# PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

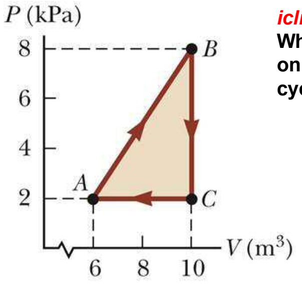
## Plan for Lecture 31:

### Chapter 21: Ideal gas equations

1. Molecular view of ideal gas
2. Internal energy of ideal gas
3. Distribution of molecular speeds in ideal gas

| | | | | | |
|---|---|---|---|---|---|
| 22 | 10/29/2012 | Kepler's laws and satellite motion | 13.4-13.6 | 13.28, 13.34 | 10/31/2012 |
| | 10/31/2012 | Review | 10-13,15 | | |
| | 11/02/2012 | Exam | 10-13,15 | | |
| 23 | 11/05/2012 | Fluid mechanics | 14.1-14.4 | 14.8, 14.24 | 11/07/2012 |
| 24 | 11/07/2012 | Fluid mechanics | 14.5-14.7 | 14.39, 14.51 | 11/09/2012 |
| 25 | 11/09/2012 | Temperature | 19.1-19.5 | 19.1, 19.20 | 11/12/2012 |
| 26 | 11/12/2012 | Heat | 20.1-20.4 | 20.3, 20.14, 20.24 | 11/14/2012 |
| 27 | 11/14/2012 | First law of thermodynamics | 20.5-20.7 | 20.26, 20.35 | 11/16/2012 |
| 8 | 11/16/2012 | Ideal gases | 21.1-21.5 | 21.10, 21.19 | 11/19/2012 |
| 29 | 11/19/2012 | Engines | 22.1-22.8 | 22.3, 22.62 | 11/26/2012 |
| | 11/21/2012 | *Thanksgiving Holiday* | | | |
| | 11/23/2012 | *Thanksgiving Holiday* | | | |
| | 11/26/2012 | Review | 14,19-22 | | |
| | 11/28/2012 | Exam | 14,19-22 | | |
| 30 | 11/30/2012 | Wave motion | 16.1-16.6 | | 12/03/2012 |
| 31 | 12/03/2012 | Sound & standing waves | 17.1-18.8 | | 12/05/2012 |
| | 12/05/2012 | Review | 1-22 | | |
| | 12/13/2012 | Final Exam -- 9 AM | | | |

**Review:**
**Consider the process described by A→B→C→A**



***iclicker exercise:***
**What is the net work done on the system in this cycle?**

A. -12000 J
B. 12000 J
C. 0

### Equation of "state" for ideal gas
### (from experiment)

8.314 J/(mol K)

$$PV = nRT$$

temperature in K

volume in m$^3$

\# of moles

pressure in Pascals

### Ideal gas -- continued

$$\text{Equation of state:} \quad PV = nRT$$

$$\text{Internal energy:} \quad E_{\text{int}} = \frac{1}{\gamma - 1} nRT = \frac{1}{\gamma - 1} PV$$

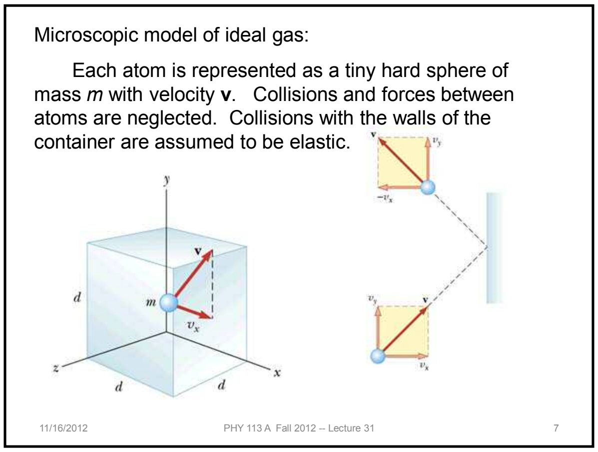
$$\gamma = \text{parameter depending on type of ideal gas}$$

$$= \begin{cases} \frac{5}{3} & \text{for monoatomic} \\ \frac{7}{5} & \text{for diatomic} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots & \end{cases}$$

**Note that at this point, the above equation for $E_{\text{int}}$ is completely unjustified…**

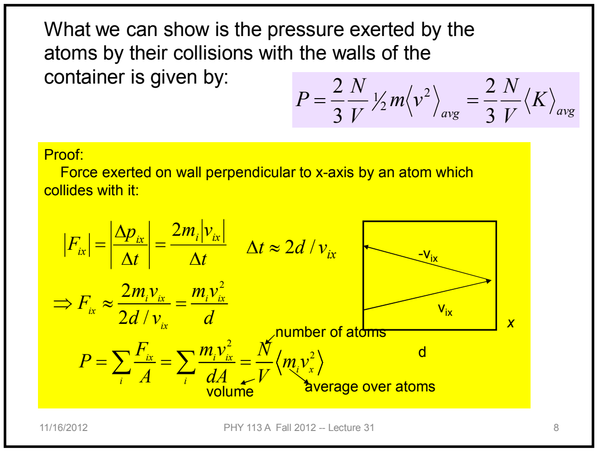
From The New Yorker Magazine, November 2003

*"Quick, get it while its molecules are still vibrating."*



Microscopic model of ideal gas:

Each atom is represented as a tiny hard sphere of mass *m* with velocity **v**. Collisions and forces between atoms are neglected. Collisions with the walls of the container are assumed to be elastic.



What we can show is the pressure exerted by the atoms by their collisions with the walls of the container is given by:

$$P = \frac{2}{3}\frac{N}{V}\tfrac{1}{2}m\langle v^2\rangle_{avg} = \frac{2}{3}\frac{N}{V}\langle K\rangle_{avg}$$

Proof:

Force exerted on wall perpendicular to x-axis by an atom which collides with it:

$$|F_{ix}| = \left|\frac{\Delta p_{ix}}{\Delta t}\right| = \frac{2m_i|v_{ix}|}{\Delta t} \qquad \Delta t \approx 2d/v_{ix}$$

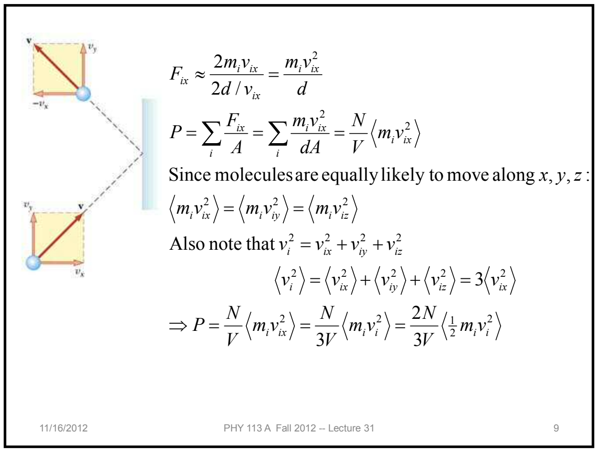
$$\Rightarrow F_{ix} \approx \frac{2m_i v_{ix}}{2d/v_{ix}} = \frac{m_i v_{ix}^2}{d}$$

$$P = \sum_i \frac{F_{ix}}{A} = \sum_i \frac{m_i v_{ix}^2}{dA} = \frac{N}{V}\langle m_i v_x^2\rangle$$

number of atoms

volume

average over atoms



$$F_{ix} \approx \frac{2m_i v_{ix}}{2d/v_{ix}} = \frac{m_i v_{ix}^2}{d}$$

$$P = \sum_i \frac{F_{ix}}{A} = \sum_i \frac{m_i v_{ix}^2}{dA} = \frac{N}{V}\langle m_i v_{ix}^2\rangle$$

Since molecules are equally likely to move along $x, y, z$:

$$\langle m_i v_{ix}^2\rangle = \langle m_i v_{iy}^2\rangle = \langle m_i v_{iz}^2\rangle$$

Also note that $v_i^2 = v_{ix}^2 + v_{iy}^2 + v_{iz}^2$

$$\langle v_i^2\rangle = \langle v_{ix}^2\rangle + \langle v_{iy}^2\rangle + \langle v_{iz}^2\rangle = 3\langle v_{ix}^2\rangle$$

$$\Rightarrow P = \frac{N}{V}\langle m_i v_{ix}^2\rangle = \frac{N}{3V}\langle m_i v_i^2\rangle = \frac{2N}{3V}\langle \tfrac{1}{2} m_i v_i^2\rangle$$

***iclicker question:***

**What should we call $\left\langle \frac{1}{2} m_i v_i^2 \right\rangle$ ?**

**A. Average kinetic energy of atom.**
**B. We cannot use our macroscopic equations at the atomic scale -- so this quantity will go unnamed.**
**C. We made too many approximations, so it is not worth naming/discussion.**
**D. Very boring.**

$$P = \frac{2N}{3V}\left\langle \tfrac{1}{2} m_i v_i^2 \right\rangle$$

Note : $N$ = number of atoms

$m_i$ = mass of atom ($6.6\times10^{-27}$ kg for He atom)

$Nm_i = nM$ where M denotes the molar mass (0.004 kg for He atom)

$n$ = number of moles of atoms

Connection to ideal gas law :

$$PV = n\left(\frac{2}{3}\left\langle \tfrac{1}{2} M v_i^2 \right\rangle\right) = nRT$$

$$\Rightarrow \left(\frac{2}{3}\left\langle \tfrac{1}{2} M v_i^2 \right\rangle\right) = RT \qquad \text{or} \left\langle \tfrac{1}{2} M v_i^2 \right\rangle = \frac{3}{2} RT$$

$\left\langle \tfrac{1}{2} M v_i^2 \right\rangle \equiv$ average kinetic energy of mole of ideal gas atoms

$$E_{\text{int}} = \frac{3}{2} nRT$$

**Average atomic velocities:**
**(note $\langle v_i \rangle = 0$)**

$$\left\langle \tfrac{1}{2} M v_i^2 \right\rangle = \frac{3}{2} RT$$

$$\left\langle v_i^2 \right\rangle = \sqrt{\frac{3RT}{M}}$$

**Relationship between average atomic velocities with T**

**For monoatomic ideal gas:**

$$E_{\text{int}} = \frac{3}{2}nRT$$

**General form for ideal gas (including mono-, di-, poly-atomic ideal gases):**

$$E_{\text{int}} = \frac{1}{\gamma - 1}nRT$$

$$\gamma = \begin{cases} \frac{5}{3} & \text{for monoatomic} \\ \frac{7}{5} & \text{for diatomic} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \end{cases}$$

Internal energy of an ideal gas:

Big leap!

$$E_{\text{int}} = N\left\{\frac{1}{2}m\left\langle v^2\right\rangle\right\} = N\frac{3}{2}k_BT \Rightarrow \frac{N}{\gamma\text{-}1}k_BT = \frac{n}{\gamma\text{-}1}RT$$

derived for monoatomic ideal gas

more general relation for polyatomic ideal gas

| Gas | γ (theory) | γ (exp) |
|---|---|---|
| He | 5/3 | 1.67 |
| $N_2$ | 7/5 | 1.41 |
| $H_2O$ | 4/3 | 1.30 |

Determination of Q for various processes in an ideal gas:

$$E_{\text{int}} = \frac{n}{\gamma\text{-}1}RT$$

$$\Delta E_{\text{int}} = \frac{n}{\gamma\text{-}1}R\Delta T = Q + W$$

Example: Isovolumetric process – (V=constant ➔ W=0)

$$\Delta E_{\text{int}\, i\to f} = \frac{n}{\gamma\text{-}1}R\Delta T_{i\to f} = Q_{i\to f}$$

In terms of "heat capacity": $Q_{i\to f} = \frac{n}{\gamma\text{-}1}R\Delta T_{i\to f} \equiv nC_V\Delta T_{i\to f}$

$$C_V = \frac{R}{\gamma\text{-}1}$$

Example: Isobaric process (P=constant):

$$\Delta E_{\mathrm{int}\, i\to f} = \frac{n}{\gamma\text{-}1} R\Delta T_{i\to f} = Q_{i\to f} + W_{i\to f}$$

In terms of "heat capacity":

$$Q_{i\to f} = \frac{n}{\gamma\text{-}1} R\Delta T_{i\to f} + P_i\left(V_f - V_i\right) = \frac{n}{\gamma\text{-}1} R\Delta T_{i\to f} + nR\Delta T_{i\to f} \equiv nC_P\Delta T_{i\to f}$$

$$\Rightarrow C_P = \frac{R}{\gamma\text{-}1} + R = \frac{\gamma R}{\gamma\text{-}1}$$

Note: $\gamma = C_P/C_V$

More examples:

Isothermal process (T=0)

$$E_{\mathrm{int}} = \frac{n}{\gamma\text{-}1} RT$$

$$\Delta E_{\mathrm{int}} = \frac{n}{\gamma\text{-}1} R\Delta T = Q + W$$

➔ $\Delta$T=0 ➔ $\Delta E_{\mathrm{int}}$ = 0 ➔ Q=-W

$$-W = \int_{V_i}^{V_f} PdV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln\left(\frac{V_f}{V_i}\right)$$

Even more examples:

Adiabatic process (Q=0)

$$\Delta E_{\mathrm{int}} = W$$

$$\frac{n}{\gamma\text{-}1} R\Delta T = -P\Delta V$$

$$PV = nRT$$

$$\Delta PV + P\Delta V = nR\Delta T$$

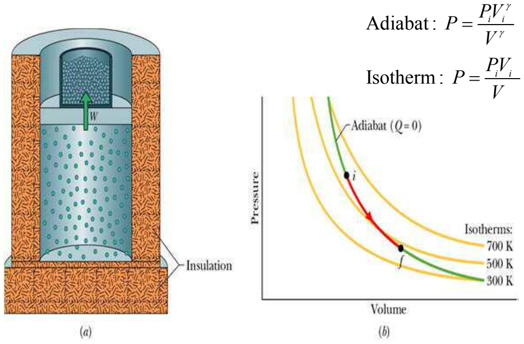
$$nR\Delta T = -(\gamma\text{-}1)P\Delta V = \Delta PV + P\Delta V$$

$$-\gamma\frac{\Delta V}{V} = \frac{\Delta P}{P}$$

$$\Rightarrow -\ln\left(\frac{V_f^\gamma}{V_i^\gamma}\right) = \ln\left(\frac{P_f}{P_i}\right) \quad \Rightarrow P_i V_i^\gamma = P_f V_f^\gamma$$

Adiabat : $P = \frac{P_i V_i^{\gamma}}{V^{\gamma}}$

Isotherm : $P = \frac{P_i V_i}{V}$



**iclicker question:**

Suppose that an ideal gas expands adiabatically. Does the temperature

(A) Increase (B) Decrease (C) Remain the same

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

$$P_i V_i = nRT_i \Rightarrow P_i = nR\frac{T_i}{V_i}$$

$$T_i V_i^{\gamma\text{-}1} = T_f V_f^{\gamma\text{-}1}$$

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma\text{-}1}$$

Review of results from ideal gas analysis in terms of the specific heat ratio $\gamma \equiv C_P/C_V$:

$$\Delta E_{\text{int}} = \frac{n}{\gamma\text{-}1} R\Delta T = nC_V \Delta T \quad ; \; C_V = \frac{R}{\gamma\text{-}1}$$

$$C_P = \frac{\gamma R}{\gamma\text{-}1}$$

For an isothermal process, $\Delta E_{\text{int}} = 0$ ➔ Q=-W

$$-W = \int_{V_i}^{V_f} PdV = nRT \ln\left(\frac{V_f}{V_i}\right) = P_i V_i \ln\left(\frac{V_f}{V_i}\right)$$

For an adiabatic process, Q = 0

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

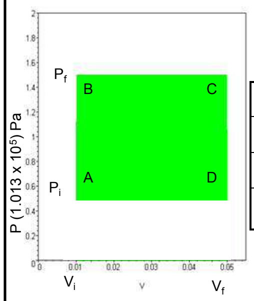
$$T_i V_i^{\gamma\text{-}1} = T_f V_f^{\gamma\text{-}1}$$

Note:

It can be shown that the work done by an ideal gas which has an initial pressure $P_i$ and initial volume $V_i$ when it expands *adiabatically* to a volume $V_f$ is given by:

$$W = -\int_{V_i}^{V_f} P dV = -\frac{P_i V_i}{\gamma - 1}\left(1 - \left(\frac{V_i}{V_f}\right)^{\gamma-1}\right)$$

Examples process by an ideal gas:



| | A→B | B→C | C→D | D→A |
|---|---|---|---|---|
| Q | $\frac{V_i(P_f - P_i)}{\gamma\text{-}1}$ | $\frac{\gamma P_f(V_f - V_i)}{\gamma\text{-}1}$ | $\frac{-V_f(P_f - P_i)}{\gamma\text{-}1}$ | $\frac{\text{-}\gamma P_i(V_f - V_i)}{\gamma\text{-}1}$ |
| W | 0 | $-P_f(V_f-V_i)$ | 0 | $P_i(V_f-V_i)$ |
| $\Delta E_{int}$ | $\frac{V_i(P_f - P_i)}{\gamma\text{-}1}$ | $\frac{P_f(V_f - V_i)}{\gamma\text{-}1}$ | $\frac{-V_f(P_f - P_i)}{\gamma\text{-}1}$ | $\frac{\text{-}P_i(V_f - V_i)}{\gamma\text{-}1}$ |

Efficiency as an engine:

$e = W_{net/} Q_{input}$