PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 24:

Chapter 13 – Fundamental force of gravity

1. **Form of gravitation force law**
2. **Gravitational potential energy**
3. **Conservation of energy**
4. **Energy associated with orbital motion**
5. **Note: We will not focus attention on elliptical orbits, black holes, and dark matter**

14	10/03/2012	Momentum and collisions	9.5-9.9	9.29,9.37	10/05/2012
	10/05/2012	Review	6-9		
	10/08/2012	Exam	6-9		
15	10/10/2012	Rotational motion	10.1-10.5	10.6, 10.13, 10.25	10/12/2012
16	10/12/2012	Torque	10.6-10.9	10.37, 10.55	10/15/2012
17	10/15/2012	Angular momentum	11.1-11.5	11.11, 11.34	10/17/2012
18	10/17/2012	Equilibrium	12.1-12.4	12.11, 12.39	10/22/2012
	10/19/2012	*Fall Break*			
19	10/22/2012	Simple harmonic motion	15.1-15.3	15.4, 15.20	10/24/2012
20	10/24/2012	Resonance	15.4-15.7	15.43, 15.43, 15.52	10/26/2012
21	10/26/2012	Gravitational force	13.1-13.3	13.6, 13.10, 13.13	10/29/2012
22	10/29/2012	Kepler's laws and satellite motion	13.4-13.6	13.28, 13.34	10/31/2012
	10/31/2012	Review	10-13,15		
	11/02/2012	Exam	10-13,15		
23	11/05/2012	Fluid mechanics	14.1-14.4		11/07/2012
24	11/07/2012	Fluid mechanics	14.5-14.7		11/09/2012
25	11/09/2012	Temperature	19.1-19.5		11/12/2012

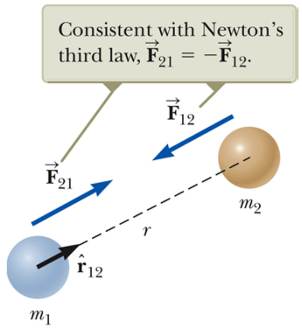
iclicker question:
Concerning prepartion for Friday's exam –

- **A. I'm good (OK with studying on my own and with the help of the already scheduled tutorials)**
- **B. I would likely attend an extra class reviewing session (probably Tuesday or Thursday)**
- **C. I would like to make an individual appointment to go over my specific questions**
- **D. None of these.**

iclicker followup question:
Concerning extra review session –

- **A. I could attend on Tuesday afternoon**
- **B. I could attend on Thursday afternoon**

Review: Universal law of gravitation
➔ Newton (with help from Galileo, Kepler, etc.) 1687

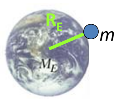


$$\mathbf{F}_{12} = \frac{G m_1 m_2 \hat{\mathbf{r}}_{12}}{r_{12}^2}$$

$$G = 6.674 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$$

10/29/2012 PHY 113 A Fall 2012 -- Lecture 24 4

Review: Gravitational force of the Earth



$$F = \frac{G M_E m}{R_E^2}$$

$$\Rightarrow g = \frac{G M_E}{R_E^2} = \frac{6.67 \times 10^{-11} \cdot 5.98 \times 10^{24}}{(6.37 \times 10^6)^2} \mathrm{m/s^2} = 9.8 \mathrm{m/s^2}$$

Note: Earth's gravity acts as a point mass located at the Earth's center.

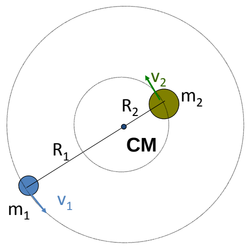
10/29/2012 PHY 113 A Fall 2012 -- Lecture 24 5

TABLE 13.2 *Useful Planetary Data*

Body	Mass (kg)	Mean Radius (m)	Period of Revolution (s)	Mean Distance from the Sun (m)
Mercury	3.30×10^{23}	2.44×10^{6}	7.60×10^{6}	5.79×10^{10}
Venus	4.87×10^{24}	6.05×10^{6}	1.94×10^{7}	1.08×10^{11}
Earth	5.97×10^{24}	6.37×10^{6}	3.156×10^{7}	1.496×10^{11}
Mars	6.42×10^{23}	3.39×10^{6}	5.94×10^{7}	2.28×10^{11}
Jupiter	1.90×10^{27}	6.99×10^{7}	3.74×10^{8}	7.78×10^{11}
Saturn	5.68×10^{26}	5.82×10^{7}	9.29×10^{8}	1.43×10^{12}
Uranus	8.68×10^{25}	2.54×10^{7}	2.65×10^{9}	2.87×10^{12}
Neptune	1.02×10^{26}	2.46×10^{7}	5.18×10^{9}	4.50×10^{12}
Pluto[a]	1.25×10^{22}	1.20×10^{6}	7.82×10^{9}	5.91×10^{12}
Moon	7.35×10^{22}	1.74×10^{6}	—	—
Sun	1.989×10^{30}	6.96×10^{8}	—	—

10/29/2012 PHY 113 A Fall 2012 -- Lecture 24 6

Review: Circular orbital motion about center of mass



$$m_1 \frac{v_1^2}{R_1} = \frac{G m_1 m_2}{(R_1+R_2)^2} = m_2 \frac{v_2^2}{R_2}$$

$$m_1 R_1 = m_2 R_2$$

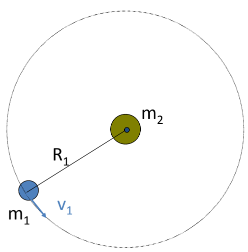
$$m_1 \frac{v_1^2}{R_1} = m_1 \left(\frac{2\pi R_1}{T_1}\right)^2 \frac{1}{R_1} = m_1 R_1 \left(\frac{2\pi}{T_1}\right)^2$$

$$T_1 = T_2 = 2\pi \sqrt{\frac{(R_1+R_2)^3}{G(m_1+m_2)}}$$

Note that if $m_2 >> m_1$ then $R_2 << R_1$

$$T_1 = T_2 = 2\pi \sqrt{\frac{(R_1+R_2)^3}{G(m_1+m_2)}} \approx 2\pi \sqrt{\frac{R_1^3}{G m_2}}$$

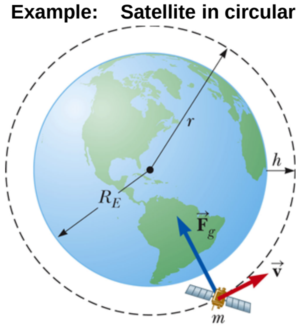
Review: Circular orbital motion about center of mass



Note that if $m_2 >> m_1$ then $R_2 << R_1$

$$T_1 = T_2 = 2\pi \sqrt{\frac{(R_1+R_2)^3}{G(m_1+m_2)}} \approx 2\pi \sqrt{\frac{R_1^3}{G m_2}}$$

Example: Satellite in circular Earth orbit



$$T = 2\pi \sqrt{\frac{(R_E+h)^3}{G M_E}}$$

Note: $M_E = 5.97 \times 10^{24} kg$

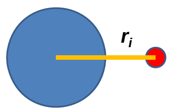
$m_S \approx 10^3 kg$

If $h = 35.83 \times 10^6 m$

$T = 8.53 \times 10^4 s = 1 \, day$

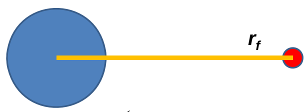
(geosynchronous)

Work of gravity:



$$W_{i \to f} = \int_{r_i}^{r_f} \mathbf{F} \cdot d\mathbf{r} \qquad \mathbf{F} = -\frac{Gm_1 m_2 \hat{\mathbf{r}}}{r^2}$$

$$W_{i \to f} = -\int_{r_i}^{r_f} \frac{Gm_1 m_2}{r^2} \, dr = \frac{Gm_1 m_2}{r} \Big|_{r_i}^{r_f}$$



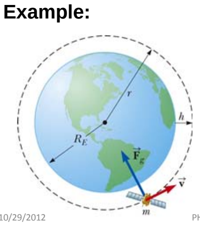
$$W_{i \to f} = -\left(-\frac{Gm_1 m_2}{r_f} - \left(-\frac{Gm_1 m_2}{r_i} \right) \right) \equiv -\left(U(r_f) - U(r_i) \right)$$

Gravitational potential energy

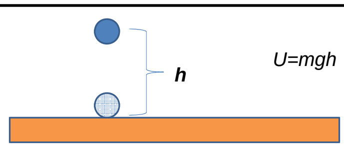
$$U_{gravity}(r) = -\int_{r_{ref}}^{r} \mathbf{F} \cdot d\mathbf{r} \qquad \mathbf{F} = -\frac{Gm_1 m_2 \hat{\mathbf{r}}}{r^2}$$

$$U_{gravity}(r) = -\int_{\infty}^{r} \frac{-Gm_1 m_2}{r'^2} \, dr' = -\frac{Gm_1 m_2}{r}$$

Example:



$$U_{gravity}(r = R_E + h) = -\frac{GM_E m_S}{R_E + h}$$

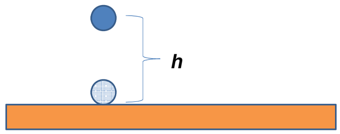


U=mgh

iclicker exercise:

We previously have said that the gravitational potential of an object of mass *m* at a height *h* is *U=mgh*. How is this related to $U_{gravity} = -\frac{GM_E m_S}{R_E + h}$?

A. No relation
B. They are approximately equal
C. They are exactly equal



$$\frac{1}{R_E+h}=\frac{1}{R_E}\frac{1}{(1+h/R_E)}\approx\frac{1}{R_E}(1-h/R_E+\cdots)=\frac{1}{R_E}-\frac{h}{R_E{}^2}+\cdots$$

$$U_{gravity}(R_E+h)-U_{gravity}(R_E)=-\frac{GM_Em}{R_E+h}-\left(-\frac{GM_Em}{R_E}\right)\approx\underbrace{\frac{GM_Em}{R_E{}^2}}_{g}h$$

➔ **Near the surface of the Earth *U=mgh* is a good approximation to the gravitation potential.**

iclicker exercise:

How much energy kinetic energy must be provided to an object of mass *m*=1000kg, initially on the Earth's surface to outer space?

A. This is rocket science and not a fair question.

B. It is not possible to escape the Earth's gravitational field.

C. We can estimate the energy by simple conservation of energy concepts.

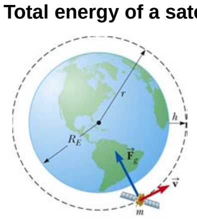
$$E=K_i+U_i=K_f+U_f=0$$

$$U_i=-\frac{GM_Em}{R_E}=-K_i$$

$$\Rightarrow K_i=\frac{GM_Em}{R_E}=\frac{6.674\times10^{-11}\cdot5.97\times10^{24}\cdot1000}{6.37\times10^{6}}J$$

$$=6.25\times10^{10}J$$

Total energy of a satellite in a circular Earth orbit



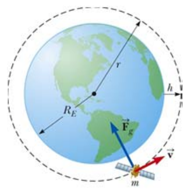
$$U_{gravity}(r=R_E+h)=-\frac{GM_Em_S}{R_E+h}$$

$$E=K+U_{gravity}\qquad K=\frac{1}{2}m_Sv^2$$

$$\text{For a circular orbit: }\quad m_S\frac{v^2}{R_E+h}=\frac{GM_Em_S}{(R_E+h)^2}$$

$$\Rightarrow K=\frac{1}{2}m_Sv^2=\frac{GM_Em_S}{2(R_E+h)}$$

Total energy of a satellite in a circular Earth orbit



$$E = K + U_{gravity}$$

$$K = \frac{GM_E m_S}{2(R_E + h)} \qquad U_{gravity} = -\frac{GM_E m_S}{(R_E + h)}$$

$$E = -\frac{GM_E m_S}{2(R_E + h)}$$

iclicker question:

Compared to the energy needed to escape the Earth's gravitational field does it take

A. more

B. less

energy to launch a satellite to orbit the Earth?

$$E = K_i + U_i = K_f + U_f = -\frac{GM_E m}{2(R_E + h)}$$

$$U_i = -\frac{GM_E m}{R_E}$$

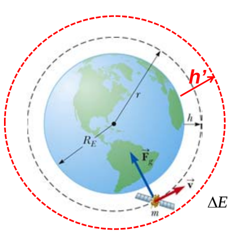
$$\Rightarrow K_i = \frac{GM_E m}{R_E} - \frac{GM_E m}{2(R_E + h)} < \frac{GM_E m}{R_E}$$

For example, $h = 560km$ (low earth orbit such as Hubble Telescope)

$$K_i = \frac{6.674\times10^{-11}\cdot 5.97\times10^{24}\cdot 1000}{6.37\times10^{6}} J - \frac{6.674\times10^{-11}\cdot 5.97\times10^{24}\cdot 1000}{2\cdot 6.93\times10^{6}} J$$

$$= 6.25\times10^{10} J\left(1 - \frac{6.37}{2\cdot 6.93}\right) = 3.38\times10^{10} J$$

Energy involved with changing orbits



$$\Delta E = E' - E = -\frac{GM_E m}{2(R_E + h')} - \left(-\frac{GM_E m}{2(R_E + h)}\right)$$

For example, $h = 560km$ and $h' = 600km$

$$\Delta E = -\frac{6.674\times10^{-11}\cdot 5.97\times10^{24}\cdot 1000}{2\times10^{6}}\left(\frac{1}{6.93} - \frac{1}{6.97}\right)$$

$$= 1.65\times10^{8} J$$

From Webassign:

5. –/0.333 points | My Notes | SerPSE8 13.P.013.

An artificial satellite circles the Earth in a circular orbit at a location where the acceleration due to gravity is 5.98 m/s^2. Determine the orbital period of the satellite.

____ min

3. –/0.5 points | My Notes | SerPSE8 13.P.034.MI.

A space probe is fired as a projectile from the Earth's surface with an initial speed of 2.02×10^4 m/s. What will its speed be when it is very far from the Earth? Ignore atmospheric friction and the rotation of the Earth.

____ m/s

4. –/0 points | My Notes | SerPSE8 13.P.039.WI.

A satellite of mass 180 kg is placed into Earth orbit at a height of 650 km above the surface.

(a) Assuming a circular orbit, how long does the satellite take to complete one orbit?

____ h

(b) What is the satellite's speed?

____ m/s

(c) Starting from the satellite on the Earth's surface, what is the minimum energy input necessary to place this satellite in orbit? Ignore air resistance but include the effect of the planet's daily rotation.

____ J