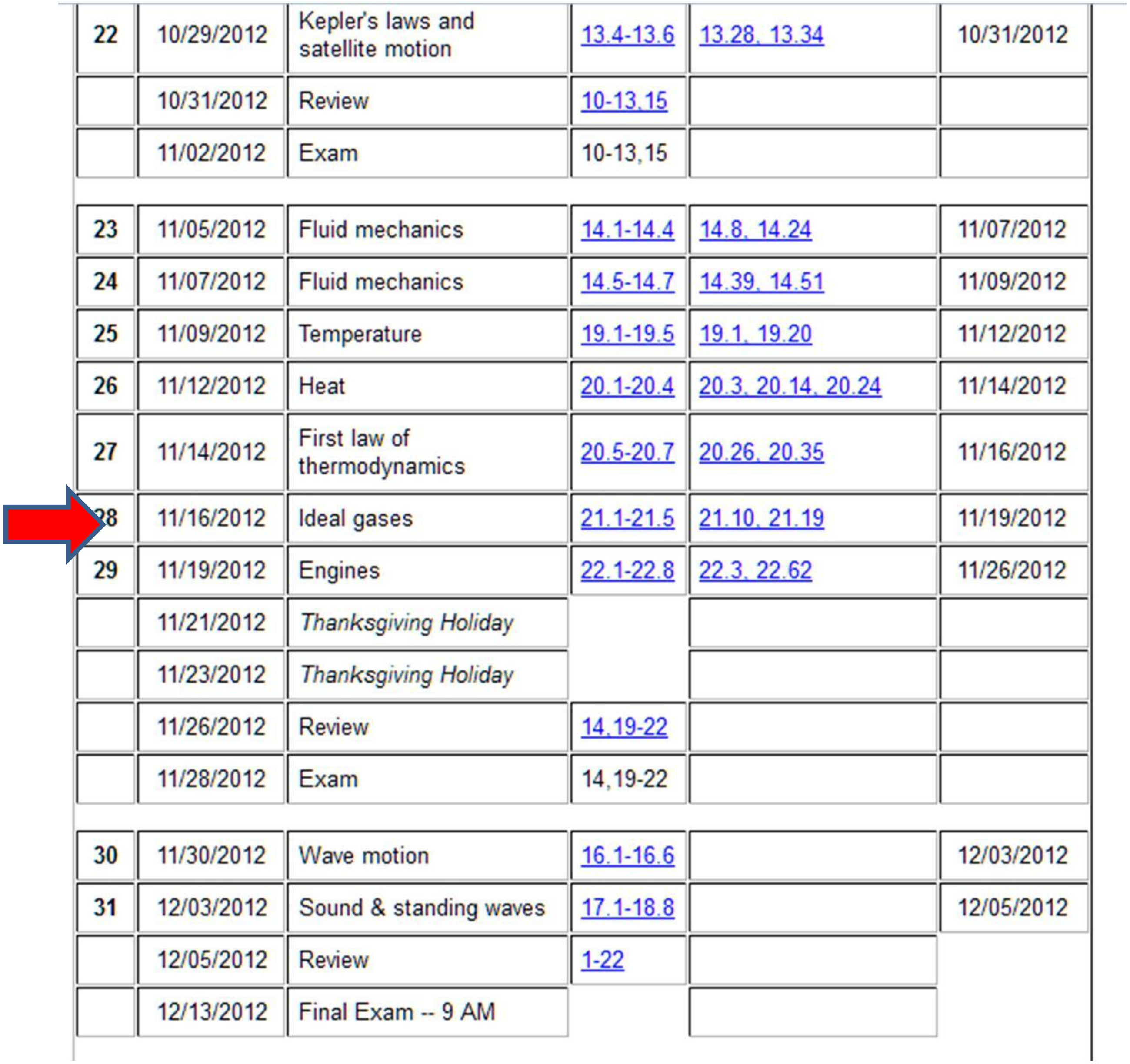
PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 31:

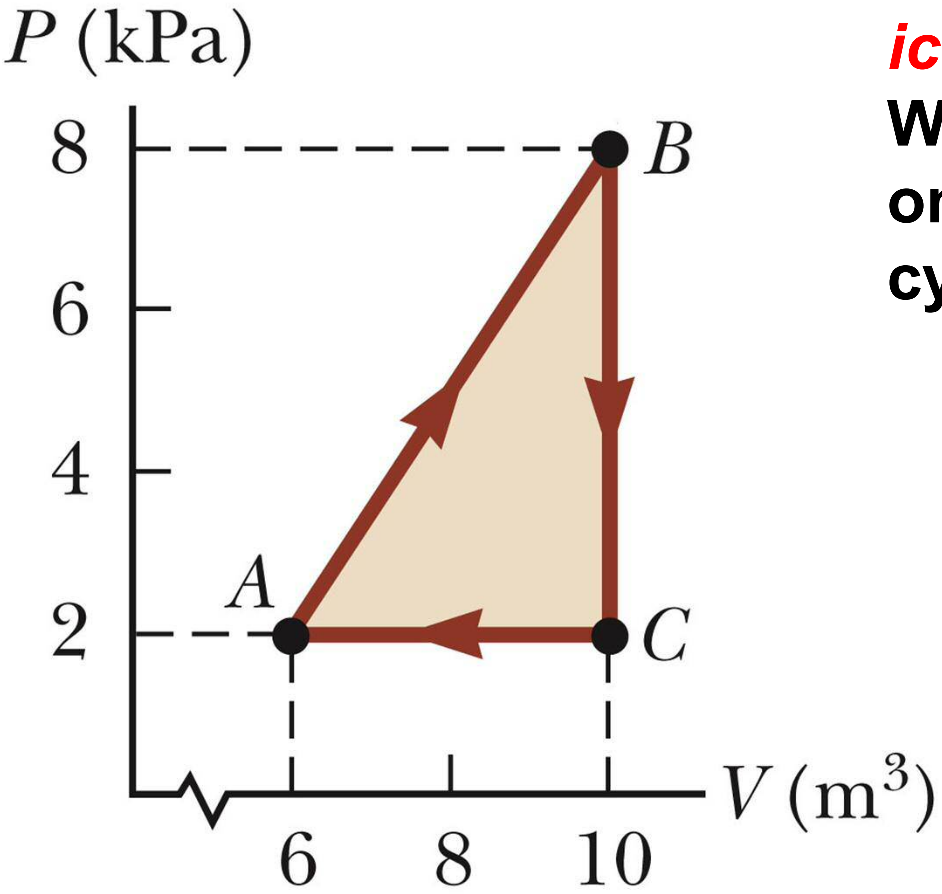
Chapter 21: Ideal gas equations

1. Molecular view of ideal gas
2. Internal energy of ideal gas
3. Distribution of molecular speeds in ideal gas



22	10/29/2012	Kepler's laws and satellite motion	13.4-13.6	13.28, 13.34	10/31/2012
	10/31/2012	Review	10-13,15		
	11/02/2012	Exam	10-13,15		
23	11/05/2012	Fluid mechanics	14.1-14.4	14.8, 14.24	11/07/2012
24	11/07/2012	Fluid mechanics	14.5-14.7	14.39, 14.51	11/09/2012
25	11/09/2012	Temperature	19.1-19.5	19.1, 19.20	11/12/2012
26	11/12/2012	Heat	20.1-20.4	20.3, 20.14, 20.24	11/14/2012
27	11/14/2012	First law of thermodynamics	20.5-20.7	20.26, 20.35	11/16/2012
28	11/16/2012	Ideal gases	21.1-21.5	21.10, 21.19	11/19/2012
29	11/19/2012	Engines	22.1-22.8	22.3, 22.62	11/26/2012
	11/21/2012	*Thanksgiving Holiday*			
	11/23/2012	*Thanksgiving Holiday*			
	11/26/2012	Review	14,19-22		
	11/28/2012	Exam	14,19-22		
30	11/30/2012	Wave motion	16.1-16.6		12/03/2012
31	12/03/2012	Sound & standing waves	17.1-18.8		12/05/2012
	12/05/2012	Review	1-22		
	12/13/2012	Final Exam -- 9 AM			

Review:
Consider the process described by A→B→C→A



iclicker exercise:
What is the net work done on the system in this cycle?

A. -12000 J
B. 12000 J
C. 0

Equation of “state” for ideal gas
(from experiment)

8.314 J/(mol K)

$$PV = nRT$$

temperature in K

volume in m^3

# of moles

pressure in Pascals

Ideal gas -- continued

$$\text{Equation of state:} \quad PV = nRT$$

$$\text{Internal energy:} \quad E_{\text{int}} = \frac{1}{\gamma - 1} nRT = \frac{1}{\gamma - 1} PV$$

$$\gamma = \text{parameter depending on type of ideal gas}$$

$$= \begin{cases} \frac{5}{3} & \text{for monoatomic} \\ \frac{7}{5} & \text{for diatomic} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \end{cases}$$

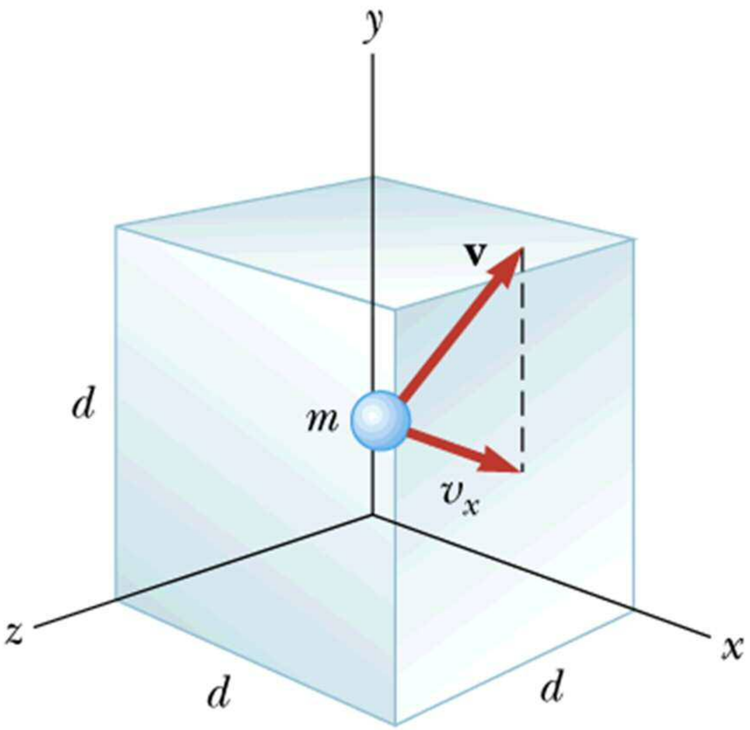
Note that at this point, the above equation for E_{int} is completely unjustified…

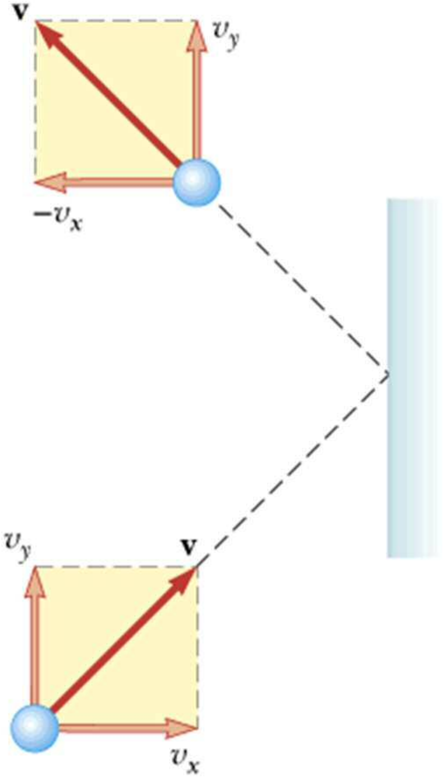
From The New Yorker Magazine, November 2003

"Quick, get it while its molecules are still vibrating."

Microscopic model of ideal gas:

Each atom is represented as a tiny hard sphere of mass *m* with velocity **v**. Collisions and forces between atoms are neglected. Collisions with the walls of the container are assumed to be elastic.





What we can show is the pressure exerted by the atoms by their collisions with the walls of the container is given by:

$$P = \frac{2}{3}\frac{N}{V}\,{}^1\!/_2\, m\left\langle v^2 \right\rangle_{avg} = \frac{2}{3}\frac{N}{V}\left\langle K \right\rangle_{avg}$$

Proof:

Force exerted on wall perpendicular to x-axis by an atom which collides with it:

$$|F_{ix}| = \left|\frac{\Delta p_{ix}}{\Delta t}\right| = \frac{2m_i|v_{ix}|}{\Delta t} \qquad \Delta t \approx 2d/v_{ix}$$

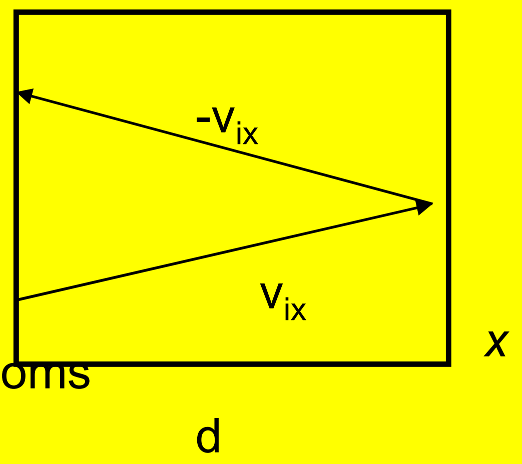
$$\Rightarrow F_{ix} \approx \frac{2m_i v_{ix}}{2d/v_{ix}} = \frac{m_i v_{ix}^2}{d}$$

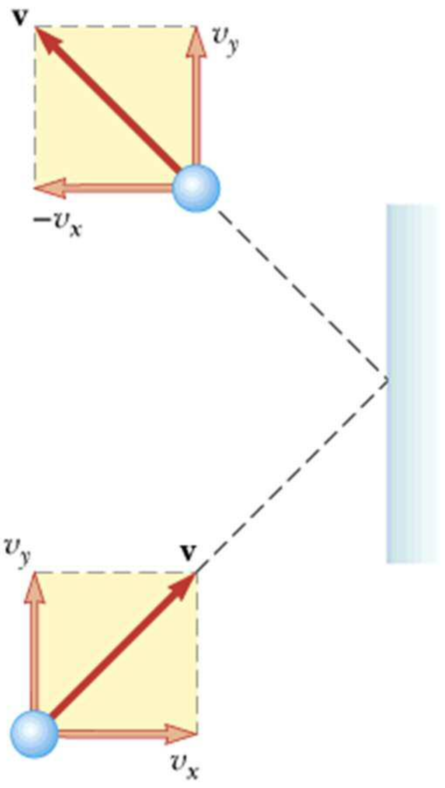
$$P = \sum_i \frac{F_{ix}}{A} = \sum_i \frac{m_i v_{ix}^2}{dA} = \frac{N}{V}\left\langle m_i v_x^2 \right\rangle$$

number of atoms

volume

average over atoms





$$F_{ix} \approx \frac{2m_i v_{ix}}{2d / v_{ix}} = \frac{m_i v_{ix}^2}{d}$$

$$P = \sum_i \frac{F_{ix}}{A} = \sum_i \frac{m_i v_{ix}^2}{dA} = \frac{N}{V}\left\langle m_i v_{ix}^2 \right\rangle$$

Since molecules are equally likely to move along x, y, z:

$$\left\langle m_i v_{ix}^2 \right\rangle = \left\langle m_i v_{iy}^2 \right\rangle = \left\langle m_i v_{iz}^2 \right\rangle$$

Also note that $v_i^2 = v_{ix}^2 + v_{iy}^2 + v_{iz}^2$

$$\left\langle v_i^2 \right\rangle = \left\langle v_{ix}^2 \right\rangle + \left\langle v_{iy}^2 \right\rangle + \left\langle v_{iz}^2 \right\rangle = 3\left\langle v_{ix}^2 \right\rangle$$

$$\Rightarrow P = \frac{N}{V}\left\langle m_i v_{ix}^2 \right\rangle = \frac{N}{3V}\left\langle m_i v_i^2 \right\rangle = \frac{2N}{3V}\left\langle \tfrac{1}{2} m_i v_i^2 \right\rangle$$

iclicker question:

What should we call $\left\langle \frac{1}{2} m_i v_i^2 \right\rangle$?

A. Average kinetic energy of atom.
B. We cannot use our macroscopic equations at the atomic scale -- so this quantity will go unnamed.
C. We made too many approximations, so it is not worth naming/discussion.
D. Very boring.

$$P=\frac{2N}{3V}\left\langle \tfrac{1}{2}m_i v_i^2\right\rangle$$

Note: N = number of atoms

m_i = mass of atom (6.6×10^{-27} kg for He atom)

$Nm_i = nM$ where M denotes the molar mass (0.004 kg for He atom)

n = number of moles of atoms

Connection to ideal gas law:

$$PV=n\left(\frac{2}{3}\left\langle \tfrac{1}{2}Mv_i^2\right\rangle\right)=nRT$$

$$\Rightarrow\left(\frac{2}{3}\left\langle \tfrac{1}{2}Mv_i^2\right\rangle\right)=RT \qquad \text{or}\ \left\langle \tfrac{1}{2}Mv_i^2\right\rangle=\frac{3}{2}RT$$

$\left\langle \tfrac{1}{2}Mv_i^2\right\rangle \equiv$ average kinetic energy of mole of ideal gas atoms

$$\boxed{E_{\text{int}}=\frac{3}{2}nRT}$$

Average atomic velocities:
(note $<v_i>=0$)

$$\left\langle \tfrac{1}{2} M v_i^2 \right\rangle = \frac{3}{2} RT$$

$$\left\langle v_i^2 \right\rangle = \sqrt{\frac{3RT}{M}}$$

Relationship between average atomic velocities with T

For monoatomic ideal gas:

$$\boxed{E_{\text{int}} = \frac{3}{2} nRT}$$

General form for ideal gas (including mono-, di-, poly-atomic ideal gases):

$$E_{\text{int}} = \frac{1}{\gamma - 1} nRT$$

$$\gamma = \begin{cases} \frac{5}{3} & \text{for monoatomic} \\ \frac{7}{5} & \text{for diatomic} \\ \dots\dots\dots\dots\dots\dots\dots\dots & \end{cases}$$

Big leap!

Internal energy of an ideal gas:

$$E_{\text{int}} = N\left\{\frac{1}{2}m\left\langle v^2\right\rangle\right\} = N\frac{3}{2}k_BT \Rightarrow \frac{N}{\gamma\text{-}1}k_BT = \frac{n}{\gamma\text{-}1}RT$$

derived for
monoatomic ideal gas

more general relation
for polyatomic ideal gas

Gas	**γ (theory)**	**γ (exp)**
He	5/3	1.67
N_2	7/5	1.41
H_2O	4/3	1.30

Determination of Q for various processes in an ideal gas:

$$E_{\text{int}} = \frac{n}{\gamma\text{-}1} RT$$

$$\Delta E_{\text{int}} = \frac{n}{\gamma\text{-}1} R\Delta T = Q + W$$

Example: Isovolumetric process – (V=constant ➔ W=0)

$$\Delta E_{\text{int}\; i\to f} = \frac{n}{\gamma\text{-}1} R\Delta T_{i\to f} = Q_{i\to f}$$

In terms of "heat capacity": $Q_{i\to f} = \frac{n}{\gamma\text{-}1} R\Delta T_{i\to f} \equiv nC_V \Delta T_{i\to f}$

$$C_V = \frac{R}{\gamma\text{-}1}$$

Example: Isobaric process (P=constant):

$$\Delta E_{\text{int}\,i\to f} = \frac{n}{\gamma\text{-}1} R\Delta T_{i\to f} = Q_{i\to f} + W_{i\to f}$$

In terms of “heat capacity”:

$$Q_{i\to f} = \frac{n}{\gamma\text{-}1} R\Delta T_{i\to f} + P_i\left(V_f - V_i\right) = \frac{n}{\gamma\text{-}1} R\Delta T_{i\to f} + nR\Delta T_{i\to f} \equiv nC_P\Delta T_{i\to f}$$

$$\Rightarrow C_P = \frac{R}{\gamma\text{-}1} + R = \frac{\gamma R}{\gamma\text{-}1}$$

Note: $\gamma = C_P/C_V$

More examples:

Isothermal process (T=0)

$$E_{\text{int}} = \frac{n}{\gamma\text{-}1} RT$$

$$\Delta E_{\text{int}} = \frac{n}{\gamma\text{-}1} R\Delta T = Q + W$$

➔ΔT=0 ➔ $\Delta E_{\text{int}} = 0$ ➔ Q=-W

$$-W = \int_{V_i}^{V_f} PdV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln\left(\frac{V_f}{V_i}\right)$$

Even more examples:

Adiabatic process (Q=0)

$$\Delta E_{\text{int}} = W$$

$$\frac{n}{\gamma\text{-}1} R\Delta T = -P\Delta V$$

$$PV = nRT$$

$$\Delta PV + P\Delta V = nR\Delta T$$

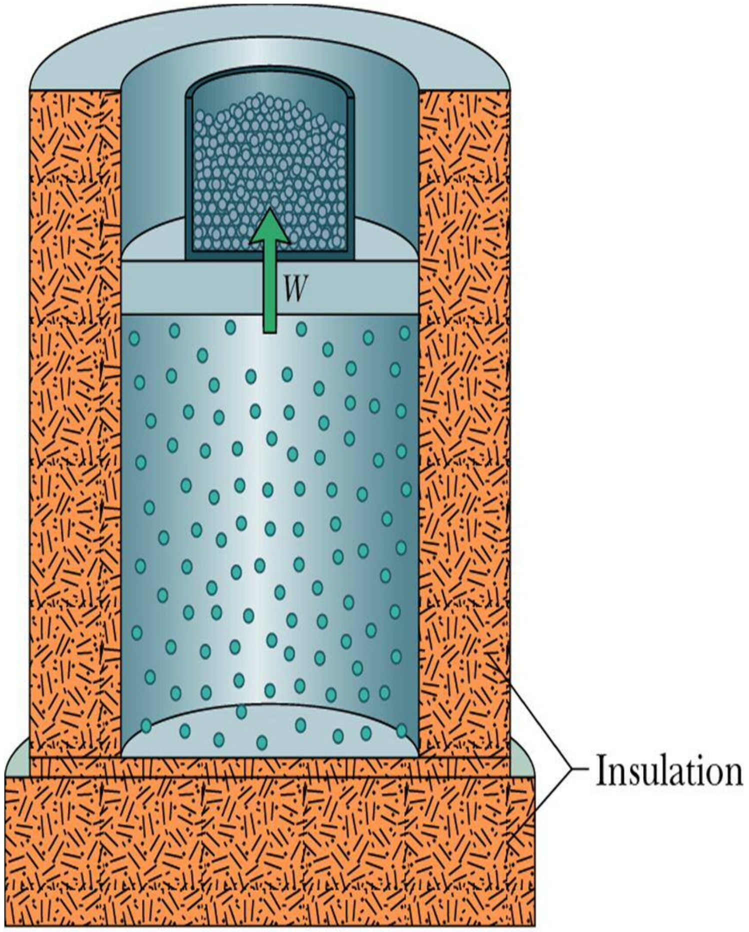
$$nR\Delta T = -(\gamma\text{-}1)P\Delta V = \Delta PV + P\Delta V$$

$$-\gamma\frac{\Delta V}{V} = \frac{\Delta P}{P}$$

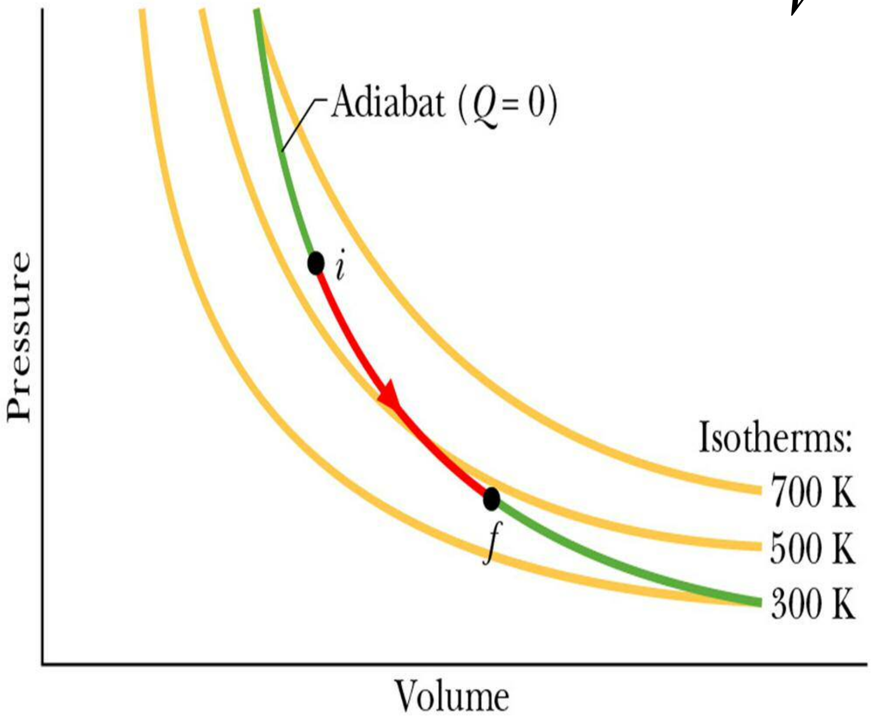
$$\Rightarrow -\ln\left(\frac{V_f^\gamma}{V_i^\gamma}\right) = \ln\left(\frac{P_f}{P_i}\right) \quad \Rightarrow P_iV_i^\gamma = P_fV_f^\gamma$$

$$\text{Adiabat}: \ P = \frac{P_i V_i^{\gamma}}{V^{\gamma}}$$

$$\text{Isotherm}: \ P = \frac{P_i V_i}{V}$$



(*a*)



(*b*)

iclicker question:

Suppose that an ideal gas expands adiabatically. Does the temperature

(A) Increase (B) Decrease (C) Remain the same

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

$$P_i V_i = nRT_i \Rightarrow P_i = nR\frac{T_i}{V_i}$$

$$T_i V_i^{\gamma\text{-}1} = T_f V_f^{\gamma\text{-}1}$$

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma\text{-}1}$$

Review of results from ideal gas analysis in terms of the specific heat ratio $\gamma \equiv C_P/C_V$:

$$\Delta E_{\text{int}} = \frac{n}{\gamma\text{-}1} R\Delta T = nC_V \Delta T \quad ; \; C_V = \frac{R}{\gamma\text{-}1}$$

$$C_P = \frac{\gamma R}{\gamma\text{-}1}$$

For an isothermal process, $\Delta E_{\text{int}} = 0$ ➔ Q=-W

$$-W = \int_{V_i}^{V_f} P dV = nRT \ln\left(\frac{V_f}{V_i}\right) = P_i V_i \ln\left(\frac{V_f}{V_i}\right)$$

For an adiabatic process, Q = 0

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

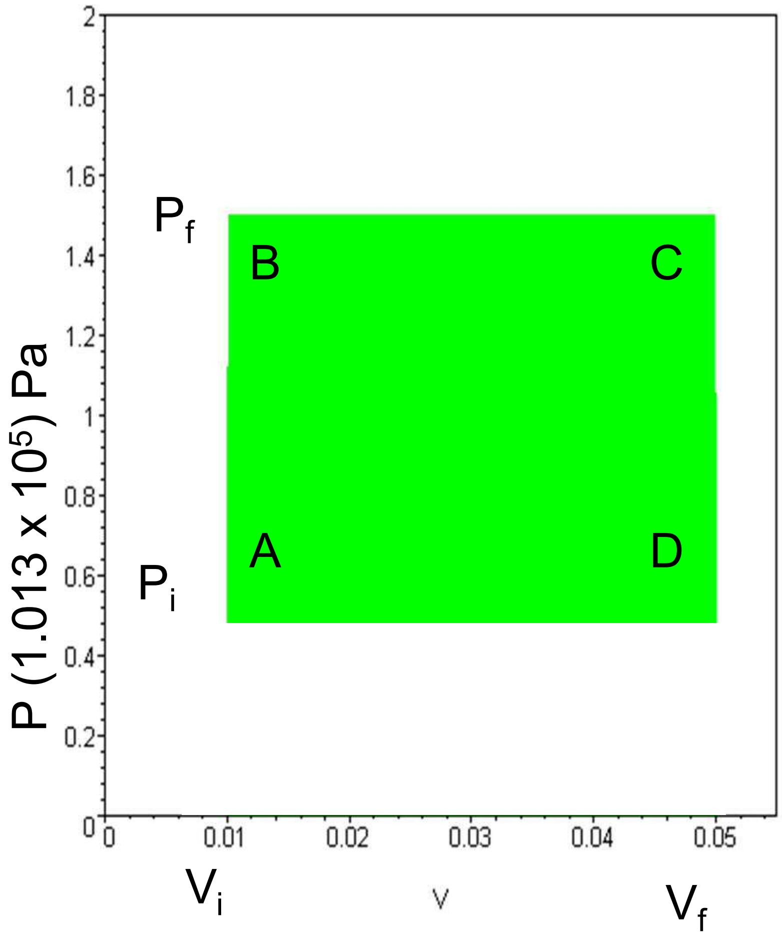
$$T_i V_i^{\gamma\text{-}1} = T_f V_f^{\gamma\text{-}1}$$

Note:

It can be shown that the work done by an ideal gas which has an initial pressure P_i and initial volume V_i when it expands *adiabatically* to a volume V_f is given by:

$$W = -\int_{V_i}^{V_f} P\,dV = -\frac{P_i V_i}{\gamma - 1}\left(1 - \left(\frac{V_i}{V_f}\right)^{\gamma - 1}\right)$$

Examples process by an ideal gas:



	A→B	B→C	C→D	D→A
Q	$\frac{V_i(P_f-P_i)}{\gamma-1}$	$\frac{\gamma P_f(V_f-V_i)}{\gamma-1}$	$\frac{-V_f(P_f-P_i)}{\gamma-1}$	$\frac{-\gamma P_i(V_f-V_i)}{\gamma-1}$
W	0	$-P_f(V_f-V_i)$	0	$P_i(V_f-V_i)$
ΔE_{int}	$\frac{V_i(P_f-P_i)}{\gamma-1}$	$\frac{P_f(V_f-V_i)}{\gamma-1}$	$\frac{-V_f(P_f-P_i)}{\gamma-1}$	$\frac{-P_i(V_f-V_i)}{\gamma-1}$

Efficiency as an engine:

$$e = W_{net/}\, Q_{input}$$