PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 25:

Review: Chapters 10-13, 15

1. Advice on how to prepare for exam
2. Review of rotational motion, angular momentum, static equilibrium, simple harmonic motion, universal gravitational force law

14	10/03/2012	Momentum and collisions	9.5-9.9	9.29,9.37	10/05/2012
	10/05/2012	Review	6-9		
	10/08/2012	Exam	6-9		
15	10/10/2012	Rotational motion	10.1-10.5	10.6, 10.13, 10.25	10/12/2012
16	10/12/2012	Torque	10.6-10.9	10.37, 10.55	10/15/2012
17	10/15/2012	Angular momentum	11.1-11.5	11.11, 11.34	10/17/2012
18	10/17/2012	Equilibrium	12.1-12.4	12.11, 12.39	10/22/2012
	10/19/2012	*Fall Break*			
19	10/22/2012	Simple harmonic motion	15.1-15.3	15.4, 15.20	10/24/2012
20	10/24/2012	Resonance	15.4-15.7	15.43, 15.43, 15.52	10/26/2012
21	10/26/2012	Gravitational force	13.1-13.3	13.6, 13.10, 13.13	10/29/2012
22	10/29/2012	Kepler's laws and satellite motion	13.4-13.6	13.28, 13.34	10/31/2012
	10/31/2012	Review	10-13,15		
	11/02/2012	Exam	10-13,15		
23	11/05/2012	Fluid mechanics	14.1-14.4		11/07/2012
24	11/07/2012	Fluid mechanics	14.5-14.7		11/09/2012
25	11/09/2012	Temperature	19.1-19.5		11/12/2012

Format of Friday's exam

What to bring:

1. **Clear, calm head**
2. **Equation sheet (turn in with exam)**
3. **Scientific calculator**
4. **Pencil or pen**

(Note: labtops, cellphones, and other electronic equipment must be off or in sleep mode.)

Timing:

May begin as early as 8 AM; must end ≤ 9:50 AM

Probable exam format

- **4-5 problems similar to homework and class examples; focus on Chapters 10-13 & 15 of your text.**
- **Full credit awarded on basis of analysis steps as well as final answer**

Examples of what to include on equation sheet

Given information on exam	Suitable for equation sheet
Universal or common constants (such as g, G, M_E, M_S, R_E…)	Basic equations from material from earlier Chapters: Newton's laws, energy, momentum, center of mass
Particular constants (such as *k,m,I…*)	Simple derivative and integral relationships, including trigonometric functions
Unit conversion factors such as Hz and rad/s	Definition of moment of inertia, torque, angular momentum, rotational kinetic energy
	Newton's law for rotational motion; combination of rotational and center of mass motion
	Equations describing simple harmonic motion and driven harmonic motion
	Newton's universal gravitation force law and corresponding gravitational potential energy
	Gravitational stable circular orbits

Possible extra review session on Thursday:

iclicker question:

Which of the following possible times would work with your schedules (vote for one)?

A. **2 PM**
B. **3 PM**
C. **4 PM**
D. **Prefer to meet individually or in small groups in my office (Olin 300).**

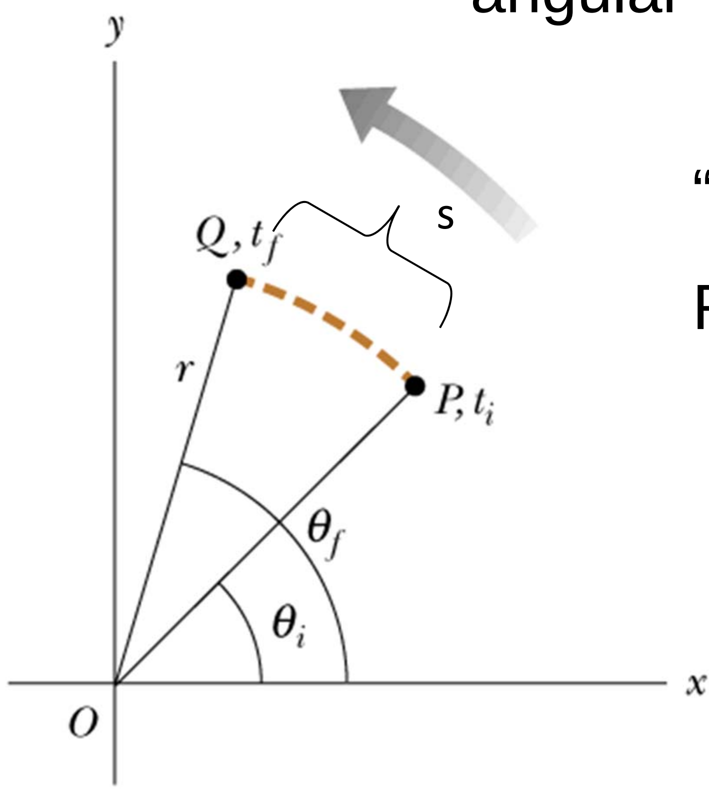
Rotations: Angular variables

angular "displacement" → $\theta(t)$

angular "velocity" → $\omega(t) = \frac{d\theta}{dt}$

angular "acceleration" → $\alpha(t) = \frac{d\omega}{dt}$

Serway, Physics for Scientists and Engineers, 5/e
Figure 10.2



"natural" angular unit = radian

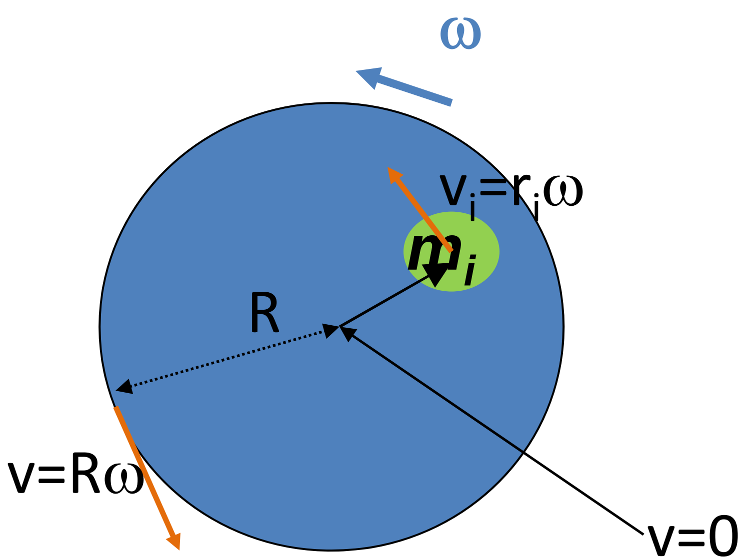
Relation to linear variables:

$$s_\theta = r\,(\theta_f - \theta_i)$$

$$v_\theta = r\,\omega$$

$$a_\theta = r\,\alpha$$

Object rotating with constant angular velocity ($\alpha = 0$)



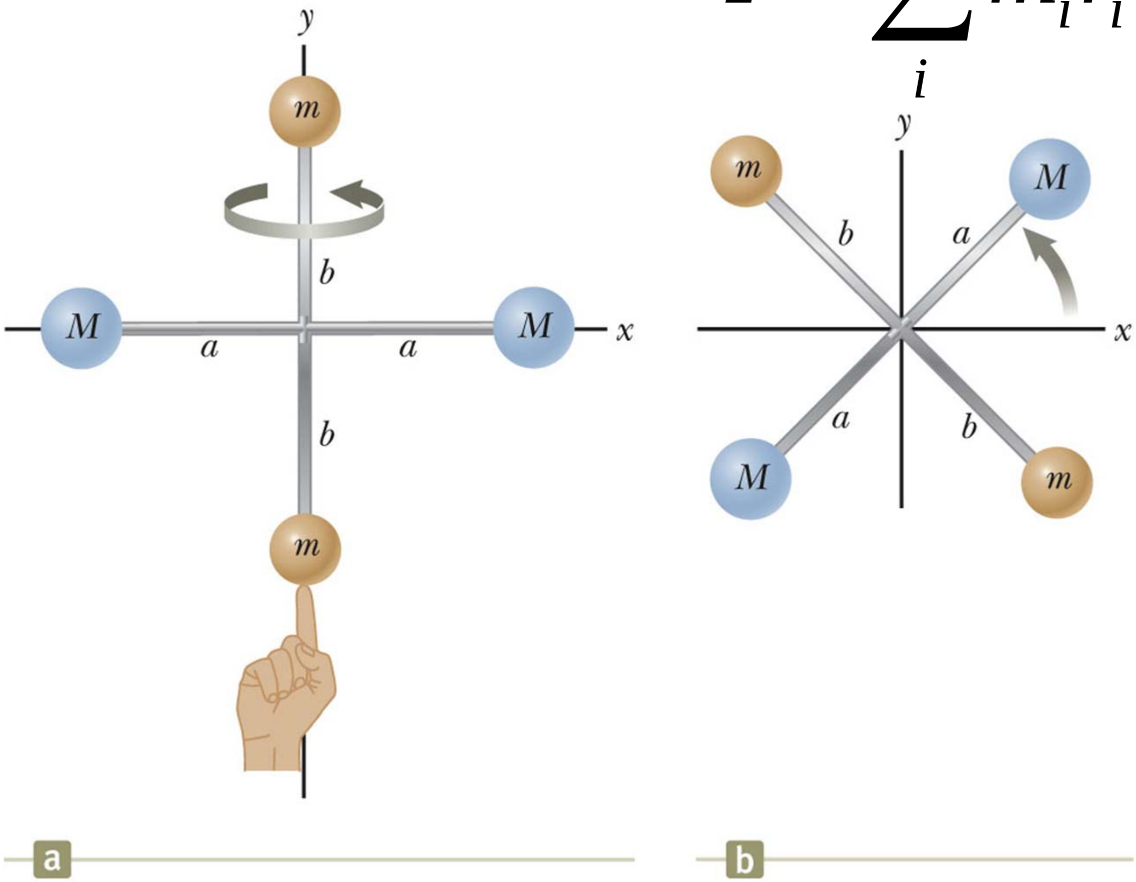
Kinetic energy associated with rotation:

$$K = \sum_i \tfrac{1}{2} m_i v_i^2 = \sum_i \tfrac{1}{2} m_i r_i^2 \omega^2 \equiv \tfrac{1}{2} I\omega^2;$$

$$\text{where}: I \equiv \sum_i m_i r_i^2$$ "moment of inertia"

Moment of inertia:

$$I \equiv \sum_i m_i r_i^2$$



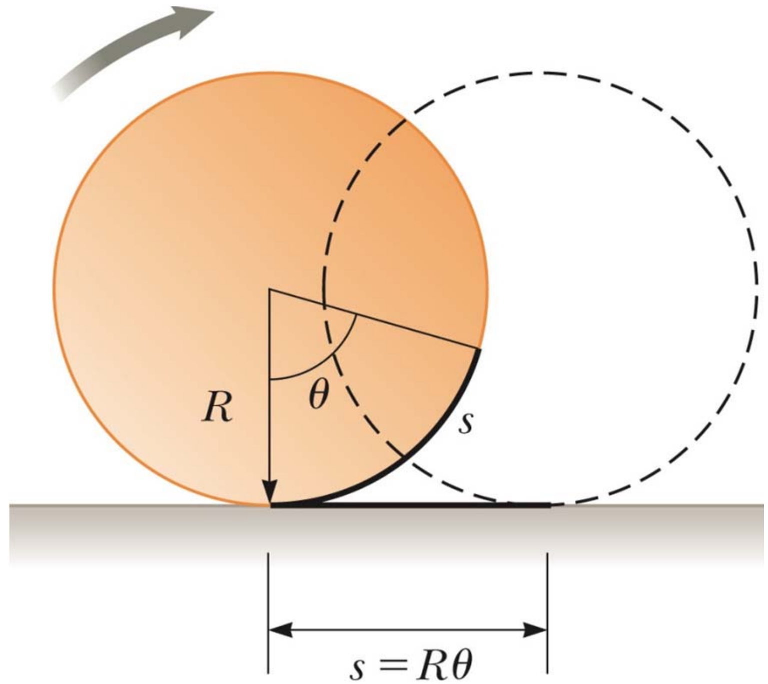
$$I = 2Ma^2$$

$$I = 2Ma^2 + 2mb^2$$

Total kinetic energy of rolling object :

$$K_{total} = K_{rolling} + K_{CM}$$

$$= \frac{1}{2} I\omega^2 + \frac{1}{2} M v_{CM}{}^2$$



Note that :

$$\omega = \frac{d\theta}{dt}$$

$$\frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega = v_{CM}$$

$$K_{total} = K_{rolling} + K_{CM}$$

$$= \frac{1}{2}\frac{I}{R^2}(R\omega)^2 + \frac{1}{2} M v_{CM}{}^2$$

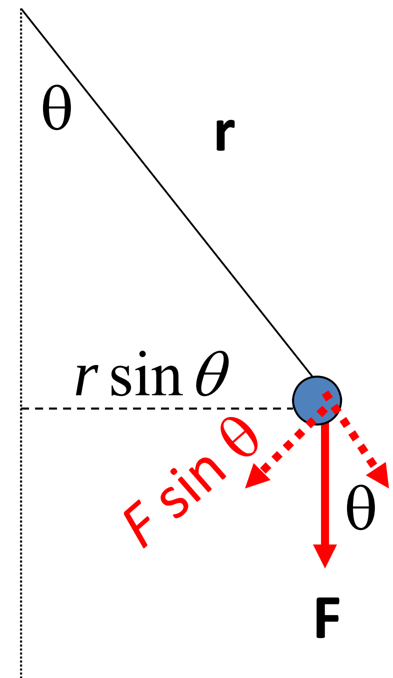
$$= \frac{1}{2}\left(\frac{I}{R^2} + M\right) v_{CM}{}^2$$

How to make objects rotate.

Define torque:

$\boldsymbol{\tau}$ = **r** x **F**

τ = rF sin θ

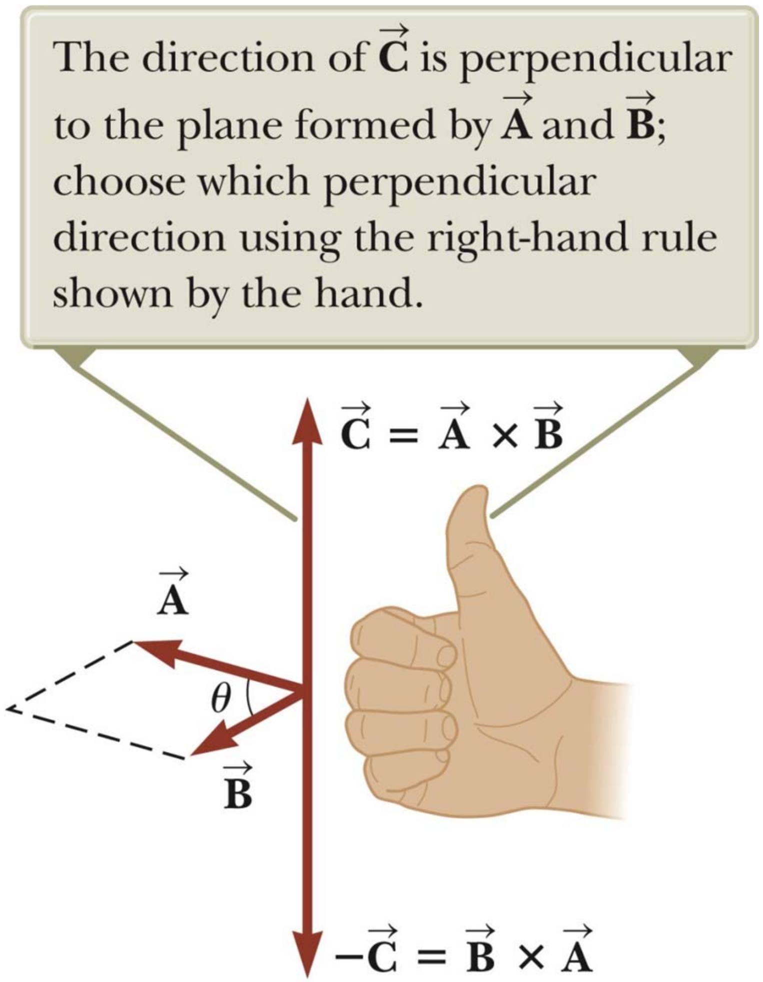


$$\mathbf{F} = m\mathbf{a}$$

$$\mathbf{r} \times \mathbf{F} \equiv \boldsymbol{\tau} = \mathbf{r} \times m\mathbf{a} = I\boldsymbol{\alpha}$$

Note: We will define and use the "vector cross product" next time. For now, we focus on the fact that the direction of the torque determines the direction of rotation.

Vector cross product; right hand rule



$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

$$|\mathbf{C}| = |\mathbf{A}||\mathbf{B}|\sin\theta$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$$

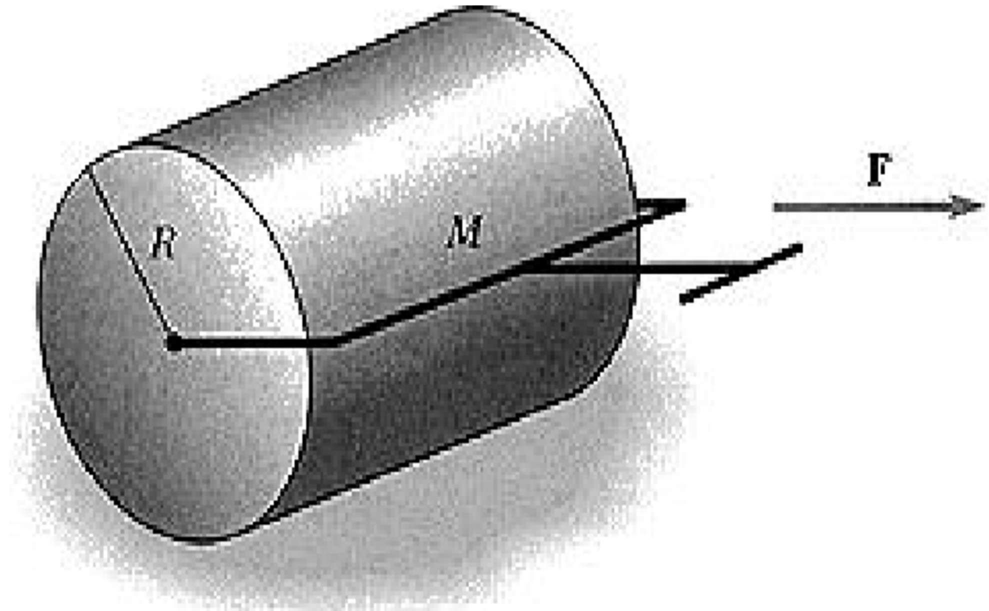
$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$

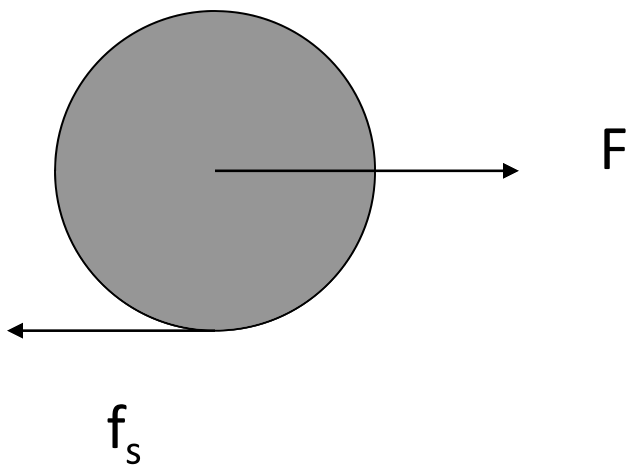
$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$$

Note that rolling motion is caused by the torque of friction:

Newton's law for torque:

$$\boldsymbol{\tau}_{total} = I\frac{d\boldsymbol{\omega}}{dt} = I\boldsymbol{\alpha}$$





$$F - f_s = Ma_{CM}$$

$$f_s R = I\alpha = Ia_{CM}/R \qquad \Rightarrow a_{CM} = \frac{f_s R^2}{I}$$

$$f_s = F\left(\frac{1}{1+\left(MR^2\right)/I}\right)$$

$$\text{For a solid cylinder,} \quad I = \tfrac{1}{2}MR^2 \qquad \Rightarrow f_s = \tfrac{1}{3}F$$

Torque and Newton's second law:

$$\mathbf{F} = m\mathbf{a}$$

$$\mathbf{r} \times \mathbf{F} \equiv \boldsymbol{\tau} = \mathbf{r} \times m\mathbf{a} = \mathbf{r} \times m\frac{d\mathbf{v}}{dt} = \mathbf{r} \times \frac{d(m\mathbf{v})}{dt} = \frac{d}{dt}(\mathbf{r} \times \mathbf{p})$$

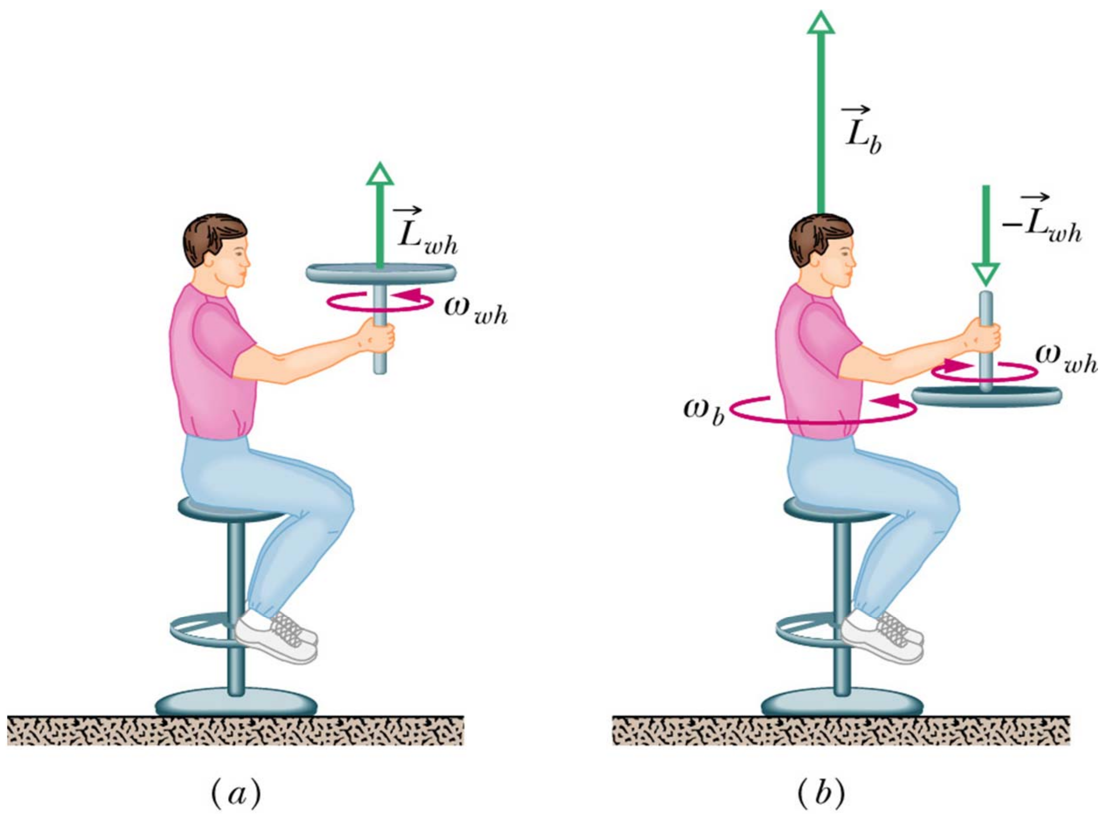
$$\mathbf{r} \times \mathbf{F} = \boldsymbol{\tau} = \frac{d}{dt}(\mathbf{r} \times \mathbf{p})$$

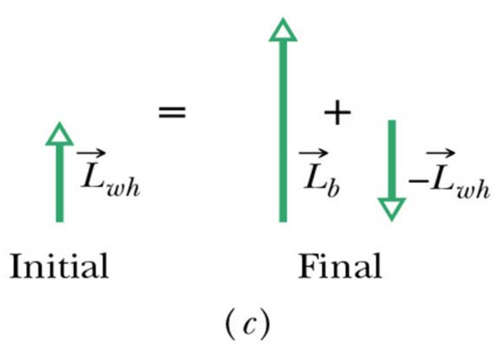
$$\text{Define:} \qquad \mathbf{L} \equiv \mathbf{r} \times \mathbf{p}$$

$$\text{Note that: if} \quad \boldsymbol{\tau} = 0 \qquad\qquad \frac{d\mathbf{L}}{dt} = 0$$

$$\text{Then:} \qquad\qquad \mathbf{L} = (\text{constant})$$

Example of conservation of angular momentum:



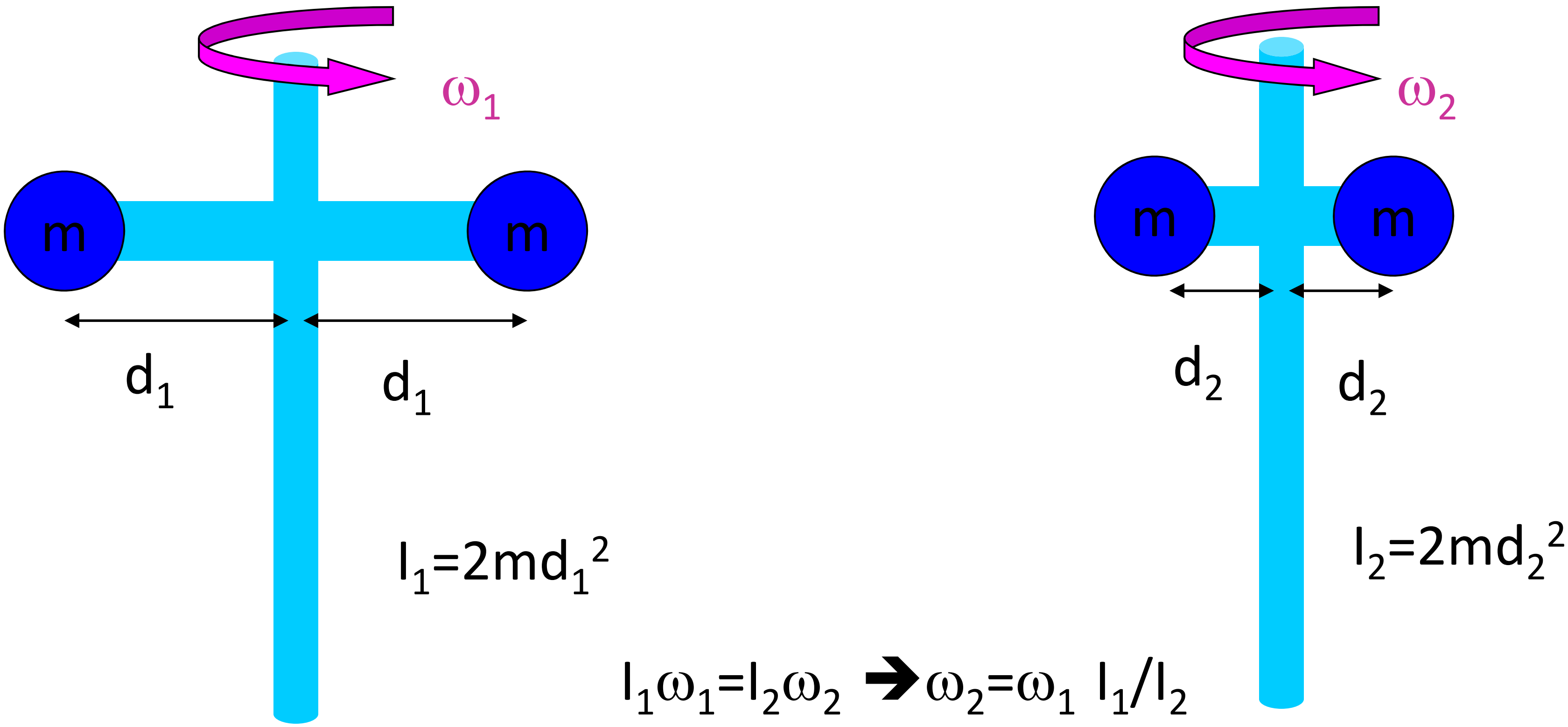


$$L_{bf} + L_{wheelf} = L_{bi} + L_{wheeli}$$

$$L_{bf} - L_{wheel} = 0 + L_{wheel}$$

$$L_{bf} = 2L_{wheel}$$

Another example of conservation of angular momentum



$I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \omega_1\, I_1/I_2$

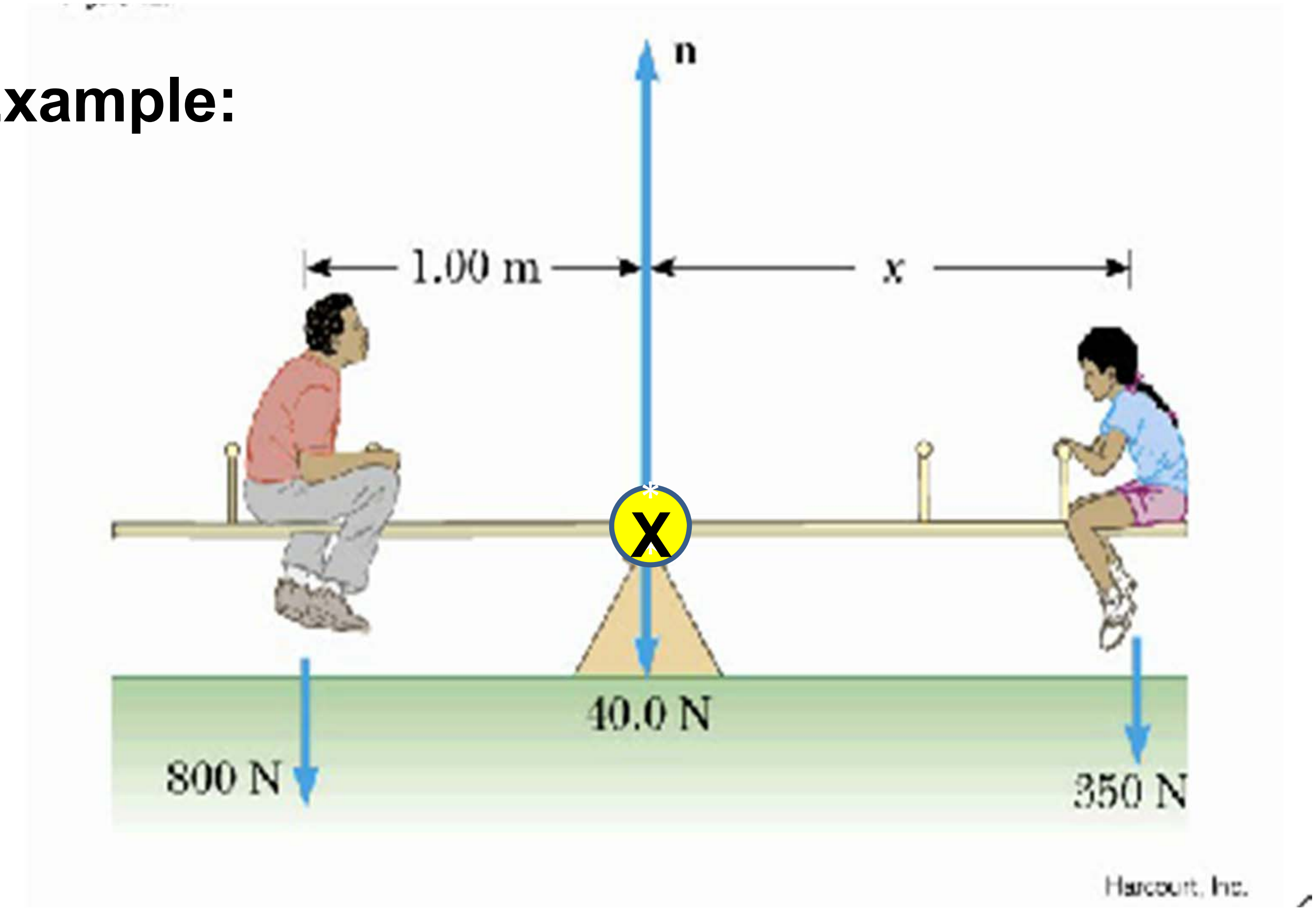
$\Rightarrow \omega_2 = \omega_1\, (d_1/d_2)^2$

Conditions for stable equilibrium

$$\text{Balance of force:} \quad \sum_i \mathbf{F}_i = 0$$

$$\text{Balance of torque:} \quad \sum_i \boldsymbol{\tau}_i = 0$$

Example:



$$\text{Forces}: \quad n - M_D g - m_c g - m_P g = 0$$

$$\text{Torques}: \quad M_D g(1m) - m_c g x = 0$$

Simple harmonic motion:

Newton's law for mass-spring system: $\qquad ma = m\frac{d^2x}{dt^2} = F = -kx$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Guess that solution for $x(t)$ has the form:

$x(t) = A\cos(\omega t + \varphi)$ where A and φ and ω are unknown constants

Condition that guess satisfies the equation:

$$\frac{d^2\left[A\cos(\omega t + \varphi)\right]}{dt^2} = -\omega^2\left[A\cos(\omega t + \varphi)\right] = -\frac{k}{m}\left[A\cos(\omega t + \varphi)\right]$$

$\Rightarrow \omega^2 = \frac{k}{m}$ (determines ω -- "natural frequency")

Summary --

Simple harmonic motion:

$$F = -kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Conveniently evaluated in *radians*

$$x(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{k}{m}}$$

Constants

Note that:

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi)$$

$$a(t) = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \varphi)$$

Energy associated with simple harmonic motion

Energy:

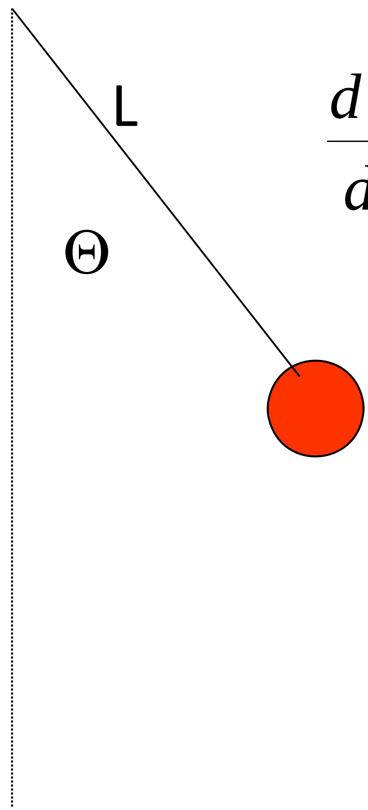
$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}m\omega^2 A^2\left(\sin(\omega t + \varphi)\right)^2 + \frac{1}{2}kA^2\left(\cos(\omega t + \varphi)\right)^2$$

But $\omega^2 = \dfrac{k}{m}$

$$\Rightarrow E = \frac{1}{2}kA^2\left[\left(\sin(\omega t + \varphi)\right)^2 + \left(\cos(\omega t + \varphi)\right)^2\right] = \frac{1}{2}kA^2$$

Simple harmonic motion for a pendulum:



$$\tau = mgL\sin\Theta = -I\alpha = -I\frac{d^2\Theta}{dt^2}$$

$$\frac{d^2\Theta}{dt^2} = -\frac{mgL}{I}\sin\Theta = -\frac{g}{L}\sin\Theta \quad (\text{since } I = mL^2)$$

Approximation for small Θ:

$$\sin\Theta \approx \Theta$$

$$\Rightarrow \frac{d^2\Theta}{dt^2} = -\frac{g}{L}\Theta$$

Solution :

$$\Theta(t) = A\cos(\omega t + \varphi); \quad \omega = \sqrt{\frac{\mathrm{g}}{\mathrm{L}}}$$

The notion of resonance:

Suppose $F=-kx+F_0 \sin(\Omega t)$

According to Newton:

$$-kx + F_0 \sin(\Omega t) = m\frac{d^2x}{dt^2}$$

Differential equation ("inhomogeneous"):

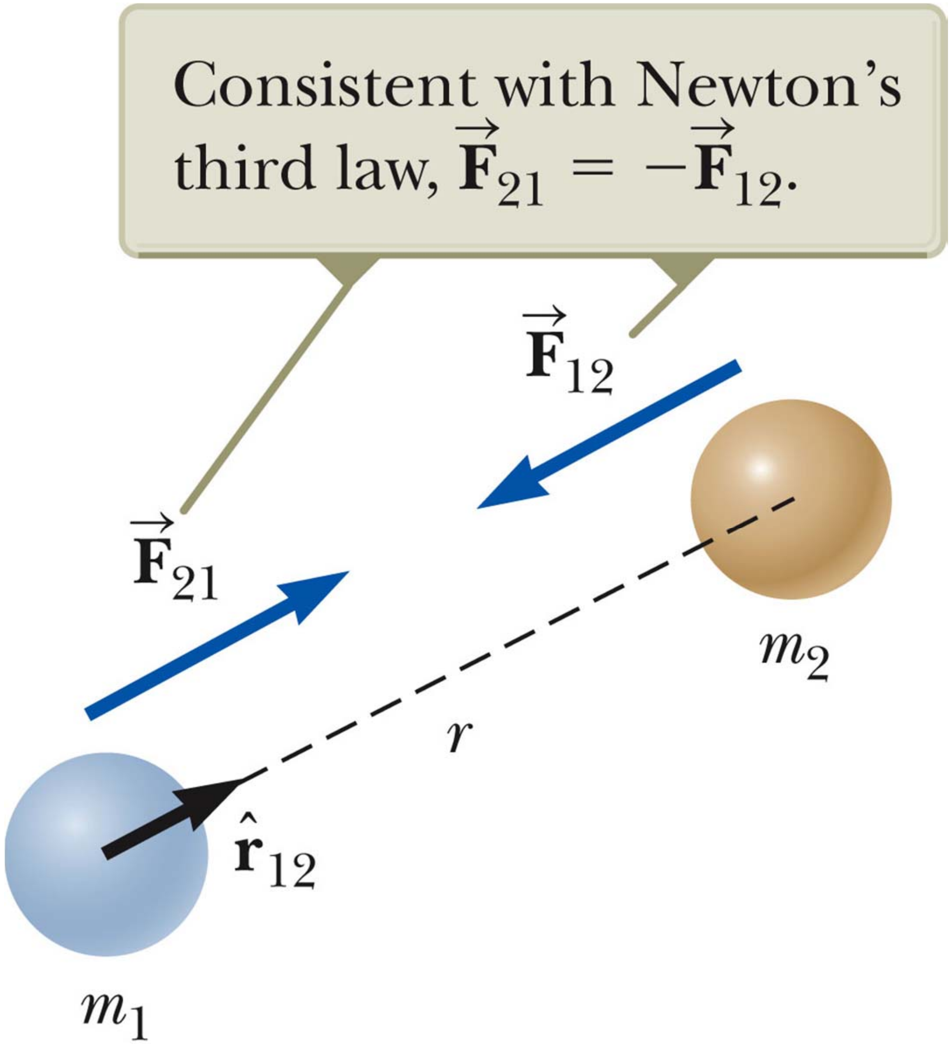
$$\frac{d^2x}{dt^2} = -\frac{k}{m}x + \frac{F_0}{m}\sin(\Omega t)$$

Solution:

$$x(t) = \frac{F_0/m}{k/m - \Omega^2}\sin(\Omega t) \equiv \frac{F_0/m}{\omega^2 - \Omega^2}\sin(\Omega t)$$

Universal law of gravitation

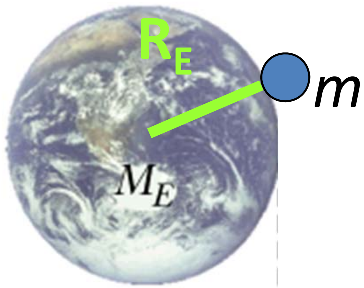
➔ Newton (with help from Galileo, Kepler, etc.) 1687



$$\mathbf{F}_{12} = \frac{G m_1 m_2 \hat{\mathbf{r}}_{12}}{r_{12}^2}$$

$$G = 6.674 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$$

Review: Gravitational force of the Earth

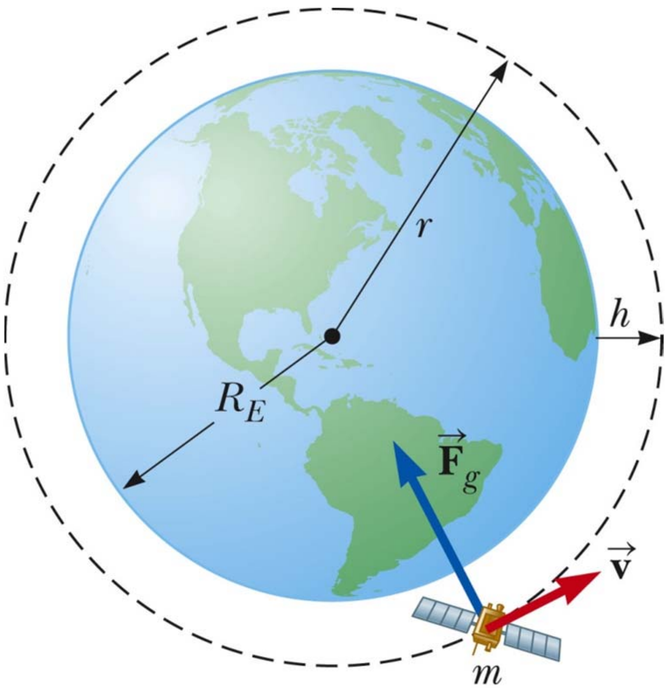


$$F = \frac{GM_E m}{R_E^2}$$

$$\Rightarrow g = \frac{GM_E}{R_E^2} = \frac{6.67\times10^{-11}\cdot 5.98\times10^{24}}{(6.37\times10^{6})^2}\,\text{m/s}^2 = 9.8\,\text{m/s}^2$$

Note: Earth's gravity acts as a point mass located at the Earth's center.

Example: Satellite in circular Earth orbit



For $m << M_E \qquad r \equiv R_E + h$

$$m\frac{v^2}{r} = m\frac{(2\pi r/T)^2}{r} = m\left(\frac{2\pi}{T}\right)^2 r = \frac{GM_E m}{r^2}$$

$$T = 2\pi\sqrt{\frac{(R_E + h)^3}{GM_E}}$$

If $h = 35.83\times 10^6\, m$

$T = 8.53\times 10^4\, s = 1\, day$

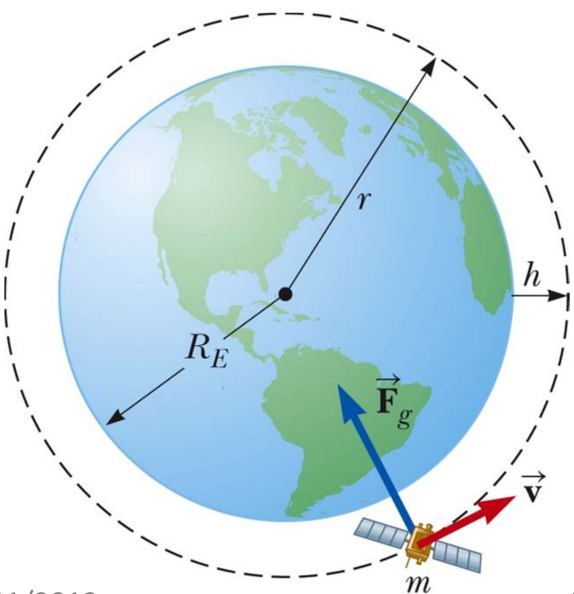
(geosynchronous)

Gravitational potential energy

$$U_{gravity}(r) = -\int_{r_{ref}}^{r} \mathbf{F} \cdot d\mathbf{r} \qquad \mathbf{F} = -\frac{Gm_1 m_2 \hat{\mathbf{r}}}{r^2}$$

$$U_{gravity}(r) = -\int_{\infty}^{r} \frac{-Gm_1 m_2}{r'^2}\, dr' = -\frac{Gm_1 m_2}{r}$$

Example:



$$U_{gravity}(r = R_E + h) = -\frac{GM_E m}{R_E + h}$$