ m/s}^2 = 9.8\text{m/s}^2$$

Stable circular orbits of gravitational attracted objects:

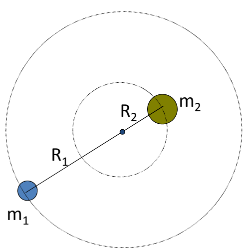


More details

If we examine the circular orbit more carefully, we find that the correct analysis is that the stable circular orbit of two gravitationally attracted masses is about their center of mass.



Center of mass: $m_1 R_1 = m_2 R_2$



Radial forces on m_1:

$$F_{r1} = \frac{G m_1 m_2}{(R_1 + R_2)^2} = m_1 a_{r1} = m_1 \frac{v_1^2}{R_1}$$

$$v_1 = \frac{2\pi R_1}{T_1}$$

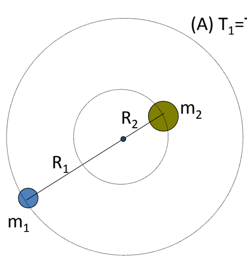
$$T_1 = 2\pi \sqrt{\frac{R_1 (R_1 + R_2)^2}{G m_2}}$$

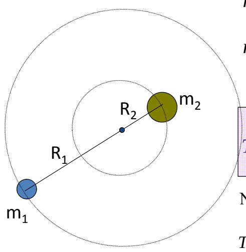
T_2 ?

iclicker question:

What is the relationship between the periods T_1 and T_2 of the two gravitationally attracted objects rotating about their center of mass? (Assume that $m_1 < m_2$.)

(A) $T_1=T_2$ (B) $T_1<T_2$ (C) $T_1>T_2$





$$m_1R_1 = m_2R_2$$

$$m_1\frac{v_1^2}{R_1} = m_1R_1\omega_1^2 = \frac{Gm_1m_2}{(R_1+R_2)^2} = m_2R_2\omega_2^2$$

$$\Rightarrow \omega_1 = \omega_2$$

$$T_1 = T_2 = 2\pi\sqrt{\frac{(R_1+R_2)^3}{G(m_1+m_2)}}$$

Note that :

$$T_1 = T_2 = 2\pi\sqrt{\frac{(R_1+R_2)^3}{G(m_1+m_2)}}$$

$$= 2\pi\sqrt{\frac{R_1(R_1+R_2)^2}{Gm_2}} = 2\pi\sqrt{\frac{R_2(R_1+R_2)^2}{Gm_1}}$$

$$T = 2\pi\sqrt{\frac{(R_1+R_2)^3}{G(m_1+m_2)}} = 2\pi\sqrt{\frac{R_1(R_1+R_2)^2}{Gm_2}} = 2\pi\sqrt{\frac{R_2(R_1+R_2)^2}{Gm_1}}$$

iclicker questions:

How is it possible that all of these relations are equal?

A. Magic.
B. Trick.
C. Algebra.

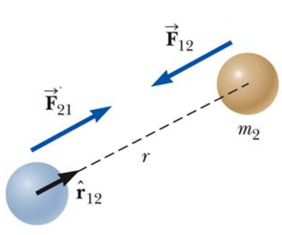
$$T = 2\pi\sqrt{\frac{(R_1+R_2)^3}{G(m_1+m_2)}} = 2\pi\sqrt{\frac{R_1\left(1+\frac{R_2}{R_1}\right)(R_1+R_2)^2}{Gm_2\left(1+\frac{m_1}{m_2}\right)}} = 2\pi\sqrt{\frac{R_1(R_1+R_2)^2}{Gm_2}}$$

because $\left(1+\frac{R_2}{R_1}\right) = \left(1+\frac{m_1}{m_2}\right)$ since $m_1R_1 = m_2R_2$

What is the physical basis for stable circular orbits?

1. Newton's second law?
2. Conservation of angular momentum? **L** = (const)

Note: Gravitational forces exert no torque



$$\mathbf{F}_{12} = \frac{Gm_1m_2\hat{\mathbf{r}}_{12}}{r_{12}^2}$$

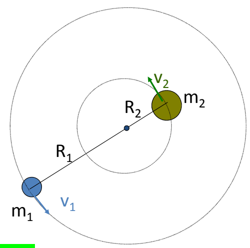
$$\boldsymbol{\tau}_{12} = \mathbf{r}_{12}\times\mathbf{F}_{12} = 0$$

$$\Rightarrow \frac{d\mathbf{L}}{dt} = 0$$

$$\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt} = 0$$
$$\Rightarrow \mathbf{L} = \text{(const)}$$

$L_1 = m_1 v_1 R_1$

$L_2 = m_2 v_2 R_2$



Question:

How are the magnitudes of L_1 and L_2 related?

$$\frac{L_1}{R_1} = \frac{L_2}{R_2}$$

Note: More generally, stable orbits can be elliptical.

Satellites orbiting earth (approximately circular orbits):

$$T = 2\pi\sqrt{\frac{R_E^3}{GM_E}}(1+h/R_E)^{3/2} = 5058(1+h/R_E)^{3/2}\, s$$

R_E ~ 6370 km

Examples:

Satellite	h (km)	T (hours)	v (mi/h)
Geosynchronous	35790	~24	6900
NOAA polar orbitor	800	~1.7	16700
Hubble	600	~1.6	16900
Inter. space station*	390	~1.5	17200

Planets in our solar system – orbiting the sun

Planet	Mass (kg)	Distance to Sun (m)	Period of orbit (years)
Mercury	3.30×10^{23}	5.79×10^{10}	0.24
Venus	4.87×10^{24}	1.08×10^{11}	0.61
Earth	5.97×10^{24}	1.496×10^{11}	1.00
Mars	6.42×10^{23}	2.28×10^{11}	1.88
Jupter	1.90×10^{27}	7.78×10^{11}	11.85
Saturn	5.68×10^{26}	1.43×10^{12}	29.43