**PHY 113 A General Physics I**
**9-9:50 AM MWF Olin 101**

**Plan for Lecture 6:**

**Chapter 4 – Motion in two dimensions, especially circular motion**

1. **Review of one and two dimensional motion**
2. **Circular motion**



| No. | Lecture Date | Topic | Text Sections | Problem Assignments | Assignment Due Date |
|---|---|---|---|---|---|
| 1 | 08/29/2012 | Units & measurement | 1.1-1.6 | 1.2,1.6,1.13,1.20 | |
| 2 | 08/31/2012 | Motion in 1d -- constant velocity | 2.1-2.3 | 2.1,2.8 | 09/07/2012 |
| 3 | 09/03/2012 | Motion in 1d -- constant acceleration | 2.4-2.8 | 2.13,2.16 | 09/07/2012 |
| 4 | 09/05/2012 | Vectors | 3.1-3.4 | 3.3,3.22 | 09/07/2012 |
| 5 | 09/07/2012 | Motion in 2d | 4.1-4.3 | 4.3,4.50 | 09/10/2012 |
| 6 | 09/10/2012 | Circular motion | 4.4-4.6 | 4.29,4.30 | 09/12/2012 |
| 7 | 09/12/2012 | Newton's laws | 5.1-5.6 | 5.1,5.13 | 09/14/2012 |
| 8 | 09/14/2012 | Newton's laws applied | 5.7-5.8 | 5.20,5.30,5.48 | 09/17/2012 |
| | 09/17/2012 | Review | 1-5 | | |
| | 09/19/2012 | Exam | 1-5 | | |
| 9 | 09/21/2012 | More applications of Newton's laws | 6.1-6.4 | | 09/24/2012 |
| 10 | 09/24/2012 | Work | 7.1-7.4 | | 09/26/2012 |



**Review**

**Position *x(t)*, Velocity *y(t)*, Acceleration *a(t)* in one dimension:**

$$v(t) = \frac{dx}{dt} \quad \Leftrightarrow \quad x(t) = \int_{t_0}^{t} v(t')dt'$$

$$a(t) = \frac{dv}{dt} \quad \Leftrightarrow \quad v(t) = \int_{t_0}^{t} a(t')dt'$$

**Special case of constant acceleration $a(t)=a_0$:**

$$\text{Suppose}: \frac{dv}{dt} = a_0 \quad \text{and} \quad v(0) = v_0, \quad x(0) = x_0$$

$$\text{Then}: \quad v(t) = v_0 + a_0 t$$

$$x(t) = x_0 + v_0 t + \tfrac{1}{2} a_0 t^2$$

**Review – continued:**
**Special case of constant acceleration *a(t)=a₀*:**

Suppose: $\frac{dv}{dt} = a_0$ and $v(0) = v_0$, $x(0) = x_0$

(initial velocity, initial position)

Then: $v(t) = v_0 + a_0 t$

$x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2$

Relationship between position, velocity, and acceleration:

$$2a_0\left(x(t) - x_0\right) = \left(v(t)\right)^2 - v_0^{\,2}$$

Result derived using algebra: $t = \frac{v(t) - v_0}{a_0}$

$$x(t) = x_0 + v_0\left(\frac{v(t) - v_0}{a_0}\right) + \frac{1}{2} a_0 \left(\frac{v(t) - v_0}{a_0}\right)^2$$



**Summary of equations –**
**one-dimensional motion with constant acceleration**

$v(t) = v_0 + a_0 t$

$x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2$

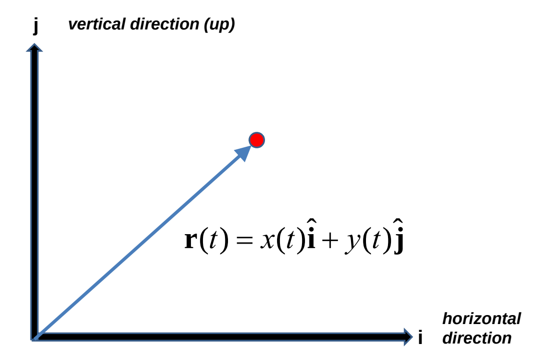
$2a_0\left(x(t) - x_0\right) = \left(v(t)\right)^2 - v_0^{\,2}$

***iclicker question:***
**Why did I show you part of the derivation of the last equation?**

A. **Because professors like to torture physics students**
B. **Because you will need to be able to prove the equation yourself**
C. **Because the "proof" helps you to understand the meaning of the equation**
D. **All of the above**
E. **None of the above**



**Review: Motion in two dimensions:**



### Vectors relevant to motion in two dimenstions

**Displacement:** **r**(*t*) = x(*t*) **i** + y(*t*) **j**

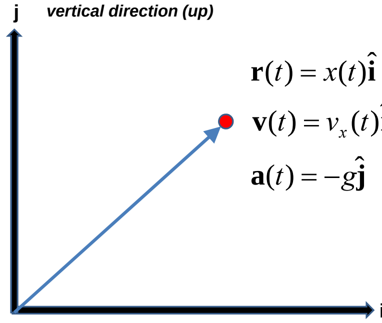
**Velocity:** **v**(*t*) = $v_x(t)$ **i** + $v_y(t)$ **j** $\quad \mathbf{v}_x = \frac{dx}{dt} \quad \mathbf{v}_y = \frac{dy}{dt}$

**Acceleration:** **a**(*t*) = $a_x(t)$ **i** + $a_y(t)$ **j** $\quad \mathbf{a}_x = \frac{dv_x}{dt} \quad \mathbf{a}_y = \frac{dv_y}{dt}$



### Projectile motion (constant acceleration)
(*reasonable approximation near Earth's surface*)

**j** ***vertical direction (up)***



$$\mathbf{r}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$$
$$\mathbf{v}(t) = v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}}$$
$$\mathbf{a}(t) = -g\hat{\mathbf{j}}$$

**i** ***horizontal direction***

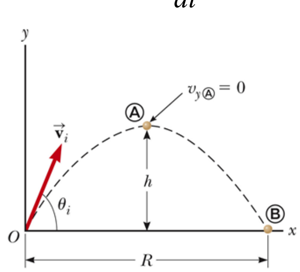


### Projectile motion (near earth's surface)

$$\mathbf{r}(t) = \mathbf{r}_i + \mathbf{v}_i t - \tfrac{1}{2} g t^2 \hat{\mathbf{j}} \qquad \mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = \mathbf{v}_i - g t \hat{\mathbf{j}}$$

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt} = -g\hat{\mathbf{j}}$$

$$\mathbf{v}_i = v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}}$$
$$v_{xi} = |\mathbf{v}_i| \cos\theta_i$$
$$v_{yi} = |\mathbf{v}_i| \sin\theta_i$$



**Projectile motion (near earth's surface)**
**Summary of component functions**

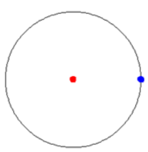
$$x(t) = x_i + v_i \cos\theta_i t \qquad y(t) = y_i + v_i \sin\theta_i t - \tfrac{1}{2} g t^2$$

$$v_x(t) = v_i \cos\theta_i \qquad v_y(t) = v_i \sin\theta_i - gt$$

$$y(x) = y_i + \tan\theta_i (x - x_i) - \tfrac{1}{2} g \left( \frac{x - x_i}{v_i \cos\theta_i} \right)^2$$



**Uniform circular motion – another example of motion in two-dimensions**

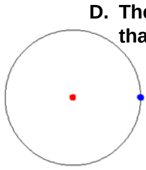


animation from
http://mathworld.wolfram.com/UniformCircularMotion.html

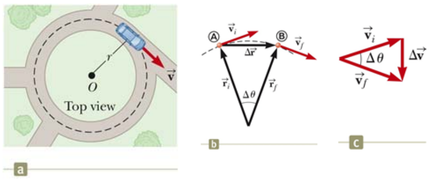


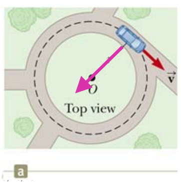
***iclicker question:***
**Assuming that the blue particle is moving at constant speed around the circle what can you say about its acceleration?**

**A. There is no acceleration**
**B. There is acceleration tangent to the circle**
**C. There is acceleration in the radial direction of the circle**
**D. There is not enough information to conclude that there is acceleration or not**



## Uniform circular motion – continued



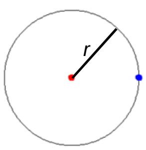




If $v_i = v_f \equiv v$, then the acceleration in the radial direction and the centripetal acceleration is :

$$\mathbf{a}_c = -\frac{v^2}{r}\hat{\mathbf{r}}$$



## Uniform circular motion – continued




$$\mathbf{a}_c = -\frac{v^2}{r}\hat{\mathbf{r}}$$

$$\mathbf{a}_c = -\left(\frac{2\pi}{T}\right)^2 r\hat{\mathbf{r}}$$

$$\mathbf{a}_c = -(2\pi f)^2 r\hat{\mathbf{r}}$$

**In terms of time period $T$ for one cycle:**

$$v = \frac{2\pi r}{T}$$

**In terms of the frequency $f$ of complete cycles:**

$$f = \frac{1}{T}; \qquad v = 2\pi f r$$



2. –/0 points My Notes | SerPSE8 4.P.028.

We found the centripetal acceleration of the Earth as it revolves around the Sun. Compute the centripetal acceleration of a point on the surface of the Earth at the equator caused by the rotation of the Earth about its axis. (The radius of the Earth is 6,371 km.)

m/s$^2$

$$\mathbf{a}_c = -\left(\frac{2\pi}{T}\right)^2 r\hat{\mathbf{r}}$$

$$T = 24\text{ hr} \times 60\text{ min/hr} \times 60\text{ s/min} = 86400\text{ s}$$

$$\mathbf{a}_c = -\left(\frac{2\pi}{86400}\right)^2 6371\times10^3 (\text{m/s}^2)\hat{\mathbf{r}}$$

$$= -0.03\text{ m/s}^2\ \hat{\mathbf{r}}$$