

PHY 113 A General Physics I

9-9:50 AM MWF Olin 101

Plan for Lecture 34:

Chapter 16 – Physics of wave motion

- 1. Examples of wave motion**
- 2. What determines the wave velocity**
- 3. Properties of periodic waves**

	11/02/2012	Exam	10-13,15		
23	11/05/2012	Fluid mechanics	14.1-14.4	14.8, 14.24	11/07/2012
24	11/07/2012	Fluid mechanics	14.5-14.7	14.39, 14.51	11/09/2012
25	11/09/2012	Temperature	19.1-19.5	19.1, 19.20	11/12/2012
26	11/12/2012	Heat	20.1-20.4	20.3, 20.14, 20.24	11/14/2012
27	11/14/2012	First law of thermodynamics	20.5-20.7	20.26, 20.35	11/16/2012
28	11/16/2012	Ideal gases	21.1-21.5	21.10, 21.19	11/19/2012
29	11/19/2012	Engines	22.1-22.8	22.3, 22.62	11/26/2012
	11/21/2012	<i>Thanksgiving Holiday</i>			
	11/23/2012	<i>Thanksgiving Holiday</i>			
	11/26/2012	Review	14,19-22		
	11/28/2012	Exam	14,19-22		
 30	11/30/2012	Wave motion	16.1-16.6	16.5, 16.22	12/03/2012
31	12/03/2012	Sound & standing waves	17.1-18.8	17.35, 18.35	12/05/2012
	12/05/2012	Review	1-22		
	12/13/2012	Final Exam -- 9 AM			

The phenomenon of wave motion

The wave equation

$y(x, t)$
position time
Wave variable

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

- What does the wave equation mean?
- Examples
- Mathematical solutions of wave equation and descriptions of waves

Example: Water waves



Source: <http://www.eng.vt.edu/fluids/msc/gallery/gall.htm>

Needs more
sophisticated
analysis:

Mechanical waves occur in continuous media. They are described by a value (y) which changes in both time (t) and position (x) and are characterized by a wave velocity c : $y=y(x-ct)$ or $y=y(x+ct)$

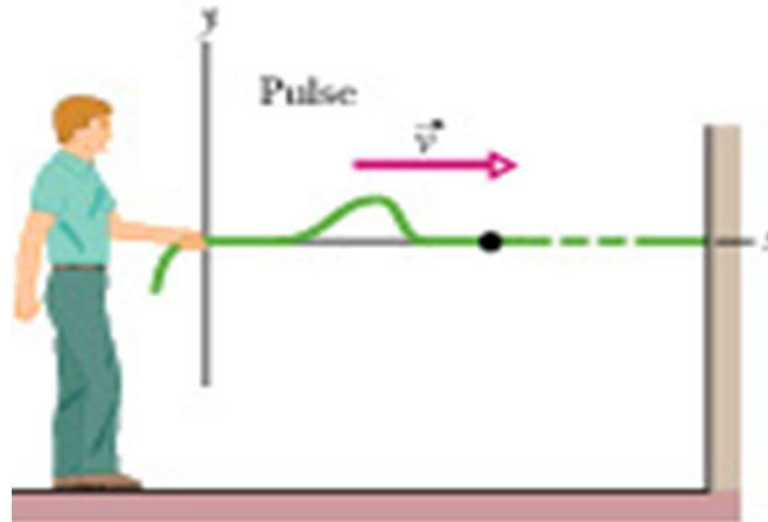
Waves on a string:

Typical values for c :

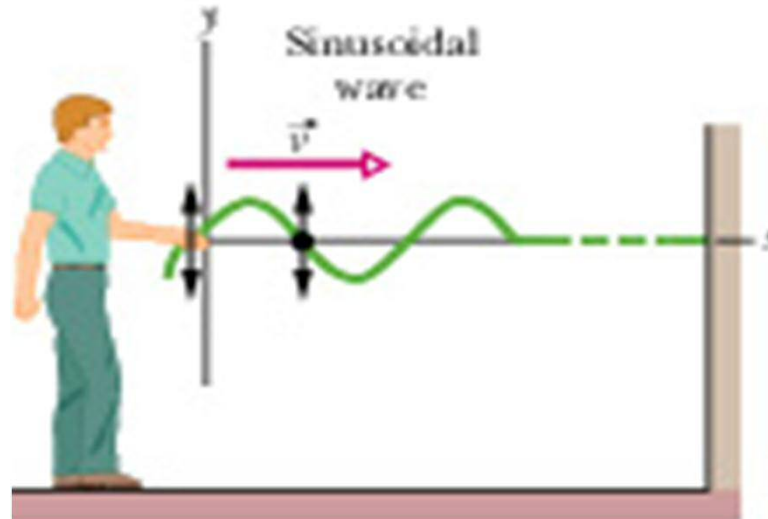
3×10^8 m/s light waves

~ 1000 m/s wave on a string

343 m/s sound in air



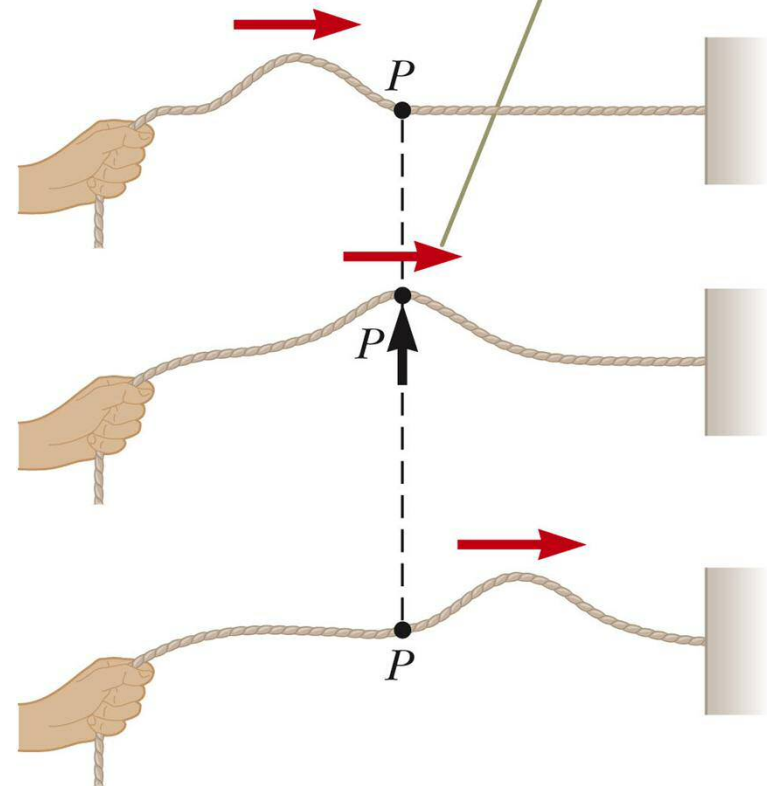
(a)



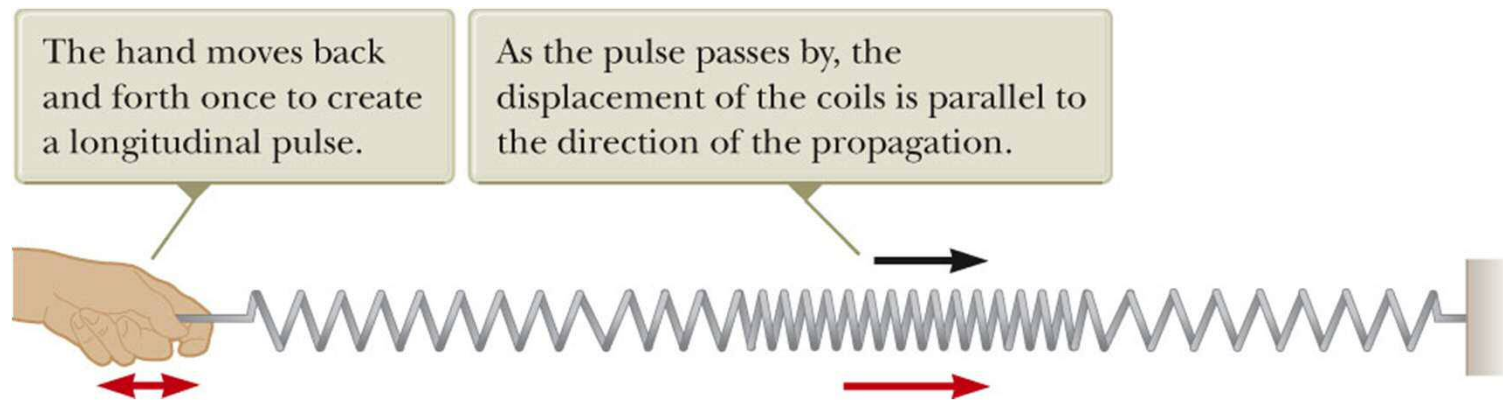
(b)

Transverse wave:

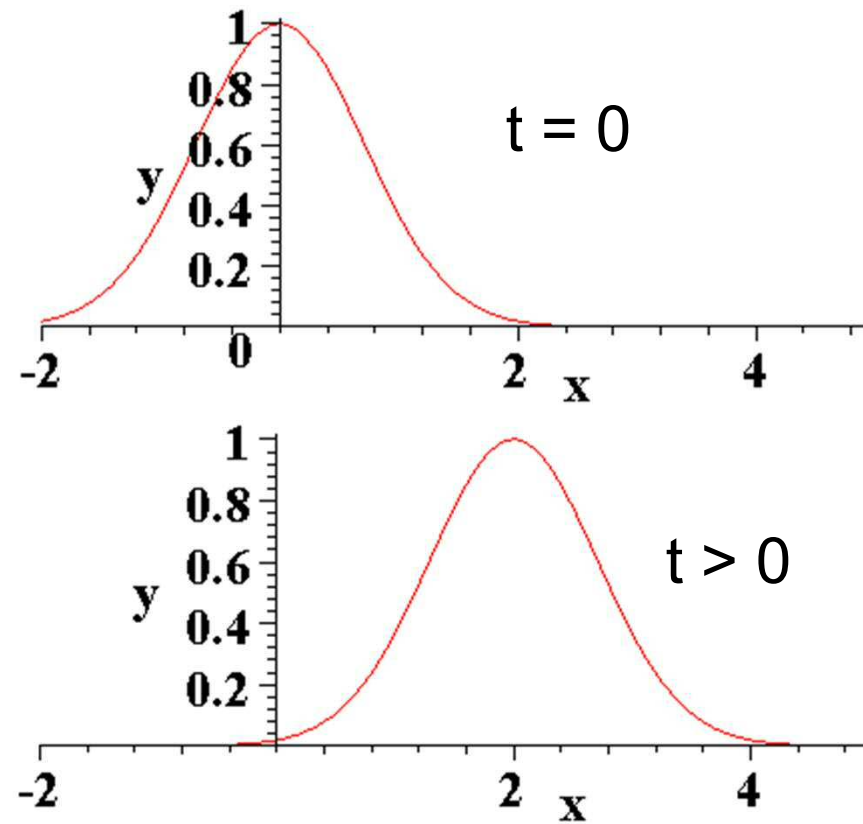
The direction of the displacement of any element at a point P on the string is perpendicular to the direction of propagation (red arrow).



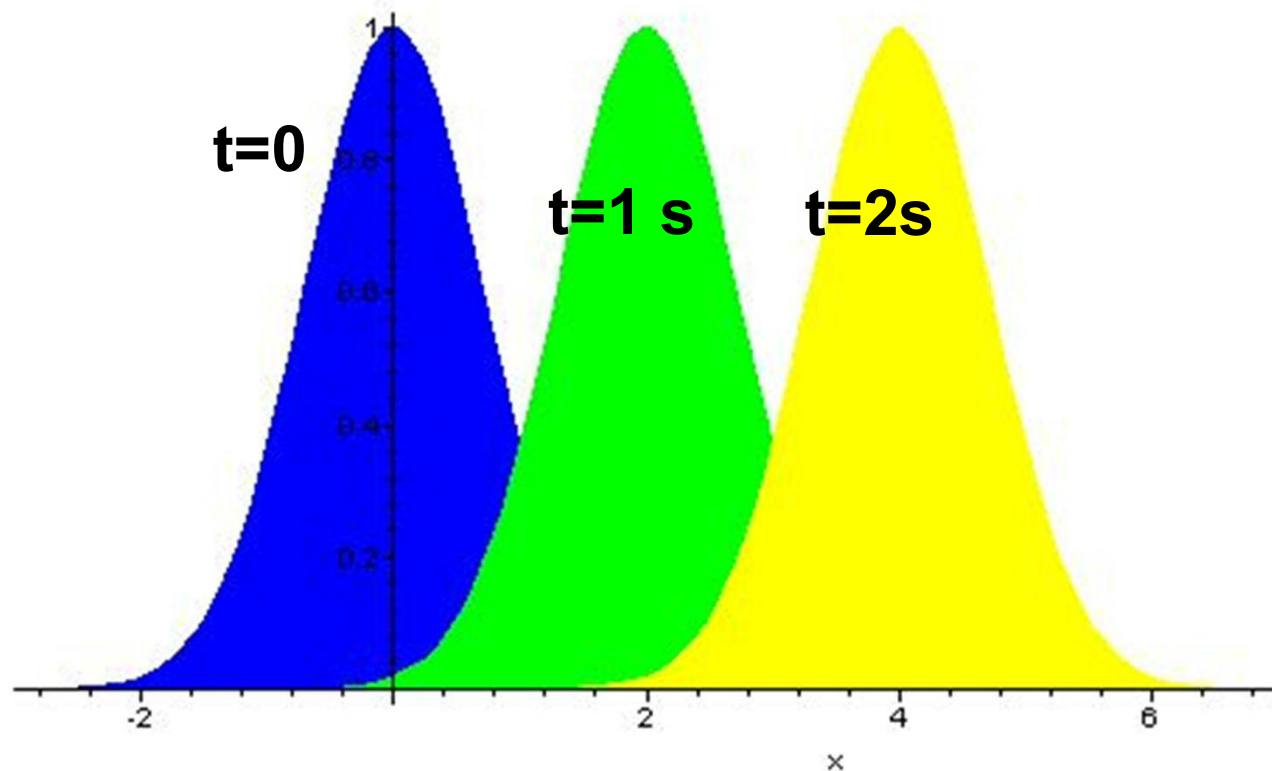
Longitudinal wave:



General traveling wave –



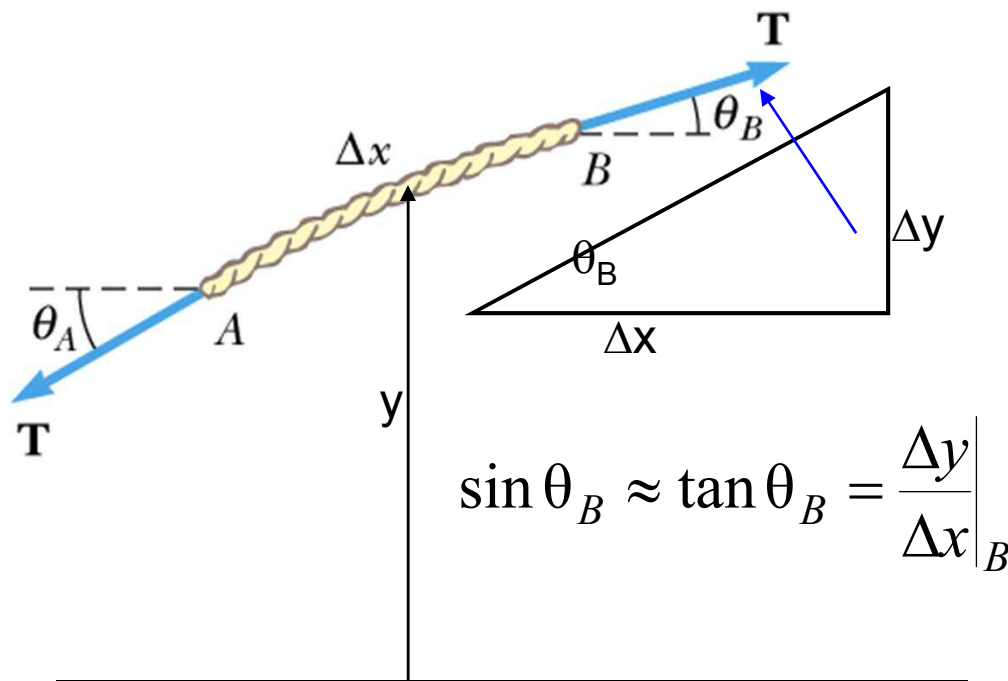
iclicker question:



The figure above shows three snap shots of a transverse wave plotted versus distance (x in meters). The snapshots were taken at $t=0$ s (blue), $t=1$ s (green), and $t=2$ s (yellow). What is the velocity of the wave? (a) 1 m/s (b) 2 m/s (c) 3 m/s (d) 4 m/s (e) 6 m/s

Basic physics behind wave motion --

example: transverse wave on a string with tension T and mass per unit length μ



$$m \frac{d^2 y}{dt^2} = T \sin \theta_B - T \sin \theta_A$$

$$m \approx \mu \Delta x$$

$$\Rightarrow \mu \Delta x \frac{d^2 y}{dt^2} \approx T \left(\left. \frac{\Delta y}{\Delta x} \right|_B - \left. \frac{\Delta y}{\Delta x} \right|_A \right)$$

$$\lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} \left(\left. \frac{\Delta y}{\Delta x} \right|_B - \left. \frac{\Delta y}{\Delta x} \right|_A \right) = \frac{\partial^2 y}{\partial x^2}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu} \frac{\partial^2 y}{\partial x^2}$$

The wave equation:

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

where $c \equiv \sqrt{\frac{T}{\mu}}$ (for a string)

Solutions: $y(x,t) = f(x \pm ct)$  function of *any* shape

Note: Let $u \equiv x \pm ct$

$$\frac{\partial f(u)}{\partial x} = \frac{\partial f(u)}{\partial u} \frac{\partial u}{\partial x}$$

$$\frac{\partial^2 f(u)}{\partial x^2} = \frac{\partial^2 f(u)}{\partial u^2} \left(\frac{\partial u}{\partial x} \right)^2 = \frac{\partial^2 f(u)}{\partial u^2}$$

$$\frac{\partial^2 f(u)}{\partial t^2} = \frac{\partial^2 f(u)}{\partial u^2} \left(\frac{\partial u}{\partial t} \right)^2 = \frac{\partial^2 f(u)}{\partial u^2} c^2$$

Examples of solutions to the wave equation:

Moving “pulse”: $y(x,t) = y_0 e^{-(x-ct)^2}$

Periodic wave: $y(x,t) = y_0 \sin(k(x-ct) + \phi)$

phase
factor

“wave vector”
not spring constant!!!

$$k = \frac{2\pi}{\lambda}$$

$$kc = \frac{2\pi}{T} = 2\pi f = \omega$$

$$y(x,t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \phi\right) \quad \frac{\lambda}{T} = c$$

Periodic traveling waves:

$$y(x, t) = y_0 \sin \left(2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) + \phi \right) \quad \frac{\lambda}{T} = c$$

Amplitude

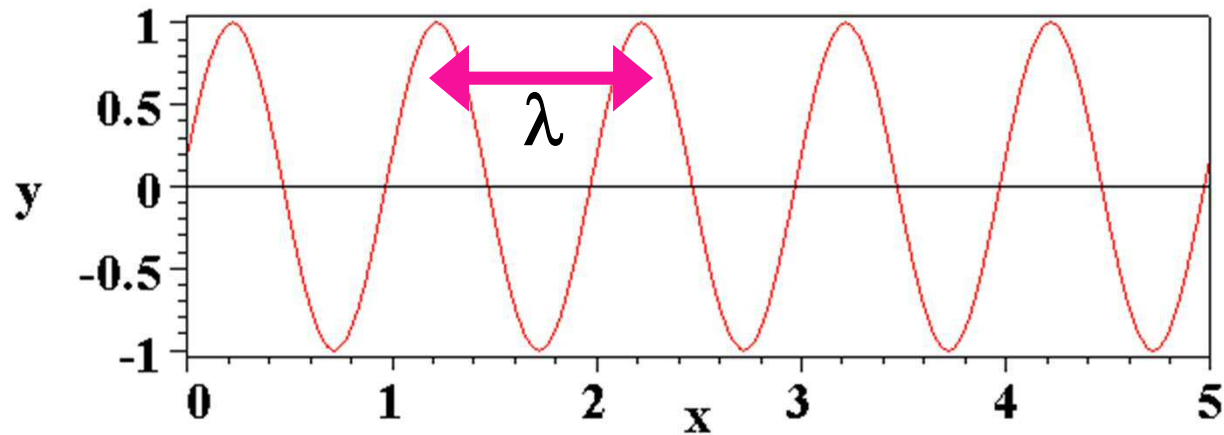
phase (radians)

velocity (m/s)

period (s); $T = 1/f$

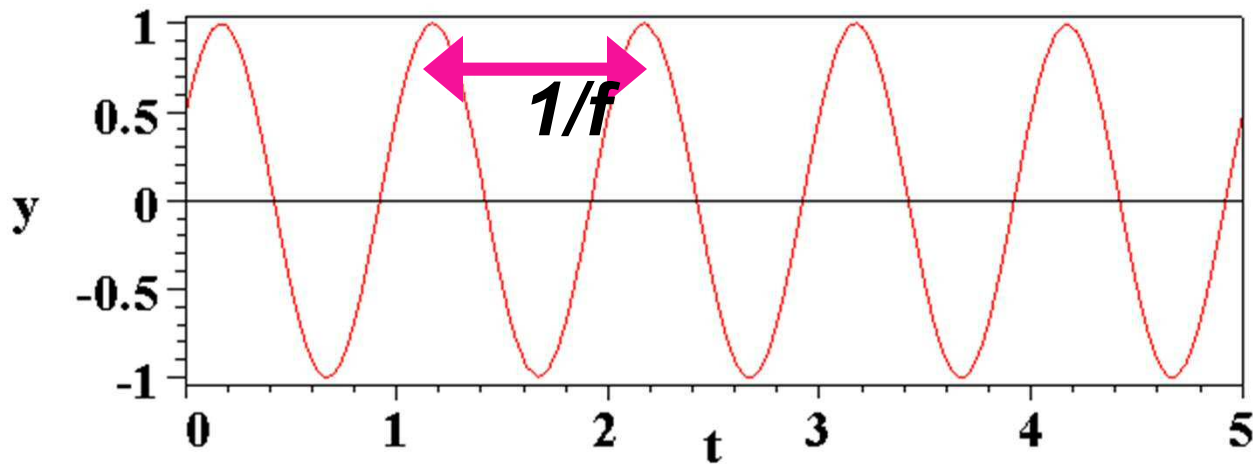
wave length (m)

Snapshot of periodic wave at $t=t_0$



$$f \lambda = c$$

Time plot of periodic wave at $x=x_0$



Combinations of waves (“superposition”)

Note that :

$$\sin A \pm \sin B = 2 \sin\left[\frac{1}{2}(A \pm B)\right] \cos\left[\frac{1}{2}(A \mp B)\right]$$

$$y_{right}(x, t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \varphi\right) \quad y_{left}(x, t) = y_0 \sin\left(2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right) + \varphi\right)$$

“Standing” wave:

$$y_{right}(x, t) + y_{left}(x, t) = 2y_0 \sin\left(\frac{2\pi x}{\lambda} + \varphi\right) \cos\left(\frac{2\pi t}{T}\right)$$

Summary of wave properties:

Wave speed c depends on the process and/or medium in which wave occurs.

Example: Wave on a string with tension T and mass/length μ : $c = \sqrt{\frac{T}{\mu}}$

Example: Sound wave in air with pressure p and density ρ : $c = \sqrt{\frac{\gamma p}{\rho}}$

Example: Light wave due to coupled electric and magnetic fields:

$$c = 2.9979 \times 10^8 \text{ m/s} \quad (\text{fundamental constant})$$