PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 27:

Chapter 14: The physics of fluids

1. Review of static fluids
2. Bernoulli's equation

22	10/29/2012	Kepler's laws and satellite motion	13.4-13.6	13.28, 13.34	10/31/2012
	10/31/2012	Review	10-13,15		
	11/02/2012	Exam	10-13,15		
23	11/05/2012	Fluid mechanics	14.1-14.4	14.8, 14.24	11/07/2012
24	11/07/2012	Fluid mechanics	14.5-14.7	14.39, 14.51	11/09/2012
25	11/09/2012	Temperature	19.1-19.5	19.1, 19.20	11/12/2012
26	11/12/2012	Heat	20.1-20.4		11/14/2012
27	11/14/2012	First law of thermodynamics	20.5-20.7		11/16/2012
28	11/16/2012	Ideal gases	21.1-21.5		11/19/2012
29	11/19/2012	Engines	22.1-22.8		11/26/2012
	11/21/2012	*Thanksgiving Holiday*			
	11/23/2012	*Thanksgiving Holiday*			
	11/26/2012	Review	14,19-22		
	11/28/2012	Exam	14,19-22		
30	11/30/2012	Wave motion	16.1-16.6		12/03/2012
31	12/03/2012	Sound & standing waves	17.1-18.8		12/05/2012
	12/05/2012	Review	1-22		
	12/13/2012	Final Exam -- 9 AM			

Review of equations describing static fluids in terms of pressure P and density ρ:

$$\text{For all fluids near Earth's surface: } \frac{dP}{dy} = -\rho g$$

$$\text{For incompressible fluid, } \rho \equiv (\text{constant}) \Rightarrow P = P_0 - \rho g (y - y_0)$$

For compressible fluid, equation is more complicated;

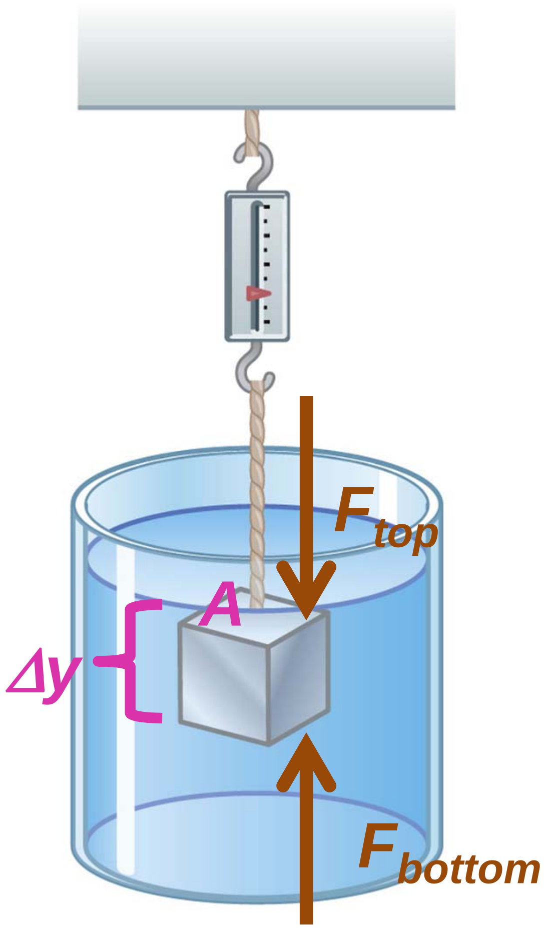
$$\text{For an "ideal gas": } \rho = P\frac{\rho_0}{P_0} \qquad \Rightarrow \frac{dP}{dy} = -P\left(\frac{\rho_0 g}{P_0}\right)$$

$$\Rightarrow P(y) = P_0 e^{-\frac{\rho_0 g}{P_0}(y - y_0)} \approx P_0 e^{-\frac{y-y_0}{8000m}} \approx P_0 e^{-\frac{y-y_0}{5mi}}$$

$$\text{For small } y - y_0, \quad P(y) \approx P_0 - \rho_0 g (y - y_0)$$

Buoyant force

$$\text{Buoyant force}: \quad F_B = \rho_{\text{fluid}} g V_{submerged}$$

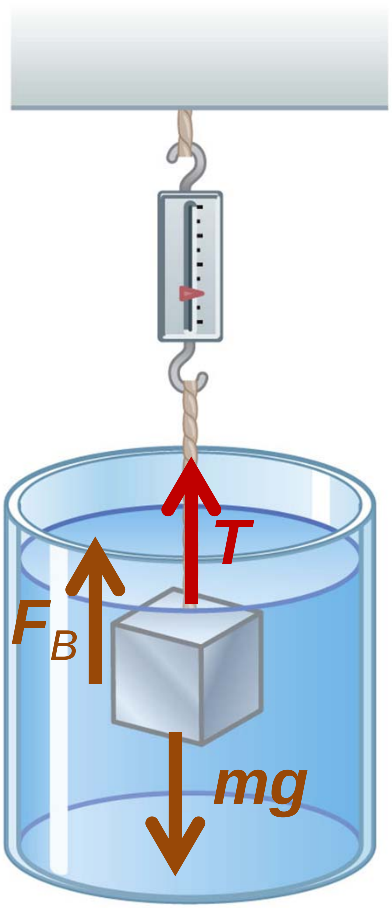


Net force on cube from fluid:

$$F_B = F_{bottom} - F_{top} = \rho_{\text{fluid}} (A\Delta y) g$$

$$\Rightarrow F_B = \rho_{\text{fluid}} V_{submerged}\, g$$

Scale reading due to buoyant force:



Scale reading :

$$T + F_B - mg = 0$$

$$F_{scale} = T = mg - F_B$$

$$F_B = \rho_{\text{fluid}} V_{submerged}\, g$$

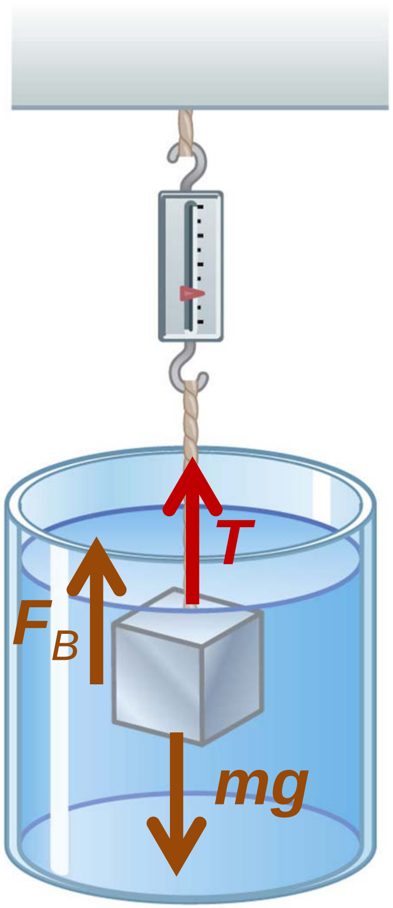
$$mg = \rho_{\text{solid}} V_{solid}\, g$$

If $\rho_{\text{fluid}} < \rho_{\text{solid}}$ $\quad V_{submerged} = V_{solid}$

Measure solid density using :

$$\frac{\rho_{solid}}{\rho_{fluid}} = \frac{mg}{F_B} = \frac{mg}{mg - F_{scale}}$$

Scale reading due to buoyant force:



$$\text{Scale reading:}$$

$$T + F_B - mg = 0$$

$$F_{scale} = T = mg - F_B$$

$$F_B = \rho_{\text{fluid}} V_{submerged}\, g$$

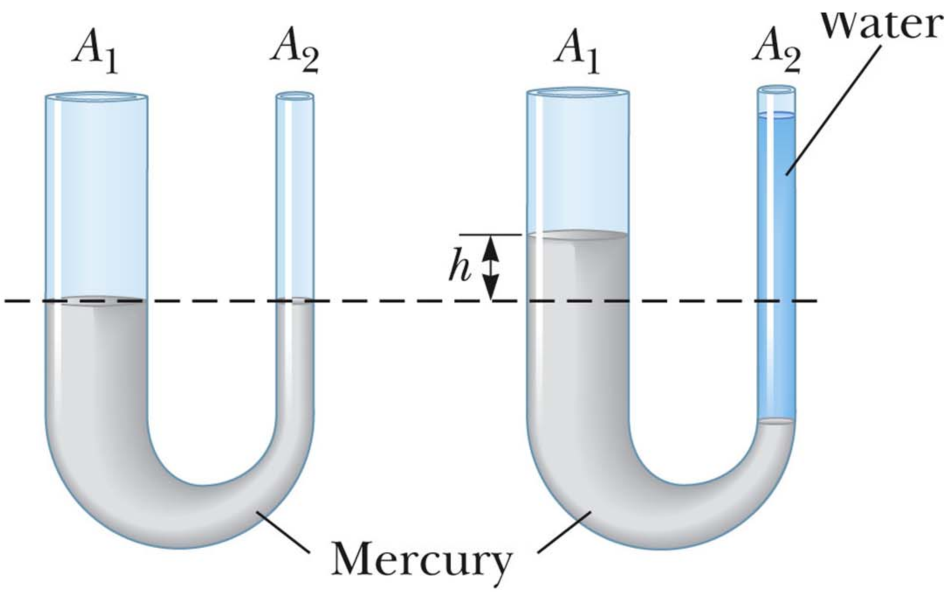
$$mg = \rho_{\text{solid}} V_{solid}\, g$$

If $\rho_{\text{fluid}} > \rho_{\text{solid}} \quad V_{submerged} < V_{solid}$

$\Rightarrow T = 0;$ Measure solid density using:

$$\frac{\rho_{solid}}{\rho_{fluid}} = \frac{V_{submerged}}{V_{solid}}$$

Mercury is poured into a U-tube as shown in Figure a. The left arm of the tube has cross-sectional area A_1 of 9.5 cm^2, and the right arm has a cross-sectional area A_2 of 5.30 cm^2. Four hundred grams of water are then poured into the right arm as shown in Figure b.

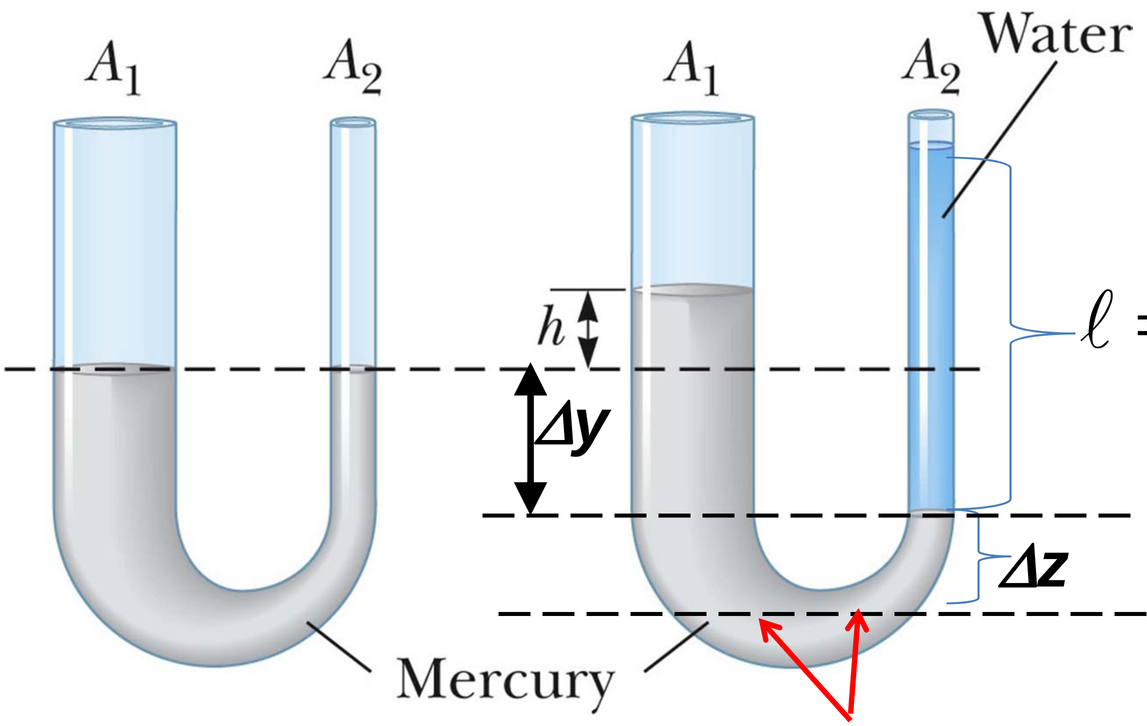


(a) Determine the length of the water column in the right arm of the U-tube.

______ cm

(b) Given that the density of mercury is 13.6 g/cm^3, what distance h does the mercury rise in the left arm?

______ cm



$$\ell = \frac{M}{\rho_{H_2O} A_2}$$

$$P_1 = \rho_{H_2O} g \ell + \rho_{Hg} g \Delta z$$

$$= \rho_{Hg} g \left(h + \Delta y + \Delta z \right)$$

$$\Rightarrow \rho_{H_2O} g \ell = \rho_{Hg} g \left(h + \Delta y \right)$$

Note that : $hA_1 = \Delta y A_2$

$$\rho_{H_2O} g \ell = \rho_{Hg} g (h + \Delta y)$$

$$A_1 h = A_2 \Delta y$$

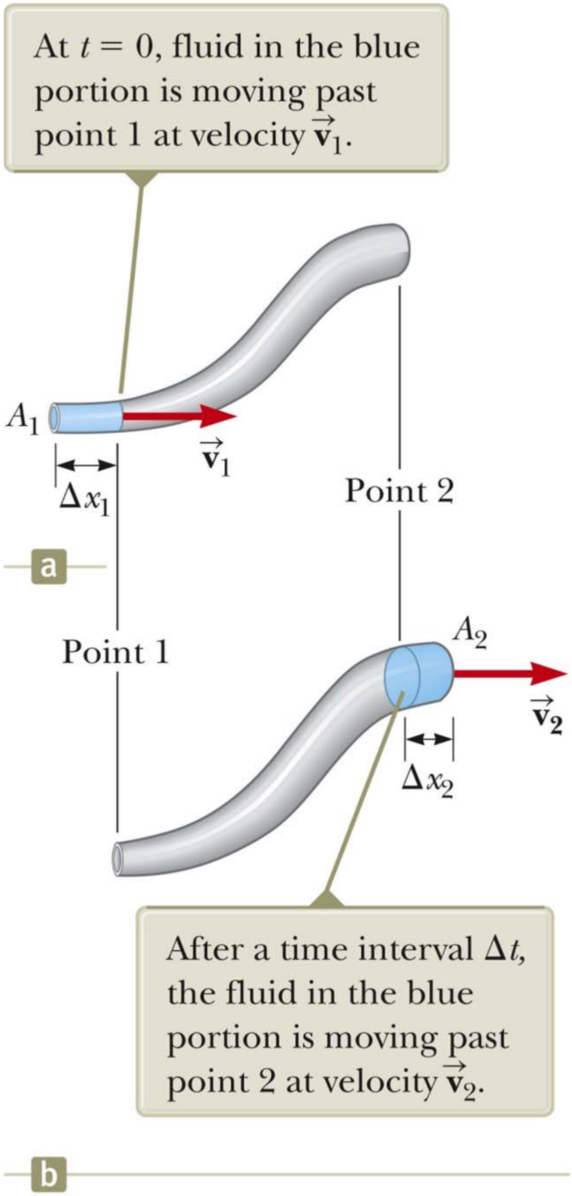
$$\Rightarrow \rho_{H_2O} g \ell = \rho_{Hg} g h \left(1 + \frac{A_1}{A_2} \right)$$

$$h = \ell \frac{\rho_{H_2O}}{\rho_{Hg}\left(1 + \frac{A_1}{A_2} \right)}$$

For $\ell = 0.2\text{ m}, \quad A_1 / A_2 = 2:$

$$h = 0.2\text{m} \frac{1000}{13600(1+2)} = 0.0049\text{m}$$

Fluid dynamics (for incompressible fluids)



For an "incompressible" fluid:

$$\rho = (\text{constant})$$

Consider a given mass of fluid

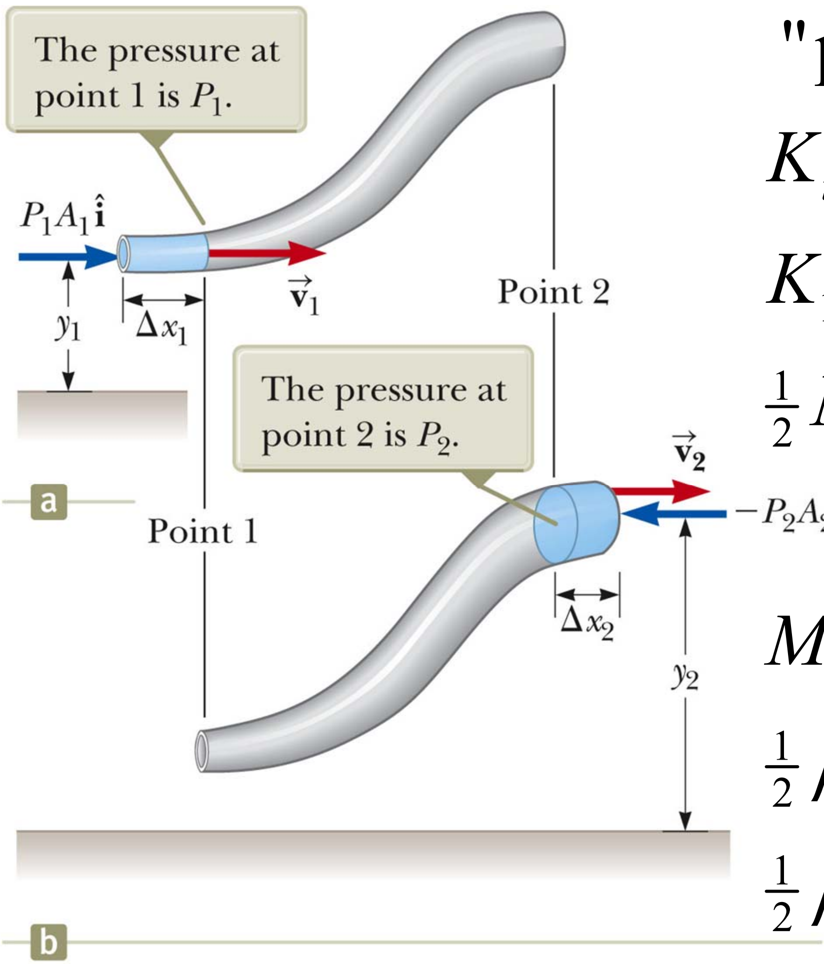
$$M = \rho V_1 = \rho V_2$$

$$M = \rho A_1 \Delta x_1 = \rho A_2 \Delta x_2$$

$$M = \rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

$$\Rightarrow A_1 v_1 = A_2 v_2$$

Approximate energy conservation in fluid dynamics
➔ Bernoulli's equation



Energy - work relationship on "piece" of fluid M

$$K_i + U_i + W_{i\to f} = K_f + U_f$$

$$K_1 + U_1 + W_{1\to 2} = K_2 + U_2$$

$$\tfrac{1}{2}Mv_1^2 + Mgy_1 - (P_2 - P_1)\Delta V = \tfrac{1}{2}Mv_2^2 + Mgy_2$$

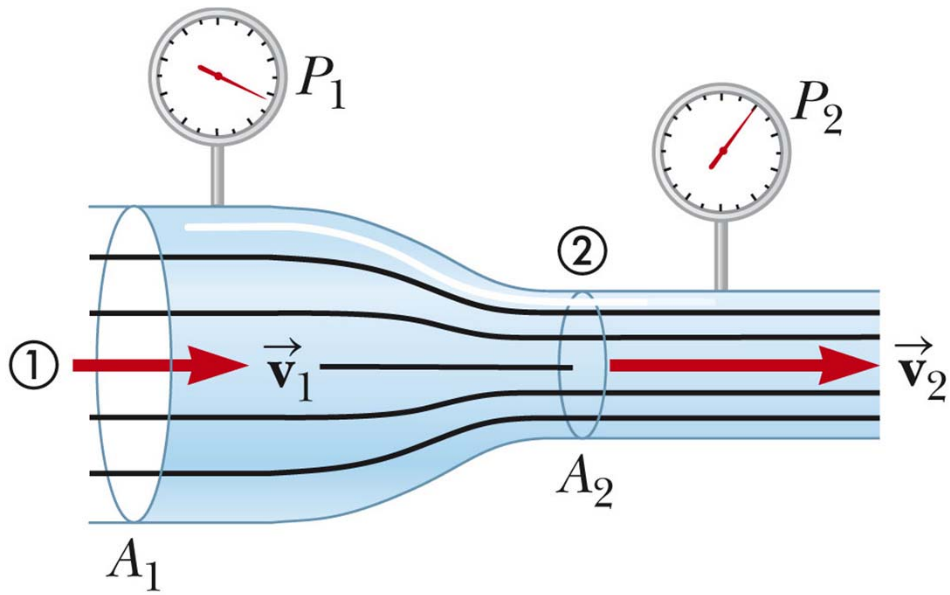
$$M = \rho \Delta V$$

$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 - (P_2 - P_1) = \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 = \tfrac{1}{2}\rho v_2^2 + \rho g y_2 + P_2$$

Bernoulli's equation:

$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 = \tfrac{1}{2}\rho v_2^2 + \rho g y_2 + P_2$$



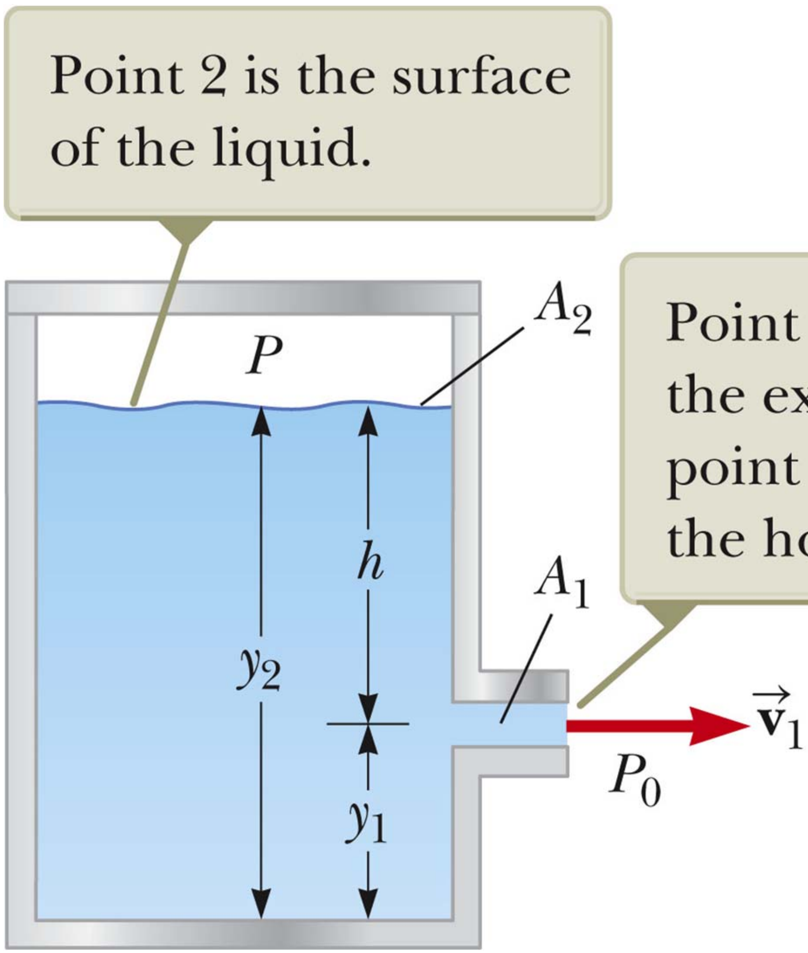
$$y_1 = y_2 \qquad A_1 v_1 = A_2 v_2$$

$$\tfrac{1}{2}\rho v_1^2 + P_1 = \tfrac{1}{2}\rho v_2^2 + P_2$$

$$\tfrac{1}{2}\rho v_2^2\left(1-\left(\frac{A_2}{A_1}\right)^2\right) = P_1 - P_2$$

Bernoulli's equation:

$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 = \tfrac{1}{2}\rho v_2^2 + \rho g y_2 + P_2$$



$$\tfrac{1}{2}\rho v_2^2 + \rho g y_2 + P =$$

$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 + P_0$$

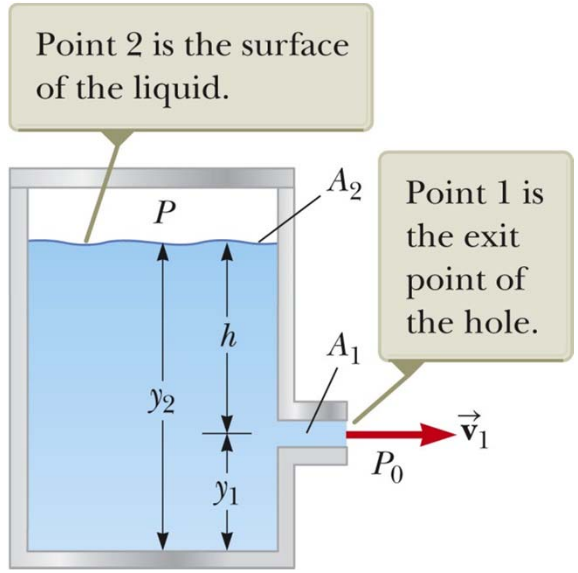
$$A_2 v_2 = A_1 v_1$$

$$v_1 = \sqrt{\frac{2\rho g h + 2(P - P_0)}{\rho\left(1 - \left(\frac{A_1}{A_2}\right)^2\right)}}$$

Possibilities:

Fire extinguisher
➔ $P >> P_0$

$$v_1 \approx \sqrt{\frac{2(P-P_0)}{\rho}}$$



$$\text{Example:} \quad P = 10P_0$$

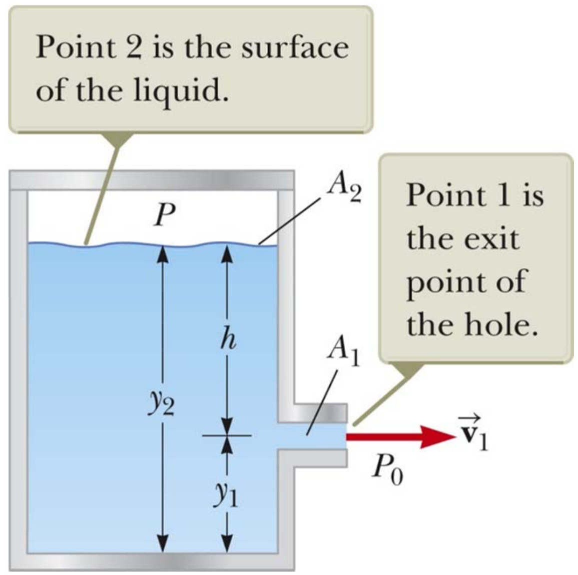
$$v_1 \approx \sqrt{\frac{18P_0}{\rho}} = \sqrt{\frac{18 \cdot 1.013 \times 10^5}{1000}}$$

$$= 42.7 \text{ m/s}$$

Possibilities:

Open top:

➔ $P=P_0$



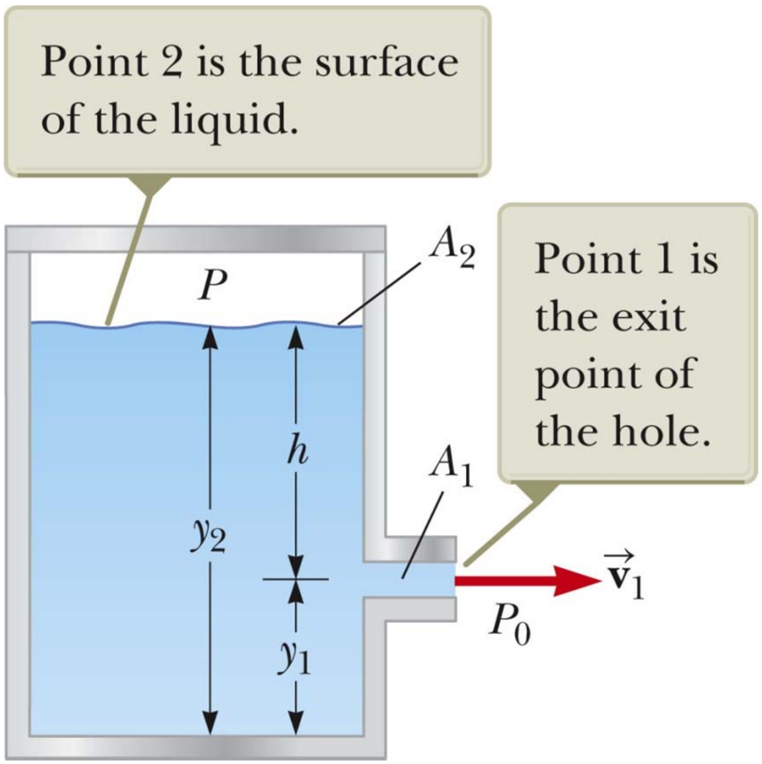
$$v_1 \approx \sqrt{2gh}$$

$$\text{Example:} \quad h = 0.5\,\text{m}$$

$$v_1 \approx \sqrt{(2)(9.8)(0.5)}\,m/s$$

$$= 3.1\,\text{m/s}$$

$$v_1 = \sqrt{\frac{2\rho g h + 2(P - P_0)}{\rho\left(1 - \left(\frac{A_1}{A_2}\right)^2\right)}}$$



iclicker question:

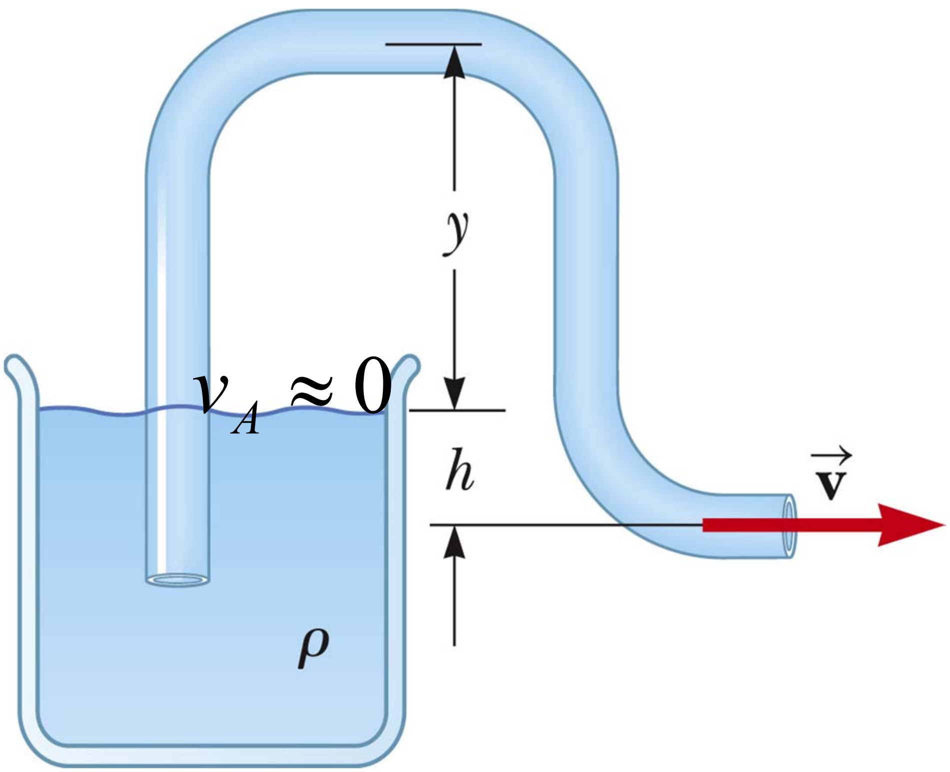
Notice that if $P \ll P_0$, v_1 could become very small. How might this happen?

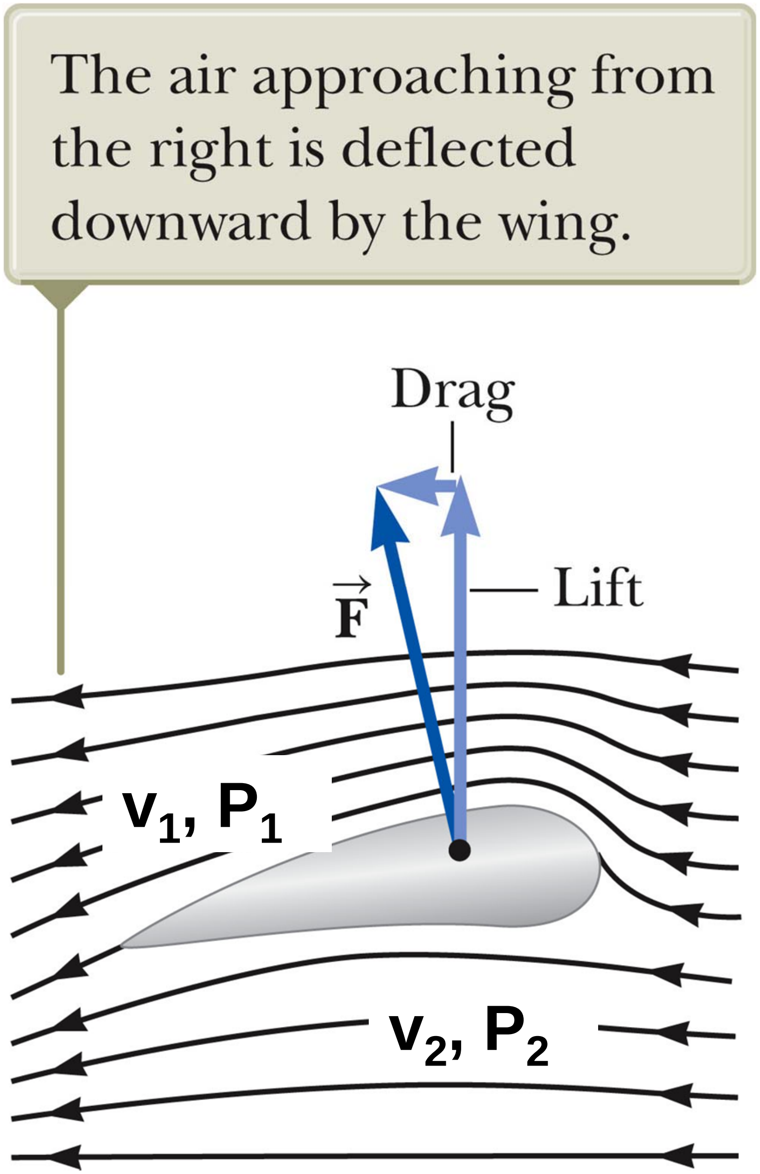
A. **Put an air-tight lid on the top.**
B. **Remove the lid on the top.**
C. **This will never happen.**

Siphon

$$\frac{1}{2}\rho v_A^2 + \rho g h + P_0 = \frac{1}{2}\rho v^2 + P_0$$

$$v = \sqrt{2gh}$$





Simplistic statement:

$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 =$$
$$\tfrac{1}{2}\rho v_2^2 + \rho g y_2 + P_2$$

$$\tfrac{1}{2}\rho\left(v_1^2 - v_2^2\right) = P_2 - P_1$$