PHY 113 A General Physics I
9-9:50 AM MWF Olin 101

Plan for Lecture 5:

Chapter 4 – Motion in two dimensions

1. **Position, velocity, and acceleration in two dimensions**
2. **Two dimensional motion with constant acceleration**

Note that in many of the Webassign problem sets there are some zero point "extra practice" problems set in the assignment for your consideration.

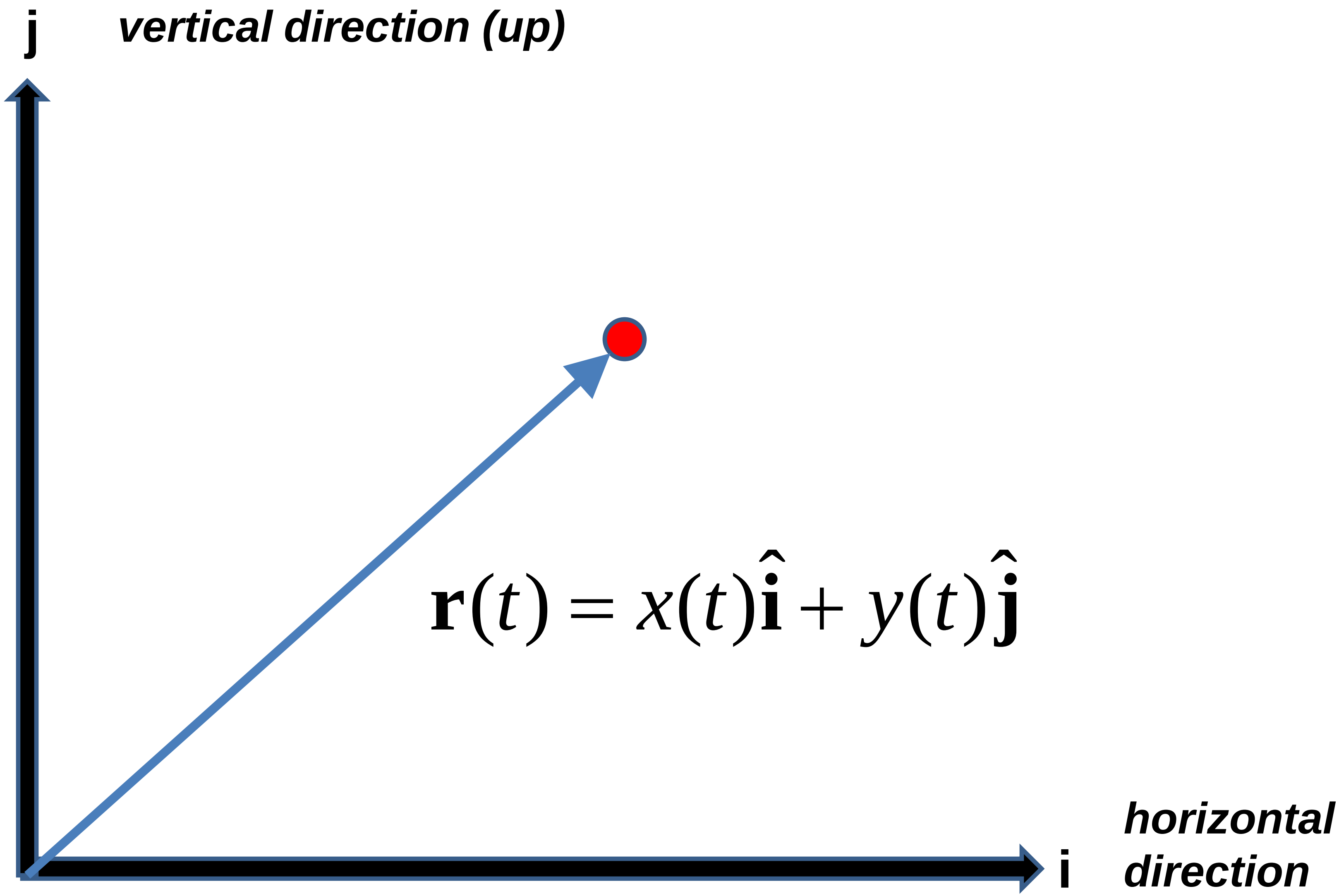
No.	Lecture Date	Topic	Text Sections	Problem Assignments	Assignment Due Date
1	08/29/2012	Units & measurement	1.1-1.6	1.2,1.6,1.13,1.20	
2	08/31/2012	Motion in 1d -- constant velocity	2.1-2.3	2.1,2.8	09/07/2012
3	09/03/2012	Motion in 1d -- constant acceleration	2.4-2.8	2.13,2.16	09/07/2012
4	09/05/2012	Vectors	3.1-3.4	3.3,3.22	09/07/2012
5	09/07/2012	Motion in 2d	4.1-4.3	4.3,4.50	09/10/2012
6	09/10/2012	Circular motion	4.4-4.6	4.29,4.30	09/12/2012
7	09/12/2012	Newton's laws	5.1-5.6		09/14/2012
8	09/14/2012	Newton's laws applied	5.7-5.8		09/17/2012
	09/17/2012	Review	1-5		
	09/19/2012	Exam	1-5		
9	09/21/2012	More applications of Newton's laws	6.1-6.4		09/24/2012

In the previous lecture, we introduced the abstract notion of a vector. In this lecture, we will use that notion to describe position, velocity, and acceleration vectors in two dimensions.

iclicker exercise:

Why spend time studying two dimensions when the world as we know it is three dimensions?

A. **Because it is difficult to draw 3 dimensions.**
B. **Because in physics class, 2 dimensions are hard enough to understand.**
C. **Because if we understand 2 dimensions, extension of the ideas to 3 dimensions is trivial.**
D. **On Fridays, it is good to stick to a plane.**



$$\mathbf{r}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$$

Vectors relevant to motion in two dimenstions

Displacement: $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}$

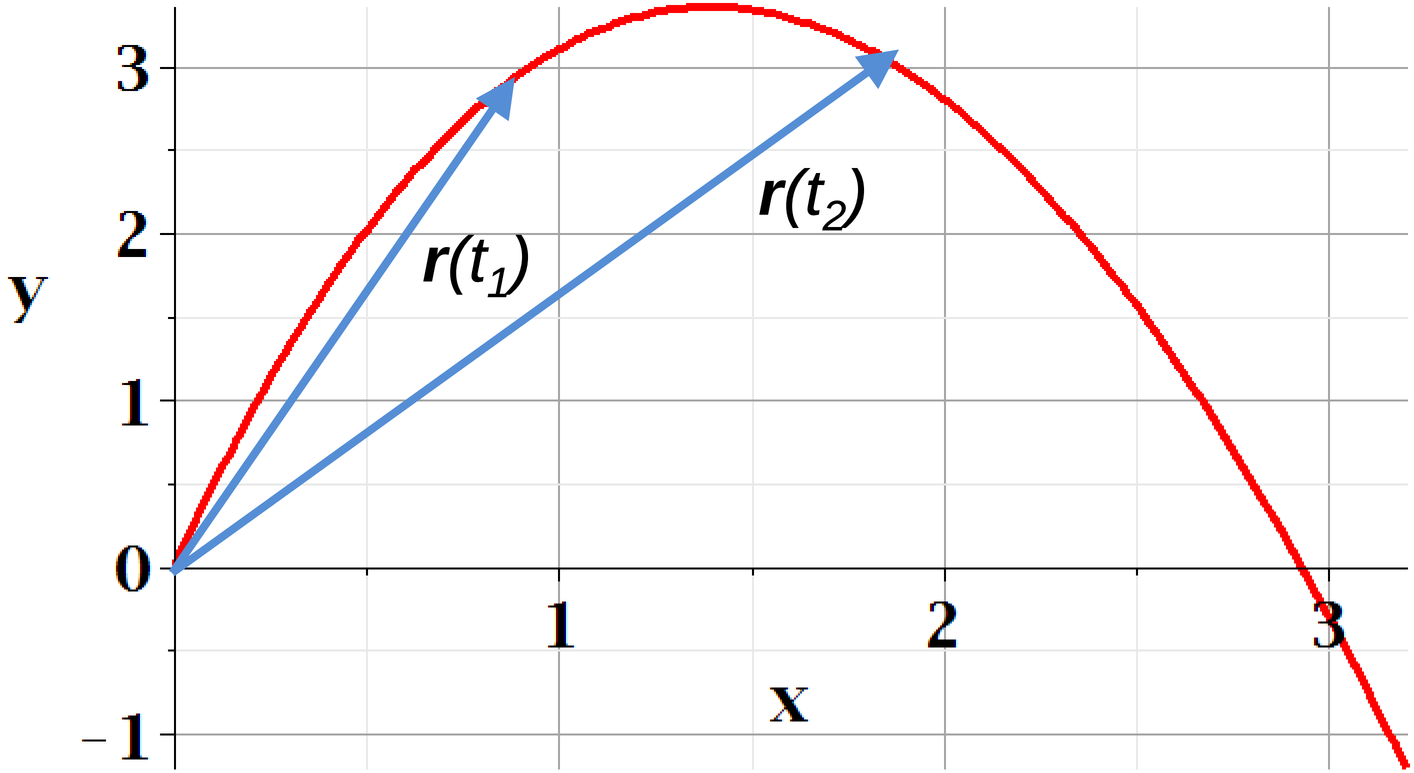
Velocity: $\mathbf{v}(t) = v_x(t)\,\mathbf{i} + v_y(t)\,\mathbf{j}$

$$\mathbf{v}_x = \frac{dx}{dt} \qquad \mathbf{v}_y = \frac{dy}{dt}$$

Acceleration: $\mathbf{a}(t) = a_x(t)\,\mathbf{i} + a_y(t)\,\mathbf{j}$

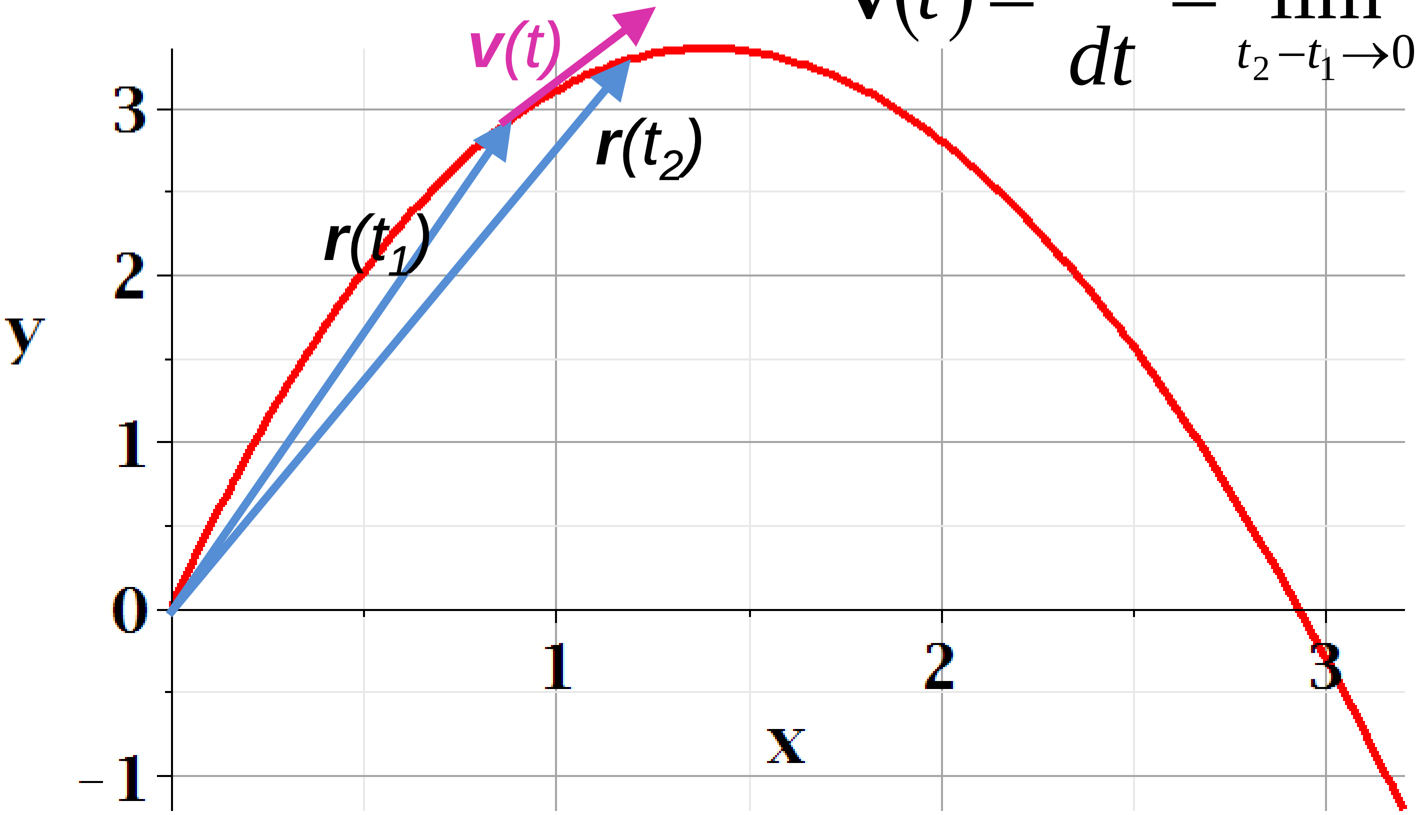
$$\mathbf{a}_x = \frac{dv_x}{dt} \qquad \mathbf{a}_y = \frac{dv_y}{dt}$$

Visualization of the position vector *r(t)* of a particle



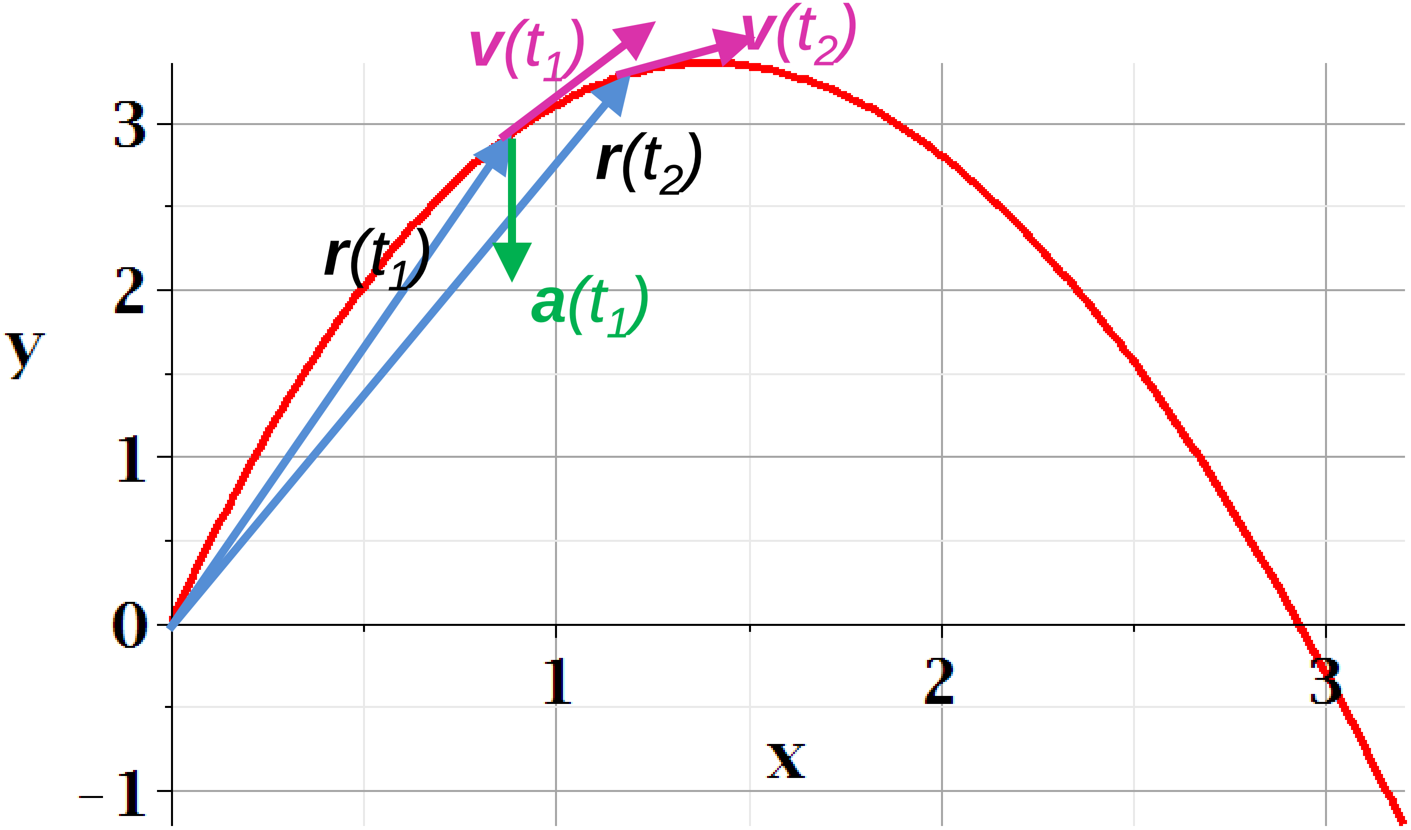
Visualization of the velocity vector *v(t)* of a particle

$$\mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = \lim_{t_2 - t_1 \to 0} \frac{\mathbf{r}(t_2) - \mathbf{r}(t_1)}{t_2 - t_1}$$

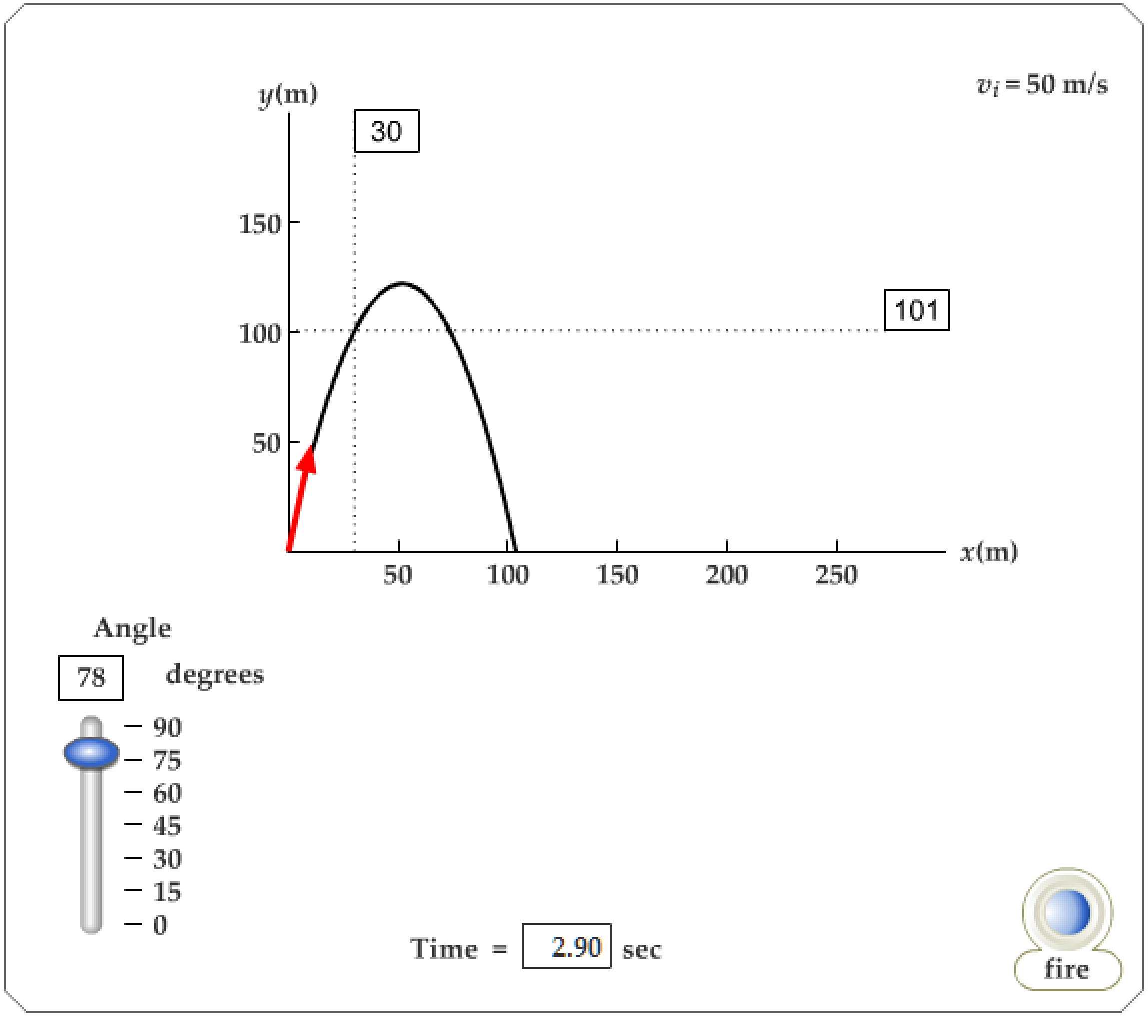


Visualization of the acceleration vector *a(t)* of a particle

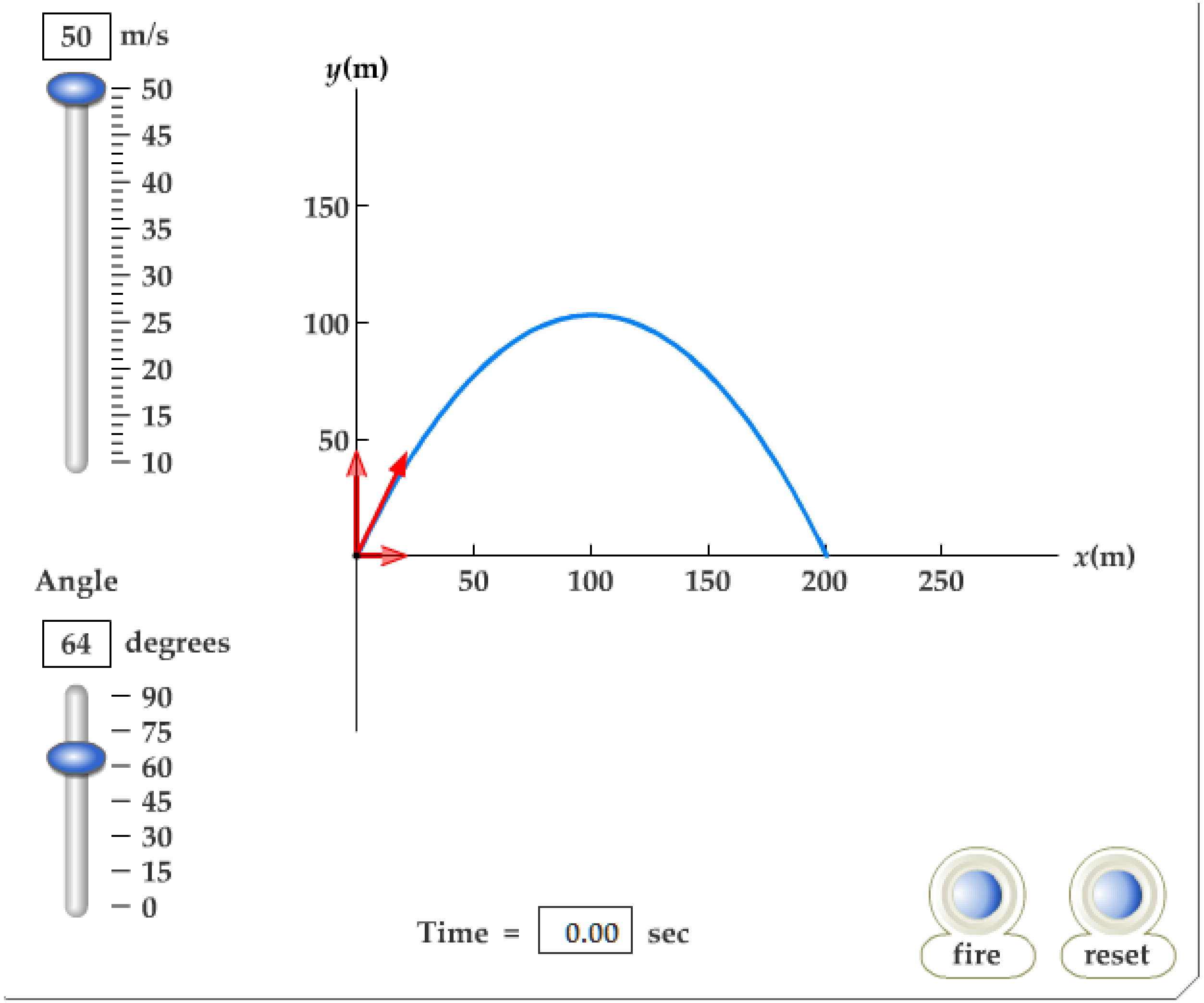
$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = \lim_{t_2 - t_1 \to 0} \frac{\mathbf{v}(t_2) - \mathbf{v}(t_1)}{t_2 - t_1}$$



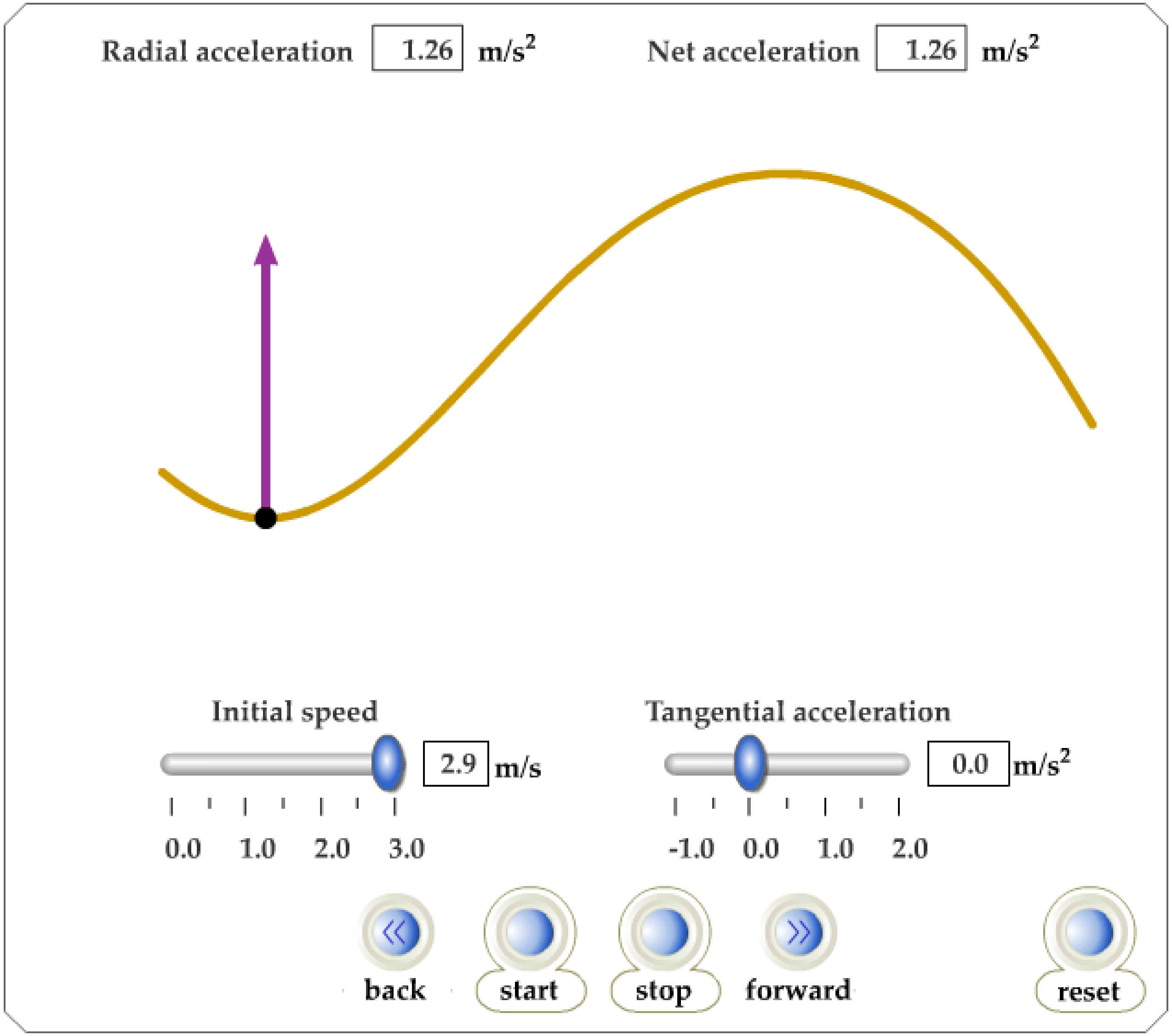
Animation of position vector components associated with trajectory motion



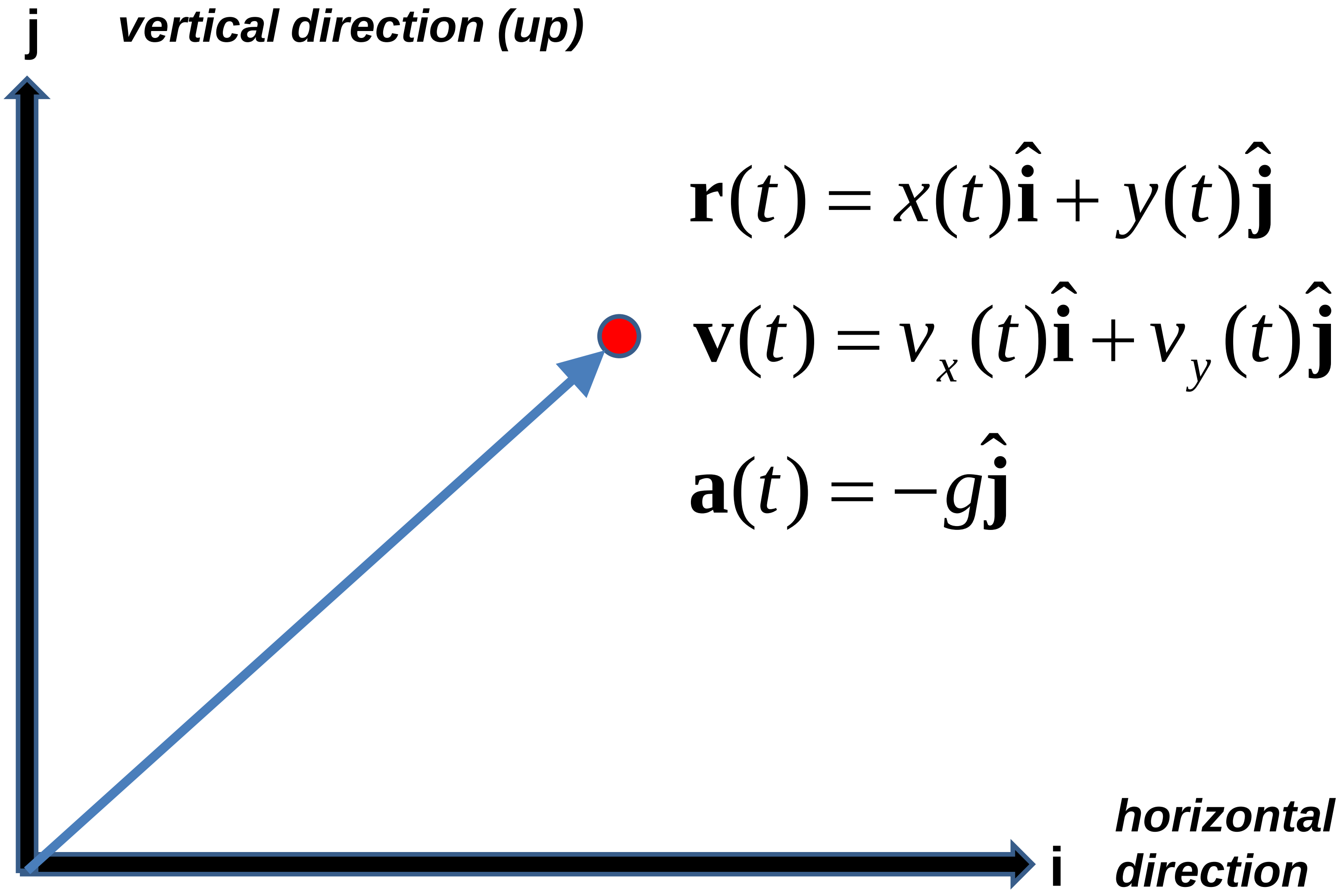
Animation of velocity vector and its components associated with trajectory motion



Animation of acceleration vector associated with motion along a path



Projectile motion (near earth's surface)



$$\mathbf{r}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$$

$$\mathbf{v}(t) = v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}}$$

$$\mathbf{a}(t) = -g\hat{\mathbf{j}}$$

Projectile motion (near earth's surface)

$$\mathbf{r}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$$

$$\mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}}$$

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = -g\hat{\mathbf{j}}$$

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = -g\hat{\mathbf{j}}$$

$$\Rightarrow \mathbf{v}(t) = \mathbf{v}_i - gt\hat{\mathbf{j}} \qquad \text{note that } \mathbf{v}(t=0) = \mathbf{v}_i$$

Projectile motion (near earth's surface)

$$\mathbf{r}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$$

$$\mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}}$$

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = -g\hat{\mathbf{j}}$$

$$\mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = \mathbf{v}_i - gt\hat{\mathbf{j}}$$

$$\Rightarrow \mathbf{r}(t) = \mathbf{r}_i + \mathbf{v}_i t - \tfrac{1}{2} g t^2 \hat{\mathbf{j}} \qquad \text{note that } \mathbf{r}(t=0) = \mathbf{r}_i$$

Projectile motion (near earth's surface)

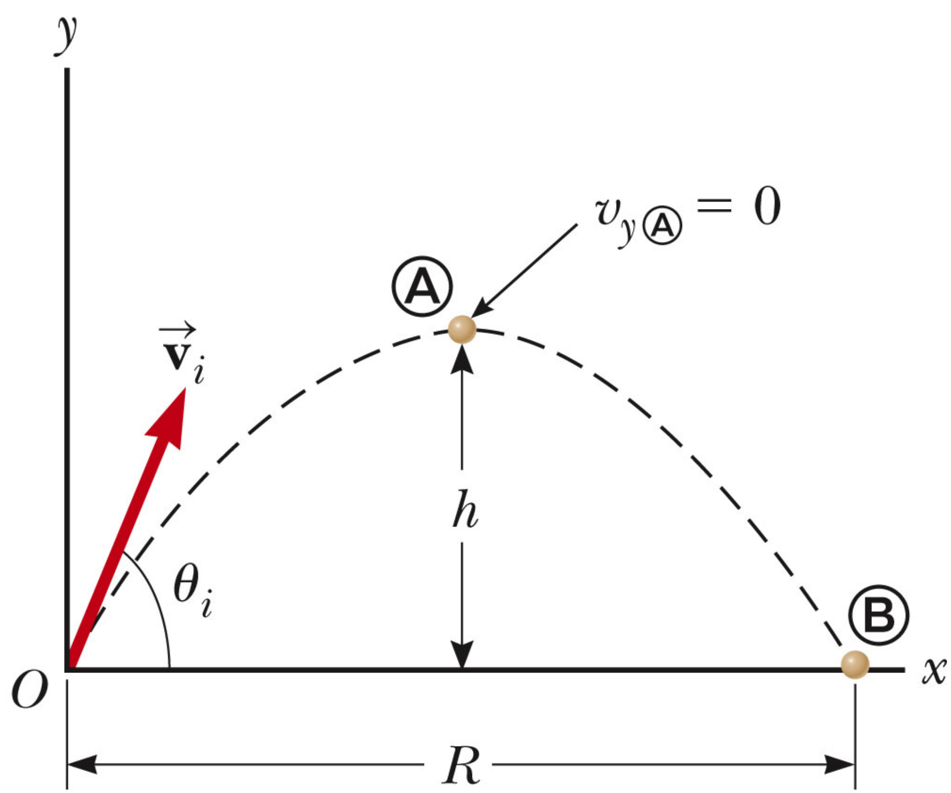
$$\mathbf{r}(t) = \mathbf{r}_i + \mathbf{v}_i t - \tfrac{1}{2} g t^2 \hat{\mathbf{j}} \qquad \mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = \mathbf{v}_i - gt\hat{\mathbf{j}}$$

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = -g\hat{\mathbf{j}}$$

$$\mathbf{v}_i = v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}}$$

$$v_{xi} = |\mathbf{v}_i| \cos\theta_i$$

$$v_{yi} = |\mathbf{v}_i| \sin\theta_i$$



Projectile motion (near earth's surface)

Trajectory equation in vector form:

$$\mathbf{r}(t) = \mathbf{r}_i + \mathbf{v}_i t - \tfrac{1}{2} g t^2 \hat{\mathbf{j}} \qquad \mathbf{v}(t) = \mathbf{v}_i - g t \hat{\mathbf{j}}$$

Trajectory equation in component form:

$$x(t) = x_i + v_{xi} t \qquad v_x(t) = v_{xi}$$

$$y(t) = y_i + v_{yi} t - \tfrac{1}{2} g t^2 \qquad v_y(t) = v_{yi} - g t$$

Aside: The equations for position and velocity written in this way are call "parametric" equations. They are related to each other through the time parameter.

Projectile motion (near earth's surface)

Trajectory equation in component form:

$$x(t) = x_i + v_{xi}t = x_i + v_i \cos\theta_i t$$

$$y(t) = y_i + v_{yi}t - \tfrac{1}{2}gt^2 = y_i + v_i \sin\theta_i t - \tfrac{1}{2}gt^2$$

$$v_x(t) = v_{xi} = v_i \cos\theta_i$$

$$v_y(t) = v_{yi} - gt = v_i \sin\theta_i - gt$$

Trajectory path *y(x)*; eliminating *t* from the equations:

$$t = \frac{x - x_i}{v_i \cos\theta_i} \qquad y(x) = y_i + v_i \sin\theta_i \frac{x - x_i}{v_i \cos\theta_i} - \tfrac{1}{2} g \left(\frac{x - x_i}{v_i \cos\theta_i} \right)^2$$

$$y(x) = y_i + \tan\theta_i (x - x_i) - \tfrac{1}{2} g \left(\frac{x - x_i}{v_i \cos\theta_i} \right)^2$$

Projectile motion (near earth's surface)
Summary of results

$$x(t) = x_i + v_i \cos\theta_i t \qquad y(t) = y_i + v_i \sin\theta_i t - \tfrac{1}{2} g t^2$$

$$v_x(t) = v_i \cos\theta_i \qquad v_y(t) = v_i \sin\theta_i - gt$$

$$y(x) = y_i + \tan\theta_i (x - x_i) - \tfrac{1}{2} g \left(\frac{x - x_i}{v_i \cos\theta_i} \right)^2$$

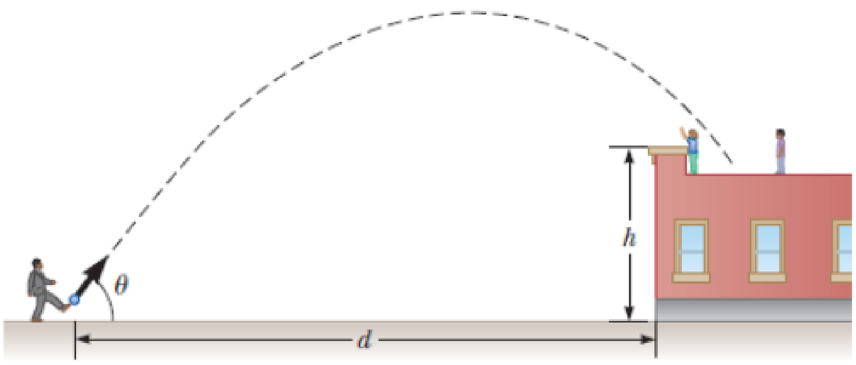
iclicker exercise:

These equations are so beautiful that

A. They should be framed and put on the wall.
B. They should be used to perfect my tennis/golf/basketball/soccer technique.
C. They are not that beautiful.

5. –/0 points   My Notes | SerPSE8 4.P.019.

A playground is on the flat roof of a city school, 5.6 m above the street below (see figure). The vertical wall of the building is h = 7.10 m high, forming a 1.5-m-high railing around the playground. A ball has fallen to the street below, and a passerby returns it by launching it at an angle of θ = 53.0°; above the horizontal at a point d = 24.0 m from the base of the building wall. The ball takes 2.20 s to reach a point vertically above the wall.



h=7.1m
θ_i=53°
d=24m=x(2.2s)

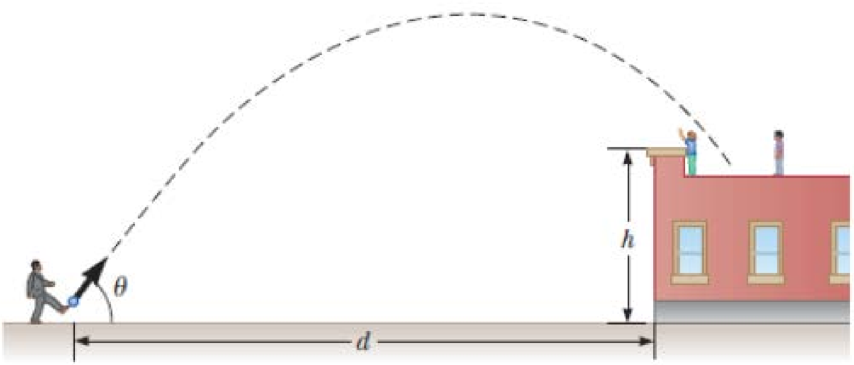
(a) Find the speed at which the ball was launched. (Give your answer to two decimal places to reduce rounding errors in later parts.)
m/s

(b) Find the vertical distance by which the ball clears the wall.
m

(c) Find the horizontal distance from the wall to the point on the roof where the ball lands.
m

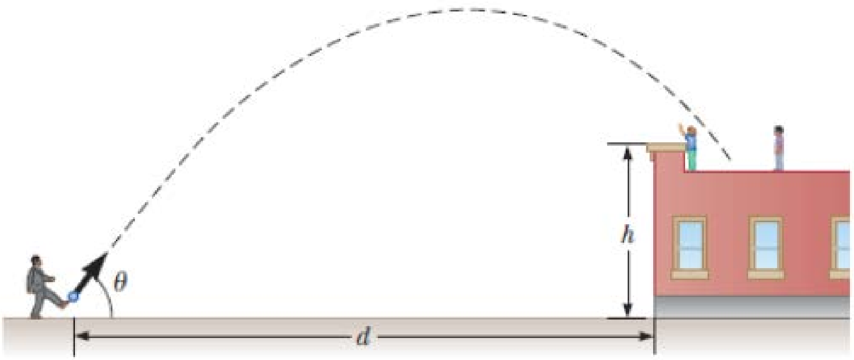
Problem solving steps

1. Visualize problem – labeling variables
2. Determine which basic physical principle(s) apply
3. Write down the appropriate equations using the variables defined in step 1.
4. Check whether you have the correct amount of information to solve the problem (same number of known relationships and unknowns).
5. Solve the equations.
6. Check whether your answer makes sense (units, order of magnitude, etc.).



$h=7.1m$
$\theta_i=53^o$
$d=24m=x(2.2s)$

(a) Find the speed at which the ball was launched.



h=7.1m
θ_i*=53º*
d=24m=x(2.2s)

(a) Find the speed at which the ball was launched.

$$x(t) = x_i + v_i \cos\theta_i t$$

$$d = x(2.2) = v_i \cos 53^o (2.2) = 24$$

$$\Rightarrow v_i = \frac{24m}{\cos 53^o (2.2s)} = 18.12698 m/s$$